ABSORPTION OF ULTRASOUND IN THE INTERMEDIATE STATE SUPERCONDUCTORS

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The ultrasonic absorption coefficient is computed in the intermediate state of a type I superconductor at low temperatures. It is assumed that the length of the electron free path l and the Larmor radius in a critical magnetic field R significantly exceed the wavelength and the thickness of the normal layers. The absorption is proportional to the first power of the sound frequency if the wavelength is much less than the thickness of the layers, and is proportional to the square of the frequency in the opposite limiting case. For $R \ll l$, oscillations of the absorption should be observed for a change in the external magnetic field, while the period of oscillation depends on the thickness of the normal layers. It is shown that the measurements of the sound absorption make it possible to determine the period of the structure of the intermediate state as a function of the magnetic field.

ONE of the most interesting quantities characterizing the intermediate state of a type I superconductor is the dimension of the period of the layer structure. Under equilibrium conditions, it is determined by the coefficient of surface tension on the boundary separating the normal and superconducting phases (see [1, 2]). Measurement of the period of the structure is therefore a method (and virtually the only method) for the determination of the surface tension.

Most of the properties of the intermediate state (thermodynamic and magnetic properties, electrical conductivity, and so forth) are practically independent of the period of the structure. For this reason, very complicated methods of direct observation of the layers on the surface of the sample have been used in the determination to date.[3-5] Inasmuch as the structure of the layers is generally distorted near the surface,[6] it is highly desirable to have the possibility of measuring the period of the structure in the depth of the specimen. In the present work, it will be shown that for this purpose one can use in principle the measurements of the ultrasonic absorption coefficient.

The problem of the sound absorption in the intermediate state has been considered earlier in a work of Bruk and the author,[7] where it was shown that there exists an absorption mechanism that is specific for the intermediate state. This mechanism is associated with the motion of the boundary of separation between the phases. From the results of [7] it is seen that this mechanism is important only for low-frequency sound and not too high a state of purity of the metal, when the skin depth in the normal phase is of the order of or greater than the period of the structure. Here we shall consider significantly higher frequencies, for which the sound wavelength is the same in order of magnitude as the thickness of the layers, and pure metals with electron free path of theorder of 10^{-2}–10^{-1} cm. We shall also assume that the temperature of the system is small in comparison with the temperature of the superconducting transition. Under these conditions, one can assume, first, that the sound absorption takes place only in the normal phase, and second, that the thickness of the normal layers is much less than the free path length of the electrons and is smaller than the Larmor radius in the critical magnetic field. So far as the relations between the latter two parameters are concerned, we shall consider the two limiting cases.

Keeping in mind the most convenient situation, from the experimental viewpoint, when the sound is propagated along the axis of a cylindrical sample located in the intermediate state in an external magnetic field perpendicular to the axis of the cylinder, we shall assume that the wave vector of the sound is perpendicular to the boundary separating the two phases.

1. We select any layer occupied by the normal phase. If the thickness of the layer is a, then one can locate the set of coordinates so that the x axis is directed along the sound wave vector, while the boundary of the layer coincides with the planes x = 0 and x = a. The magnetic field, which is equal in magnitude to the critical field H_c, is assumed to be directed along the z axis.

For a deformation u_{ik} produced by the sound wave, the energy of the electron is equal to (see [8])

$$\varepsilon = \varepsilon_0(\mathbf{p}) + \lambda_{ik}(\mathbf{p})u_{ik}, \tag{1}$$

where $\varepsilon_0(\mathbf{p})$ is the energy in the absence of the sound, $\mathbf{p}$ is the quasimomentum, and $\lambda_{ik}(\mathbf{p})$ is the deformation potential. Furthermore, the sound propagation in the metal is accompanied by the generation of an electric field acting on the electron. Inasmuch as the sound wavelength is always greater than the Debye screening radius, the longitudinal part of the field is determined from the condition of electrical neutrality. The transverse electric field must satisfy the Maxwell equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{j}, \tag{2}$$

where $\mathbf{j}$ is the current density produced under the action of the deformation and the electric field, and ω is the sound frequency.

