STRUCTURE OF ISOBARIC ANALOG STATES IN HEAVY NUCLEI

D. F. ZARETSKIĬ and M. G. URIN

Moscow Engineering-physics Institute

Submitted February 9, 1967

Zh. Eksp. Teor. Fiz. 53, 324–330 (July, 1967)

The microscopic structure of analog states in heavy nuclei are treated by the interacting-quasiparticle method. The relation between the microscopic approach and the description by means of isotopic spin is established. The "optical" width of the analog states is estimated.

1. INTRODUCTION

As shown in [1], analog states in heavy nuclei can be regarded from the microscopic point of view as collective excitations of the proton-neutron hole type with zero total angular momentum. Since analog states are usually described by using the formalism of isotopic spin [2], a connection must be established between this description and the microscopic approach.

There exist also other collective states of the proton-neutron hole type, which differ in energy and in isospin from the analog states. Allowance for the self-consistent Coulomb potential gives rise to small admixtures of these states to the analog state. In other words, a mixing of the states with different isospins takes place. The values of the coefficients that determine the magnitude of the admixture of states with different isospin in the analog state are important for the clarification of the problem of the width of the analog state and for the calculation of certain nuclear reactions.

In Sec. 2 we consider the analog states with the aid of the Hamiltonian for the quasiparticles. This approach makes it possible to avoid the simplifying assumptions made in [1], and to clarify the question of the classification of the states of the nucleus with the aid of isotopic spin.

In Sec. 3 we consider, using methods of the theory of finite Fermi systems [3], other collective states of the proton-neutron hole type. We determine the energy and wave functions of some of these states in the quasiclassical approximation. Their width and isotopic spin are discussed.

The method of calculating the corrections to the wave function and the density matrix of the analog state, which result from the variable part of the self consistent Coulomb potential, is formulated in Sec. 4. The magnitudes of these corrections are estimated in the quasiclassical approximation.

2. ANALOG STATES AND THE HAMILTONIAN FOR THE QUASIPARTICLES

To describe the low-excited states, the nucleus can be regarded as a gas of interacting quasiparticles with an effective Hamiltonian which has, in accord with the theory of finite Fermi systems, the form ($\hbar = c = M = 1$) [3]

$$H = -\sum_i \frac{\Delta_i}{2M_i^*} + \sum_i U(\mathbf{r}_i) + \frac{1}{2}\sum_i \varphi(r_i)(1-\tau_i^{(3)}) + H_{int}. \quad (1)$$

Here M^* is the effective mass of the quasiparticles (we shall henceforth assume for neutrons and protons $M_n^* = M_p^* = 1$); $U(\mathbf{r}_i)$ is the nuclear self-consistent potential and is the same for the neutrons and the protons; $\varphi(\mathbf{r}_i)$ is the self-consistent Coulomb potential acting on the protons; H_{int} is the quasiparticle interaction Hamiltonian. To describe the excitations with small momentum transfer in the particle-hole channel we can use the following expression for H_{int} [3]

$$H_{int} = \frac{1}{4}\frac{V}{\rho_0}\sum_{ik}{}' [(f + g\sigma_i\sigma_k) + (f' + g'\sigma_i\sigma_k)\tau_i\tau_k]\delta(\mathbf{r}_i - \mathbf{r}_k), \quad (2)$$

where V is the volume of the nucleus; ρ_0 coincides with the density of the single-particle levels near the Fermi boundary for a rectangular potential well; f, g, f′, and g′ are phenomenological constants. In the case of scattering of quasiparticles with transfer of arbitrary angular momentum, expression (2) does not hold for H_{int}, and it can only be stated that H_{int} remains an isotopic scalar.

The operator H_{int} contains, besides the terms describing the scattering of the quasiparticles, also terms that make a contribution to the self-

consistent field; this contribution is proportional to N – Z. These terms can be separated in the Hartree-Fock approximation. It is easy to estimate them by assuming that the density of the excess neutrons is constant over the volume of the nucleus (and equal to $(N - Z)V^{-1}$). Then, according to (1) and (2),

$$U_n(\mathbf{r}) = U(\mathbf{r}) + {}^1/_2 f'(N-Z)\rho_0^{-1},$$
$$U_p(\mathbf{r}) = U(\mathbf{r}) - {}^1/_2 f'(N-Z)\rho_0^{-1} + \varphi(r). \quad (3)$$

Inside the nucleus, where the nucleon density is constant, the self-consistent potential (3) coincides with the expression used in the shell model [4]. The constant f′ then turns out to be close to the value obtained from the Weizsacker formula [3].

It is convenient to separate from the Coulomb potential $\varphi(r)$ the part ΔE_C which is constant over the volume of the nucleus and coincides with the Coulomb energy per proton:

$$\Delta E_C = \frac{1}{2}\frac{\partial}{\partial Z}\int \varphi(r)\rho_z(r)\,dV, \quad (4)$$

where $\rho_Z(r)$ is the proton density. If $\rho_Z(r) = ZV^{-1}$ ($r < R$, R = radius of nucleus), then $\Delta E_C = ({}^6/_5)Ze^2R^{-1}$, and $\varphi(r)$ can be represented in the form $\varphi(r) = \varphi_0(r) + \Delta\varphi(r)$, where

$$\varphi_0(r) = \begin{cases} \frac{6}{5}\frac{Ze^2}{R}, & r < R \\ \frac{Ze^2}{r}, & r > R \end{cases};$$

$$\Delta\varphi(r) = \begin{cases} \frac{3}{10}\frac{Ze^2}{R}\left(1 - \frac{5}{3}\frac{r^2}{R^2}\right), & r < R \\ 0, & r > R. \end{cases} \quad (5)$$

To study the nuclear excitation spectrum we can use in first approximation the Hamiltonian H′, which is obtained from (1) when $\Delta\varphi = 0$. The corrections connected with $\Delta\varphi$ will be taken into account subsequently by perturbation theory. The Hamiltonian H′ obtained in this manner satisfies the following commutation relations, which are valid for all $r_i < R$:

$$[H', T^{(-)}] = \Delta E_C T^{(-)}; \quad [H', T^{(+)}] = -\Delta E_C T^{(+)}, \quad (6)$$

where $T^{(\pm)} = ({}^1/_2)\sum_i \tau_i^{(\pm)}$). It follows from (6) that there exists an excited state $|a\rangle$ (analog state) with energy ΔE_C and with wave function

$$|a\rangle = 2(N-Z)^{-1/2}T^{(-)}|0\rangle, \quad (7)$$

where $|0\rangle$ is the wave function of the ground state for the Hamiltonian H′, and satisfies in accordance with (6) the relation

$$T^{(+)}|0\rangle = 0. \quad (8)$$

The Hamiltonian H′ commutes with the total isospin operator T^2 (for all $r_i < R$). Therefore a definite isospin can be ascribed to the eigenfunctions of this Hamiltonian. In particular, it follows from (6)—(8) that the states $|0\rangle$ and $|a\rangle$ have isospins and projections respectively equal to (T_0, T_0) and $(T_0, T_0 - 1)$ for $T_0 = ({}^1/_2)(N - Z)$.

To calculate the effects connected with the excitation of the analog states, it is necessary to determine the quasiparticle states, that is, the spectrum of the single-quasiparticle excitations of the system with the Hamiltonian H′. In the case of sufficiently large N – Z, the self-consistent potential for the quasiparticles is determined by an expression of the form (3) with $\Delta\varphi \equiv 0$. The wave functions of the neutrons and the protons $\varphi_\lambda^{n,p}$ can be regarded here as identical in the region of the discrete spectrum, accurate to boundary effects.

In conclusion we note that the expressions for the energy and for the wave function of the analog state, obtained in [1] using the concrete form of H_{int}, as well as the assumptions concerning the constancy of the density of the excess neutrons over the volume of the nucleus, coincide with the corresponding expressions of the present section.