Considering first the case $R \gg l$ (R is the Larmor radius of the electron in the critical magnetic field,

l is the free path length), when the magnetic field in the layer has no effect on the motion of the electrons, we can write the kinetic equation for $0 \le x \le a$ in the form

$$v_x \partial\chi / \partial x + \nu\chi = e v_i E_i - \Lambda_{ik} \dot{u}_{ik}. \quad (3)$$

Here, as usual,[8] the distribution function of the electrons is sought in the form $f = f_0 + \chi \partial f_0/\partial\varepsilon$ (f_0 is the equilibrium value of the function), $\mathbf{v} = \partial\varepsilon_0/\partial\mathbf{p}$ is the velocity of the electron, e is its charge, ν the collision frequency, $\Lambda_{ik}(\mathbf{p}) = \lambda_{ik} - \overline{\lambda}_{ik}$, where the bar indicates averaging over the Fermi surface. In Eq. (3), we have neglected the arbitrary, time-dependent function χ, inasmuch as the sound velocity s is much less than the velocity of the electrons and the sound wave can therefore be regarded as quasistatic. Moreover, the derivatives $\partial\chi/\partial y$ and $\partial\chi/\partial z$ are equal to zero, which is clear from symmetry consideration.

We shall represent the function χ in the form of a sum of two functions $\chi = \varphi + \psi$, where φ and ψ satisfy the Eq. (3) in the case in which the right-hand side of it contains only terms with u_{ik} and E, respectively. The first of them is equal to

$$\varphi = C(\mathbf{v}) \exp\left\{-\frac{x\nu}{v_x}\right\} - \frac{\Lambda_{ik}}{v_x} \exp\left\{-\frac{x\nu}{v_x}\right\} \int_0^x \dot{u}_{ik}(x') \exp\left\{\frac{x'\nu}{v_x}\right\} dx', \quad (4)$$

where $C(\mathbf{v})$ is an arbitrary function dependent on the character of the reflection of the electrons from the surface layer. For its determination, we need to write down the boundary conditions for $x = 0$. As is shown in [9], these conditions in the low temperature case under study have the form

$$\chi(\mathbf{v}) + \chi(-\mathbf{v}) = 0 \quad \text{for } x = 0, a. \quad (5)$$

The functions φ and ψ must obviously satisfy the same boundary conditions, whence we find

$$C(\mathbf{v}) = \frac{\Lambda_{ik}}{2v_x \operatorname{sh}(a\nu/v_x)} \left[\int_0^a \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(a - x')\right\} dx' - \int_0^a \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(x' - a)\right\} dx'\right]$$

and thus

$$\varphi = \frac{\Lambda_{ik}}{2v_x \operatorname{sh}(a\nu/v_x)} \exp\left\{-\frac{\nu x}{v_x}\right\} \left[\int_0^a \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(a - x')\right\} dx' - \int_0^x \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(x' + a)\right\} dx' - \int_x^a \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(x' - a)\right\} dx'\right].$$

For what follows, it is convenient to extend the function $\dot{u}_{ik}(x)$ to the region $-a \le x \le 0$, so that

$$\dot{u}_{ik}(-x) = -\dot{u}_{ik}(x). \quad (6)$$

We then have

$$\varphi = -\frac{\Lambda_{ik}}{2v_x \operatorname{sh}(a\nu/v_x)} \exp\left\{-\frac{\nu x}{v_x}\right\} \left[\int_{-a}^x \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(x' + a)\right\} dx' + \int_x^a \dot{u}_{ik}(x') \exp\left\{\frac{\nu}{v_x}(x' - a)\right\} dx'\right]. \quad (7)$$

For $v_x > 0$, using the obvious identity

$$\exp\left\{\frac{\nu a}{v_x}\right\} \Big/ 2 \operatorname{sh}\frac{\nu a}{v_x} = \sum_{n=0}^{\infty} \exp\left\{-\frac{2a\nu n}{v_x}\right\},$$

we can rewrite Eq. (7) in the form

$$\varphi(v_x > 0) = -\frac{\Lambda_{ik}}{v_x} \exp\left\{-\frac{\nu x}{v_x}\right\} \sum_{n=0}^{\infty} \left[\int_x^a \dot{u}_{ik}(x') \times \exp\left\{\nu \frac{x' - 2a(n+1)}{v_x}\right\} dx' + \int_{-a}^x \dot{u}_{ik}(x') \exp\left\{\nu \frac{x' - 2an}{v_x}\right\} dx'\right]$$