3. COLLECTIVE EXCITATIONS OF THE PROTON-NEUTRON HOLE TYPE

As shown in [1], in heavy nuclei ($(N - Z) \gg A^{1/3}$) the analog state is one of the possible collective states of the proton-neutron hole type. The self-field corresponding to this excitation is constant over the volume of the nucleus ($v_a(r) = \text{const}$). There exist also excitations whose self-fields differ from a constant (for example $v_2 = a + br^2$, $v_3 = c + dr^2 + fr^4$, etc.). In the quasiclassical approximation, for a rectangular potential well with infinite walls, we can verify that, for example, $v_2(r) = \text{const}\,(1 - 5\,r^2/3R^2)$ is a self-field, that is, it satisfies the homogeneous equation whose solution is equivalent to the approximate diagonalization of the Hamiltonian H′,

$$v_2(r) = \frac{f'V}{\rho_0}\sum_{\lambda\lambda'}\varphi_\lambda^{p*}\varphi_{\lambda'}^{n}\frac{n_\lambda^p - n_{\lambda'}^n}{\varepsilon_\lambda^p - \varepsilon_{\lambda'}^n - \omega_2}(v_2)_{\lambda'\lambda}. \quad (9)$$

Here $n_\lambda^{n,p}$ and $\epsilon_\lambda^{n,p}$ are the occupation numbers and the excitation energies of the quasiparticles (neutrons and protons). Indeed, if we use the

relations obtained in the study of giant monopole resonance [5],

$$\sum_{\lambda}|\varphi_{\lambda}(\mathbf{r})|^2(n_{\lambda}^{n}-n_{\lambda}^{p})\,(v_2)_{\lambda\lambda'}=c\frac{N-Z}{V}v_2(r);$$

$$\sum_{\lambda\neq\lambda'}\varphi_{\lambda}^{*}\varphi_{\lambda'}\frac{n_{\lambda}-n_{\lambda'}}{\varepsilon_{\lambda}-\varepsilon_{\lambda'}}(v_2)_{\lambda'\lambda}=-(1-c)\frac{\rho_0}{V}v_2(r);\ c=\frac{4}{9},\tag{10}$$

then we get in accordance with (9) and (10), in the approximation $\Delta\epsilon_0 \ll \omega^0$ ($\omega^{02}=42\epsilon^0/MR^2$; $\epsilon_0=(3/4)A\rho_0^{-1}$ is the energy of the Fermi boundary), two natural frequencies corresponding to the same self-field $v_2(r)$:

$$\omega_2=\Delta\varepsilon_{01}\frac{1+f'}{1+f'(1-c)}=\frac{\Delta E_c}{1+f'(1-c)},$$

$$\omega_{2'}=\omega^0\sqrt{1+f'(1-c)},\tag{11}$$

where $\Delta\epsilon_0=(4/3)\epsilon_0(N-Z)/A$ is the difference in the depths of the potential wells for the neutrons and protons. It is natural to set the state $|2\rangle$ with energy $\omega_2 < \Delta E_C$ in correspondence with one of the possible configuration states (using Lane's terminology [2]).

The expressions for the wave functions of the states with energies ω_2 and ω_2' can be obtained with the aid of the relation given in [1], using the explicit form of $v_2(r)$. For example, for the state $|2\rangle$ we have

$$|2\rangle\equiv\mathfrak{A}_2^{+}|0\rangle,\ \mathfrak{A}_2|0\rangle=0,\ \mathfrak{A}_2^{+}=\sum_{\lambda\lambda'}C_{\lambda\lambda'}^{(2)}a_{\lambda}^{p+}a_{\lambda'}^{n};$$

$$C_{\lambda\lambda'}^{(2)}=-B^{(2)}\frac{1}{\varepsilon_{\lambda}^{p}-\varepsilon_{\lambda'}^{n}-\omega_2}(v_2)_{\lambda\lambda'},$$

$$B^{(2)}=\left[\sum_{\lambda\lambda'}|(v_2)_{\lambda\lambda'}|^2\frac{n_{\lambda'}^{n}-n_{\lambda}^{p}}{(\varepsilon_{\lambda}^{p}-\varepsilon_{\lambda'}^{n}-\omega_2)^2}\right]^{-1/2}.\tag{12}$$

Thus, to describe states of the proton-neutron hole type with small total angular momentum, the Hamiltonian H' can be written in the form

$$H'=\sum_{s}\omega_s\mathfrak{A}_s^{+}\mathfrak{A}_s,\tag{13}$$

where the operators $\mathfrak{A}_S$ and $\mathfrak{A}_S^{+}$ satisfy, at the same accuracy with which equation (9) is valid, the boson commutation rules

$$[\mathfrak{A}_s,\mathfrak{A}_{s'}^{+}]=\delta_{ss'}.\tag{14}$$

A sufficient condition for the applicability of equation (9), and consequently also (13) and (14), is the requirement

$$\rho_{\lambda\lambda'}^{s}=\langle s|a_{\lambda}^{p+}a_{\lambda'}^{n}|0\rangle=(C_{\lambda\lambda'}^{s})^{*}(n_{\lambda'}^{n}-n_{\lambda}^{p})\ll 1.\tag{15}$$

Thus, for example, for the state $|2\rangle$ this condition is equivalent, in accord with (12), to the inequality $(N-Z)^{-1/2}\ll 1$.

We note that the case $s=a$ corresponds to the analog state which, as shown in Sec. 1, is obtained as a result of exact diagonalization of the Hamiltonian H', that is $\mathfrak{A}_a^{+}|0\rangle = 2(N-Z)^{-1/2}T^{(-1)}|0\rangle$.

It is of interest to determine the value of the isotopic spin for a collective state that does not coincide with the analog state. Let us calculate the mean value

$$\langle T_0,T_0|\mathfrak{A}_sT^2\mathfrak{A}_s^{+}|T_0,T_0\rangle=(T_0-1)^2+2\langle T_0,T_0|\mathfrak{A}_sT^{(+)}T^{(-)}\mathfrak{A}_s|T_0,T_0\rangle+2\langle T_0,T_0|\mathfrak{A}_sT^{(-)}T^{(+)}\mathfrak{A}_s|T_0,T_0\rangle.$$

Using the relation $[T^{(+)},T^{(-)}]=(1/2)T^{(3)}$, and also (8) and (14), we get for any $s\neq a$

$$\langle 0|\mathfrak{A}_sT^2\mathfrak{A}_s^{+}|0\rangle=T_0(T_0-1).\tag{16}$$

Thus, the isotopic spin of an arbitrary collective state $|s\rangle$ that does not coincide with $|a\rangle$ is equal to the isotopic spin of the ground state of the nucleus produced as a result of replacing the neutron by a proton in the target nucleus.

Apart from isotopic spin, the analog state differs from other collective states of the proton-neutron hole type in the absence of an "optical" width. By "optical" width W_{opt} of a single-quasiparticle state we mean the width connected with the decay of this state into more complicated configurations with the same energy. This means that the Hamiltonian H' cannot in general be exactly diagonalized with the aid of the single-quasiparticle states. The "optical" width increases with increasing excitation energy $(E-\epsilon_0)$, and at sufficiently large $(E-\epsilon_0)\ll\epsilon_0$ we have $W_{opt}=\alpha(E-\epsilon_0)^2$, where $\alpha\sim 1/\epsilon_0$ [3].

Inasmuch as the analog state corresponds to exact diagonalization of the Hamiltonian H', its "optical" width is equal to zero. Other collective states of the proton-neutron hole type are obtained as a result of approximate diagonalization of the Hamiltonian. The corresponding exact eigenfunctions contain, besides configurations of the particle-hole type, also more complicated configurations (two particles—two holes, etc.). Therefore the "optical" width of the states $s\neq a$ is of the order of the "optical" width of the quasiparticles with excitation energy $\Delta\epsilon_0$.

4. COULOMB CORRECTIONS

The foregoing analysis was based on the use of a Hamiltonian H', in which no account was taken of the time-dependent part of the self consistent Coulomb potential $\Delta\varphi$. Allowance for $\Delta\varphi$ leads

to the appearance of corrections to the wave function and energy of the analog state. In the first approximation of perturbation theory, the corrections to the wave functions are determined by the relation

$$|a\rangle' = |a\rangle + \sum_{s \neq a} \frac{\langle s|\Delta\varphi|a\rangle}{E_a - E_s}|s\rangle. \quad (17)$$