If we now continue $\dot{u}_{ik}(x)$ periodically for all values of x

$$\dot{u}_{ik}(x + 2a) = \dot{u}_{ik}(x), \quad (8)$$

we then obtain

$$\varphi(v_x > 0) = -\frac{\Lambda_{ik}}{v_x} \exp\left\{-\frac{\nu x}{v_x}\right\} \sum_{n=0}^{\infty} \int_{x-2a(n+1)}^{x-2an} \dot{u}_{ik}(x') \exp\left\{\frac{\nu x'}{v_x}\right\} dx'$$
$$= -\frac{\Lambda_{ik}}{v_x} \exp\left\{-\frac{\nu x}{v_x}\right\} \int_{-\infty}^x \dot{u}_{ik}(x') \exp\left\{\frac{\nu x'}{v_x}\right\} dx'. \quad (9)$$

For $v_x < 0$, use of the identity

$$\exp\left\{-\frac{\nu a}{v_x}\right\} \Big/ 2 \operatorname{sh}\frac{a\nu}{v_x} = -\sum_{n=0}^{\infty} \exp\left\{\frac{2a\nu n}{v_x}\right\}$$

gives

$$\varphi(v_x < 0) = \frac{\Lambda_{ik}}{v_x} \exp\left\{-\frac{x\nu}{v_x}\right\} \int_x^{\infty} \dot{u}_{ik}(x') \exp\left\{\frac{\nu x'}{v_x}\right\} dx'. \quad (10)$$

It is easy to obtain similar formulas for the function ψ if we continue in accord with the formulas

$$\mathbf{E}(-x) = \mathbf{E}(x), \quad \mathbf{E}(x + 2a) = \mathbf{E}(x) \quad (11)$$

the electric field for all values of the parameter x:

$$\psi(v_x > 0) = \frac{e v_i}{v_x} \exp\left\{-\frac{\nu x}{v_x}\right\} \int_{-\infty}^x E_i(x') \exp\left\{\frac{\nu x'}{v_x}\right\} dx',$$
$$\psi(v_x < 0) = -\frac{e v_i}{v_x} \exp\left\{-\frac{\nu x}{v_x}\right\} \int_x^{\infty} E_i(x') \exp\left\{\frac{\nu x'}{v_x}\right\} dx'. \quad (12)$$

It is interesting to note that Eqs. (9), (10), and (12) are formally identical to the solutions of Eq. (3) for an infinite normal metal. This fact is a characteristic mark of the law of reflection of the electrons from the boundary separating the phases, which is described by the boundary conditions (5).

As is seen from (8) and (11), the electric field and the deformation tensor[1)] are periodic functions of the coordinate x. Thus the functions φ and ψ will possess the same property if we so continue them that Eqs. (9), (10), and (12) remain in force for all values of x. Then, transforming to the Fourier components in accord with the formulas

$$F(x) = \sum_{n=-\infty}^{+\infty} F_n e^{iq_n x}, \quad q_n = \frac{\pi n}{a},$$
$$F_n = \frac{1}{2a} \int_{-a}^{a} F(x) e^{-iq_n x} dx, \quad (13)$$

we get, for all values of $\mathbf{v}$

$$\varphi_n = -\frac{\Lambda_{ik} \dot{u}_{ik,n}}{iq_n v_x + \nu} \quad \psi_n = \frac{e v_i E_{i,n}}{iq_n v_x + \nu} \quad (14)$$

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1)We emphasize that we are concerned with the functions $u_{ik}(z)$ and $\mathbf{E}(z)$, which have been introduced in purely formal fashion, and which are identical with the deformation tensor and the electric field actually present in the sample for $0 < z < a$ only.

With the help of these latter relations, we can easily calculate the electric current density:

$$\mathbf{j} = \frac{e}{4\pi^3}\int \mathbf{v}\chi \frac{dS}{v},$$

where dS is an element of the Fermi surface. Then, substituting (14), we get

$$\mathbf{j} = \mathbf{j}^{(\varphi)} + \mathbf{j}^{(\psi)},$$

$$j_{i,n}^{(\varphi)} = -\frac{e}{4\pi^3}\dot{u}_{lm,n}\int\frac{dS}{v}\frac{v_i\Lambda_{lm}}{iq_nv_x+\nu}, \quad j_{i,n}^{(\psi)} = \frac{e}{4\pi^3}E_{k,n}\int\frac{dS}{v}\frac{ev_iv_k}{iq_nv_x+\nu}. \tag{15}$$

For small ν and $n \neq 0$, we can assume that

$$\frac{1}{iq_nv_x+\nu} = \mathrm{P}\frac{1}{iq_nv_x} + \frac{\pi}{|q_n|v}\delta\left(\frac{v_x}{v}\right).$$

where P is the symbol for the principal value of the integral. From (15) we then get for $n \neq 0$

$$j_{\alpha,n}^{(\varphi)} = \frac{ie}{4\pi^3 q_n}D_{\alpha lm}\dot{u}_{lm,n}, \quad j_{x,n}^{(\varphi)} = 0,\ j_{\alpha,n}^{(\psi)} = \frac{e^2}{4\pi^2|q_n|}B_{\alpha\beta}E_{\beta,n},\ j_{x,n}^{(\psi)} = 0. \tag{16}$$

Here

$$D_{\alpha lm} = \int\frac{dS}{v}\frac{n_\alpha}{n_x}\Lambda_{lm}, \quad B_{\alpha\beta} = \int dS\, n_\alpha n_\beta \delta(n_x),$$

$\mathbf{n} = \mathbf{v}/v$, the indices α and β run over the values y and z. We have taken it into account that the tensor Λ_{ik} satisfies the condition

$$\int \Lambda_{ik}\frac{dS}{v} = 0,$$

as is seen from the definition of this tensor.

For the determination of the electric field, we shall begin with the Maxwell equations (2), which, in our geometry, can be written in the form

$$\frac{\partial^2 E_\alpha}{\partial x^2} + \frac{4\pi i\omega}{c^2}j_\alpha = 0.$$

Transforming to the Fourier components in these equations, we get

$$\left(\frac{\pi e}{|q_n|}B_{\alpha\beta} - \frac{\pi^2c^2q_n^2}{ie\omega}\delta_{\alpha\beta}\right)E_{\beta,n} + \frac{\pi^2c^2}{ie\omega a}[E_\alpha'(+0) - (-1)^n E_\alpha'(a-0)] = -\frac{4\pi^3}{e}j_{\alpha,n}^{(\varphi)}. \tag{17}$$

The presence of terms in the square brackets of Eq. (17) is associated with the discontinuity in the functions $E_\alpha'(x)\ dE_\alpha\ dx$ for $x = 0, \pm a, \pm 2a, \ldots$

It is easy to see that in the frequency range of interest, when $s/\omega \sim a$, the term with q_n^2 in the left-hand side of (17) can be neglected. Actually, its ratio to the first term is, in order of magnitude, equal to $(\delta/a)^3$, where $\delta \sim (c^2/e^2p_F^2\omega)^{1/3}$ is the skin depth. This ratio is small for all reasonable values of the layer thickness a.

We now estimate the magnitude of the E_α'. This can be done if we note that the magnetic field on the boundary between phases should be equal to the critical value. Under the action of the deformation brought about by the sound wave, the critical field changes by an amount $u_{ik}\partial H_c/\partial u_{ik} \sim H_c u_{ik}$. It is therefore clear that the oscillating part of the magnetic field on the boundary is of the order of $H_c u_{ik}$, whence (using the Maxwell equation curl $\mathbf{E} = -c^{-1}\partial \mathbf{H}/\partial t$) we find $E_\alpha' \sim H_c\dot{u}_{ik}/c$. The ratio of the terms with E_α' to the right-hand side of (17) is equal in magnitude to the value of $cH_c/asep_F^3 \sim (T_c/\omega_D)(\partial_L/a)$, where T is the critical temperature, ω is the Debye frequency of the phonons, and ∂_L is the London penetration depth of the magnetic field in the superconductor. Thus we can also neglect these terms.