The calculation of the matrix elements $\langle s|\Delta\varphi|a\rangle = \langle 0|[\mathfrak{A}_s[\Delta\varphi, \mathfrak{A}_a^+]]|0\rangle$ with the aid of the explicit expression for the operators $\mathfrak{A}_s$, $\mathfrak{A}_a^+$, and $\Delta\varphi$ leads to the following expression:

$$\langle s|\Delta\varphi|a\rangle = \frac{1}{\sqrt{N-Z}} \sum_{1,2} \frac{n_1^p - n_2^n}{\varepsilon_1^p - \varepsilon_2^n - \omega_s} v_{21}^s (\Delta\varphi)_{12} B^s. \quad (18)$$

Using (9), we can rewrite (18) in the form

$$\langle s|\Delta\varphi|a\rangle = \frac{1}{\sqrt{N-Z}} B^s \frac{\rho_0}{f'V} \int v^s(r)\Delta\varphi\, d\mathbf{r}. \quad (19)$$

Since $\Delta\varphi \sim v_2(r)$, it follows that the matrix element (19) differs from zero only for the states $|2\rangle$ and $|2'\rangle$. Taking (17) and (19) into account, we get

$$|a\rangle' = |a\rangle + \frac{\rho_0 \Delta E_C}{21 f' \sqrt{N-Z}} \left[\frac{B^{(2)}|2\rangle}{\omega_a - \omega_2} + \frac{B^{(2')}|2'\rangle}{\omega_a - \omega_{2'}}\right] = |a\rangle + \alpha_2|2\rangle + \alpha_{2'}|2'\rangle. \quad (20)$$

Let us estimate the values of the admixture coefficients of the states $|2\rangle$ and $|2'\rangle$ in (20). If $\Delta\epsilon_0 \ll \omega^0$, then the ratio $\alpha_{2'}/\alpha_2 \sim \sqrt{\Delta\epsilon_0/\omega^0} \ll 1$. In the same approximation we have, in accordance with (10)—(12),

$$B^{(2)} \cong \frac{\omega_2 - \Delta\varepsilon_0}{\sqrt[4]{21} c (N-Z)} \qquad \alpha_2 \cong \frac{1}{4} \frac{1}{f'(1-c)} \sqrt{\frac{4}{21}} c. \quad (21)$$

Assuming that $f' \cong 1.5$ and $c = 4/9$, we get $\alpha_2 \cong 0.1$. Thus, the correction to the wave function of the analog state is practically the same for the region of nuclei of interest to us, and does not exceed 10%.

Let us estimate now the correction to the energy of the analog state

$$\delta\Delta E_C = \sum_{s=2,2'} \frac{|\langle s|\Delta\varphi|a\rangle|^2}{E_s - E_a}. \quad (22)$$

It can be verified that the states $|2\rangle$ and $|2'\rangle$ give comparable contributions, but with opposite signs, to (22):

$$\delta\Delta E_C = \alpha_2^2(\omega_a - \omega_2) + \alpha_{2'}^2(\omega_a - \omega_{2'}). \quad (23)$$

Using the same values of f' and c as before, we obtain the estimate $|\delta\Delta E_C/\Delta E_C| \lesssim 0.005$. Thus, the energy shift of the analog state relative to ΔE_C does not exceed 100 keV even in the heaviest nuclei.

To calculate the effects connected with the excitation of the analog states, it is necessary to know the density matrix ρ'

$$\rho_{12}' = \langle a'|a_1^p + a_2^n|0'\rangle, \quad (24)$$

Where $|a'\rangle$ is determined from formula (20) and $|0'\rangle$ is the wave function of the ground state of the target nucleus

$$|0'\rangle = |0\rangle + \sum_i \frac{\langle i|\Delta\varphi|0\rangle}{E_0 - E_i}|i\rangle. \quad (25)$$

Here $|i\rangle$ are all the possible wave functions corresponding to excitations of the neutron (proton)-neutron (proton) hole with zero total angular momentum. We note that the diagonal corrections of the density matrix are determined only by the corrections to the wave function of the analog state.