Taking all the above into account, we find from (17)

$$E_{\alpha,n} = \frac{|q_n|}{\pi ea}\frac{1-(-1)^n e^{ika}}{k^2-q_n^2}(B^{-1})_{\alpha\beta}D_{\beta lm}\dot{U}_{lm}. \tag{18}$$

In writing down (18), we have used the explicit dependence of the tensor $\dot{u}_{ik}(x) = \dot{U}_{ik}e^{ikx}$ on the coordinates (U_{ik} does not depend on x) for $0 < x < a$ and Eqs. (6), (8), and (13), according to which

$$\dot{u}_{ik,n} = \frac{\dot{U}_{ik}}{ia}\frac{q_n}{k^2-q_n^2}[e^{ika}(-1)^n - 1]. \tag{19}$$

We shall be interested in the time average of the energy absorbed in a unit volume in the intermediate state,

$$\overline{Q} = (a+b)^{-1}\int_0^a Q(x)\,dx,$$

where b is the thickness of the superconducting layer,

$$Q(x) = \frac{1}{(2\pi)^3}\int\frac{dS}{v}\nu|\chi|^2$$

is the energy absorbed in a unit volume of normal phase (see [8, 10]). Expressing it in terms of the Fourier components of the function χ, we obtain

$$\overline{Q} = \frac{1}{(2\pi)^3(a+b)}\int\frac{\nu dS}{v}\left[a\sum_{n=-\infty}^{+\infty}|\chi_n|^2 + \sum_{m\neq n}\chi_n\chi_m^*\frac{(-1)^{n+m}-1}{i(q_n-q_m)}\right]. \tag{20}$$

As is seen from (14), (18), and (19), the function $\chi_n(\mathbf{v})$ satisfies the condition $\chi_{-n}(-\mathbf{v}) = \chi_n(\mathbf{v})$. Here the second term in (20) is equal to zero, since the expression under the summation and integral signs changes sign for the simultaneous substitution $n \to -n$, $m \to -m$, $\mathbf{v} \to -\mathbf{v}$. We thus have

$$\overline{Q} = \frac{\eta}{(2\pi)^3}\int\frac{\nu dS}{v}\sum_n|\chi_n|^2 = \frac{\eta}{(2\pi)^3}\int\frac{\nu dS}{v}\sum_n\{|\varphi_n|^2 + |\psi_n|^2\}, \tag{21}$$

where $\eta = a/(a+b)$ is the concentration of the normal phase. In writing down the last expression, it was taken into account that the expressions $\varphi_n\psi_n^*$ and $\psi_n\varphi_n^*$ are odd in n.

The sound absorption coefficient Γ is equal to the energy $\overline{Q}$ multiplied by the reciprocal of the energy flux in the sound wave $2/\rho S\omega^2|\mathbf{u}_0|^2$ (ρ is the density of the metal, $\mathbf{u}_0$ the displacement vector). Substituting in (21) the expression for the sum which follows from (14), (18), and (19),

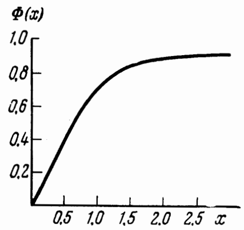
$$|\varphi_n|^2 + |\psi_n|^2 = \frac{2}{\nu va^2}\frac{|q_n|}{(k^2-q_n^2)^2}[1-(-1)^n\cos ka] \times\left[\pi|\Lambda_{ik}\dot{U}_{ik}|^2 + \frac{1}{\pi}|v_\alpha(B^{-1})_{\alpha\beta}D_{\beta lm}\dot{U}_{lm}|^2\right]\delta(n_x),$$

we get

$$\Gamma/\Gamma^{(N)} = \eta\Phi(ka/\pi), \tag{22}$$

where

$$\Gamma^{(N)} = \frac{1}{4\pi^2\rho\omega^3|\mathbf{u}_0|^2}\int\frac{dS}{v^2}\delta(n_x)\left[|\Lambda_{ik}\dot{U}_{ik}|^2 + \frac{1}{\pi^2}|v_\alpha(B^{-1})_{\alpha\beta}D_{\beta lm}\dot{U}_{lm}|^2\right]$$



is the sound absorption coefficient in the normal phase (see [10]),

$$\Phi(x)\equiv\frac{4x}{\pi^2}\sum_{n=1}^{\infty}\frac{n}{(n^2-x^2)^2}[1-(-1)^n\cos\pi x]$$
$$=\frac{1}{\pi^2}\Big\{\pi^2-\frac{1}{x^2}-2\psi'(1+x)-\frac{\cos\pi x}{2}\Big[\psi'\Big(\frac{(1+x)}{2}\Big)-\psi'\Big(1+\frac{x}{2}\Big)-\frac{2}{x^2}\Big]\Big\}. \qquad (23)$$

Here $\psi'(x)$ is the second derivative of the logarithm of the Γ function. The graph of the function $\Phi(x)$ obtained as the result of numerical computation is shown in the drawing.

When the sound wavelength is much less than the thickness of the normal layers $ka \gg 1$, the natural result $\Gamma/\Gamma^{(N)}=\eta$ is obtained from (22), inasmuch as $\Phi(\infty)=1$. In this case, the coefficient Γ, as also $\Gamma^{(N)}$, is proportional to the first power of the frequency.

In the opposite limiting case, one can set $\Phi(x) = 7\zeta(3)x/\pi^2$ ($\zeta(x)$ is the Riemann zeta function) and thus

$$\frac{\Gamma}{\Gamma^{(N)}}=\frac{7\zeta(3)}{\pi^3}\eta ka,$$

i.e., the absorption is proportional to the square of the sound frequency and is the same in order of magnitude as the absorption in the normal metal with electron free path length equal to the thickness of the layers a (see [8]).

2. Now let $R \ll l$. In place of (3) we should consider the kinetic equation with account of the magnetic field:

$$v_x\frac{\partial\chi}{\partial x}+\Omega\frac{\partial\chi}{\partial\tau}+\nu\chi=ev_iE_i-\Lambda_{ik}\dot{u}_{ik}, \qquad (24)$$

where Ω is the cyclotron frequency in the field H_c, τ is the dimensionless time of rotation of the electron in the magnetic field. Just as in the case above, we continue the functions χ, $\dot{u}_{ik}$, and E_i periodically over all x so that the conditions (6), (8), and (11) are satisfied; we also rewrite Eq. (24) in terms of the Fourier components:

$$\Omega\partial\chi_n/\partial\tau+(iq_nv_x+\nu)\chi_n=ev_iE_{i,n}-\Lambda_{ik}\dot{u}_{ik,n}. \qquad (25)$$

The latter equation is identical with the kinetic equation for the bulk of the normal metal. Using the results of the work of Gurevich,[10] we can immediately write down the solution:

$$\chi_n=\frac{\Omega}{2\pi\nu}\frac{e^{iq_nB(\tau)}}{|q_n|^{1/2}}\Big[\Lambda_{ik}^{(1)}\dot{u}_{ik,n}\Big(\frac{2\pi}{v_{x_1}'}\Big)^{1/2}\exp\Big\{-i\Big(B_1q_n+\frac{\pi}{4}\frac{q_n}{|q_n|}\Big)\Big\}+\Lambda_{ik}^{(2)}\dot{u}_{ik,n}\Big(\frac{2\pi}{|v_{x2}'|}\Big)^{1/2}\exp\Big\{-i\Big(B_2q_n-\frac{\pi}{4}\frac{q_n}{|q_n|}\Big)\Big\}\Big]. \qquad (26)$$

Here $B(\tau)=c[p_y(\tau)-p_y(0)]/eH_c$, the prime denotes time differentiation, the indices 1 and 2 show that the value of the corresponding quantities is taken at points τ_1 and τ_2 such that $v_x(\tau_{1,2})=0$. For simplicity, it is assumed that there are two such points, and $v_{x1}'>0$, $v_{x2}'<0$. Noting that in the substitution $\mathbf{v}\to-\mathbf{v}$ the points 1 and 2 change places, it is easy to show by means of (19) that the solution (26) satisfies the relation $\chi_n(\mathbf{v})=\chi_{-n}(-\mathbf{v})$. It then follows that the boundary conditions (5) for the function χ are satisfied automatically.