The correction to the density matrix of the analog state can also be obtained by using the equation for the self-field (vertex). This method, which is of independent interest, is described briefly below. The equation for the proper field $\tilde{v}^s(r)$ with allowance for $\Delta\varphi$ is

$$\tilde{v}^s = \frac{f'V}{\rho_0}\langle \tilde{G}^p \tilde{G}^n \tilde{v}^s\rangle, \quad (26)$$

where the brackets $\langle \ldots \rangle$ denote integration with respect to the fourth variable ϵ. The Green's function $\tilde{G}^{p(n)}(\epsilon)$ can be obtained by perturbation theory, for example,

$$(\delta G^p)_{\lambda\lambda'} \equiv (\tilde{G}^p - G^p)_{\lambda\lambda'} = (G^p \tau^p G^p)_{\lambda\lambda'}, \quad (27)$$

where $(G^p)_{\lambda\lambda'} = \delta_{\lambda\lambda'}(\epsilon - \epsilon_\lambda^p + i\alpha\epsilon_\lambda)^{-1}$ is the unperturbed Green's function.

The static vertex $\tau^{p(n)}(\Delta\varphi)$ which results from the action of the continuous field $\Delta\varphi$ on the system can be obtained from equations that are well known in the theory of finite Fermi systems. Taking (27) into account, the equation for $\tilde{v}^a$ takes the form

$$\tilde{v}^{(s)} = \frac{f'V}{\rho_0}\langle G^p G^n \tilde{v}^s\rangle + \frac{f'V}{\rho_0}\langle \delta G^p G^n v^s\rangle + \frac{f'V}{\rho_0}\langle G^p \delta G^n v^s\rangle. \quad (28)$$

In the case of the analog state $\tilde{v}^a = v^a + \delta v^a$. Therefore in first order in δv^a we have

$$\frac{\delta v^a(r)}{v^a} = \frac{f'V}{\rho_0}\left\langle G^p G^n \frac{\delta v^a}{v^a}\right\rangle + \frac{f'V}{\rho_0}\langle \delta G^p G^n\rangle + \frac{f'V}{\rho_0}\langle G^p \delta G^n\rangle. \quad (29)$$

The solution of the inhomogeneous equation (29) determines the correction to the density matrix

$$\frac{\delta\rho}{v^a} = \left\langle G^p G^n \frac{\delta v^a}{v^a}\right\rangle + \langle \delta G^p G^n\rangle + \langle G^p \delta G^n\rangle. \quad (30)$$

we can verify, for example, that the diagonal corrections to the density matrix, obtained from

formulas (30) and (24), coincide. If the corrections to the wave function or to the density matrix of the analog state are known, then we can estimate the "optical" width of the same state W_{opt}^{a}:

$$W_{opt}^{a} \approx \alpha_2^2 W_{opt}(\Delta\varepsilon_0) \ll W_{opt}(\Delta\varepsilon_0), \qquad (31)$$

where $W_{opt}(\Delta\epsilon_0)$ is the "optical" width of the single-qusiparticle level with energy $\Delta\epsilon_0$. Since $W_{opt} \sim (\Delta\epsilon_0)^2$, then the "optical" width of the analog state increases with increasing Z and A like $[(A - 2Z)/A]^2$.

5. CONCLUSION

In our preceding analysis we limited ourselves to states analogous to the ground state. If $|0_1\rangle$, $|0_2\rangle$ ... and E_1, E_2 ... are the wave functions and energies of the first, second, etc. excited states of the target nucleus, then, according to (6), we have $2(N - Z)^{-1/2} T^{(-1)} |0_1\rangle$, $2(N - Z)^{-1/2} T^{(-1)} |0_2\rangle$, ..., and $E_1 + \Delta E_C$, $E_2 + \Delta E_C$, ..., are the wave functions and energies of the corresponding analog states. The "optical" width of these states, as in the case of the state analogous to the ground state, is much smaller than the "optical" width of the single-quasiparticle levels at the corresponding energies. We note that the width of the single-quasiparticle states can be estimated from the imaginary part of the optical potential.

We did not take into account the Coulomb scattering amplitude of the protons in the Hamiltonian for the quasiparticles (1). To calculate the "optical" width of the analog state it is necessary, generally speaking, to take into account the Coulomb amplitude, since its contribution to the width can be comparable with (31).

The authors consider it their pleasant duty to thank V. M. Galitskiĭ, A. B. Migdal, B. L. Birbrair, and A. A. Lushnikov for useful discussions.

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Translated by J. G. Adashko
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