By means of (26), we easily find

$$\nu|\chi_n|^2=\frac{\Omega}{2\pi\nu|q_n|}\Big[\frac{|\Lambda_{ik}^{(1)}\dot{u}_{ik,n}|^2}{|v_{x1}'|}+\frac{|\Lambda_{ik}^{(2)}\dot{u}_{ik,n}|^2}{|v_{x2}'|}\Big]-\frac{\Omega}{\pi\nu|q_n|}\Lambda_{ik}^{(1)}\Lambda_{lm}^{(2)}\dot{u}_{ik,n}\dot{u}_{lm,n}^{*}\frac{\sin[(B_2-B_1)|q_n|]}{|v_{x1}'v_{x2}'|^{1/2}}.$$

Substituting this in (21), we can express the absorption coefficient Γ in the form of a sum $\Gamma_0+\Delta\Gamma$, where Γ_0 corresponds to the first two terms on the right side of the last equation and is a monotonic function of the external magnetic field, $\Delta\Gamma$ is an oscillating addition. We have for

$$\Gamma_0=\eta\Phi(ka/\pi)\Gamma_0^{(N)}(H_\kappa), \qquad (27)$$

where

$$\Gamma_0^{(N)}(H_\kappa)=\frac{2}{(2\pi)^4k\rho s\omega^2|\mathbf{u}_0|^2}\int\frac{dS}{v}\Big(\frac{\Omega}{\nu}\Big)\Big[\frac{|\Lambda_{ik}^{(1)}\dot{U}_{ik}|^2}{|v_{x_1}'|}+\frac{|\Lambda_{ik}^{(2)}\dot{U}_{ik}|^2}{|v_{x_2}'|}\Big]$$

is the monotonic part of the sound absorption in the normal metal, $\Phi(x)$ is a function defined by Eq. (23) and shown in the drawing.

The oscillating part of the absorption is equal to (see [10])

$$\Delta\Gamma\sim\Gamma_0^{(N)}(H_\kappa)\eta ka\Big(\frac{a}{R}\Big)^{1/2}\sum_{n=1}^{\infty}\frac{n^{1/2}[1-(-1)^n\cos ka]}{(n^2-k^2a^2/\pi^2)^2}\sin\Big[\frac{\pi R_{ext}}{a}n\pm\frac{\pi}{4}\Big]. \qquad (28)$$

Here $R_{ext}=c(p_y^{(2)}-p_y^{(1)})_0/eH_c$, the index 0 indicating that the value of $p_y^{(2)}-p_y^{(1)}$ should be taken at the extremal value of p_z, i.e., for p_z satisfying the equation

$$\frac{d}{dp_z}(p_y^{(2)}-p_y^{(1)})=0;$$

the sign + or − in (28) depends on the sign of the derivative of p_z from the left side of the latter equation.

It is seen from (28) that for $ka\sim1$ the oscillating part of the absorption is smaller than the monotonic part by the factor $(R/a)^{1/2}$. The quantity $\Delta\Gamma$, considered as a function of the external field H is an almost periodic function with a slowly changing period

$$\delta H=2a^2\Big/\frac{\partial a}{\partial H}R_{ext}. \qquad (29)$$

3. In conclusion, we shall discuss the possibility of the use of the sound absorption measurements for the determination of the period of the structure.

If $R \ll l$, then Eq. (22) offers the possibility (from the measured ratio $\Gamma/\Gamma^{(N)}$, and with the help of the explicit form of the function $\Phi(ka/\pi)$ and the relation $\eta=2H/H_c-1$ (for a cylinder in a perpendicular field) of determining the thickness of the normal layers a as a function of the external magnetic field. The period of the structure $d(H)=a(H)/\eta(H)$ is then easily determined.

In the case $R \ll l$, measurement of the monotonic part of the absorption gives the same information, as is seen from (27). Moreover, there is an additional possibility here, associated with the presence of oscillations in the absorption. Use of the relation (29) for the period of oscillation also permits us to determine the function d(H). The quantity R_{ext} entering into (29) can be found from measurement of the absorption oscillations in the region $H > H_c$, where the period of the oscillations is determined by the ratio of R_{ext} to the sound wavelength (see [10]).

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76