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THEORY OF A DOMAIN WALL IN METALS UNDER CONDITIONS OF THE DE HAAS–VAN ALPHEN EFFECT

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The relation between the field strength H and induction B is not linear under conditions of the de Haas–van Alphen effect and instability regions are possible on the H(B) state diagram.[3] A domain structure should exist in this case in samples with a nonzero magnetization factor. The domain wall whose shape is determined by the "inhomogeneity energy" is investigated. The induction distribution in the transition layer is determined and the surface tension at the domain interfaces is calculated.

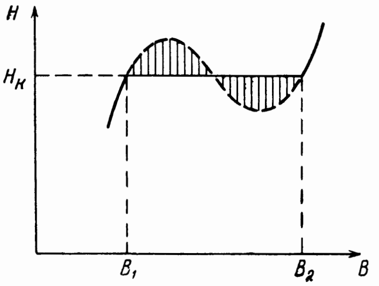
1. IN the theory of the de Haas–van Alphen effect (see, for example, [1,2]), the magnetization M is usually calculated as a function of the magnetic field H with the aid of the thermodynamic relation $M = -V^{-1}\partial\Omega/\partial H$ (V is the volume of the system). In this case the thermodynamic potential Ω is assumed equal to $-T \ln \mathrm{Sp} \exp(-\mathcal{H}/T)$, where T is the temperature and $\mathcal{H}$ is the Hamiltonian of the system in an external field H. This, strictly speaking, is incorrect,since the field acting on the charges is the induction B. If account is taken of the difference between H and B, and the corresponding substitution is made in the formulas derived in the theory, then Ω as a function of the induction B will, as before, have the meaning of the thermodynamic potential. The part of this quantity that depends on the magnetic field, calculated per unit volume, is equal to the work of the magnetization $(1/4\pi)\int \mathbf{H}d\mathbf{B}$, minus the field energy $B^2/8\pi$. Thus,

$$\frac{1}{V}\frac{\partial\Omega}{\partial\mathbf{B}} = -\mathbf{M}(\mathbf{B}) \qquad (1)$$

and the oscillations of the magnetization are described by the same formulas as before, in which, however, H must be replaced by B.[3]

The difference between H and B becomes significant only when the amplitude of the magnetization oscillations is not small compared with their period. In this case, for specified value of the field H, there can exist several values of the induction B[3]—see the figure. (The dependence of H on B is always single-valued.)

At the extremum points on the H(B) curve, the derivative $\partial M/\partial B$ is equal to $1/4\pi$, so that the condition for the occurrence of ambiguity is the inequality $(\partial M/\partial B)_{\max} > 1/4\pi$. The region of the curve between the points B_1 and B_2, which are determined from the condition that the shaded areas be equal, corresponds to the unstable states. The points of the extremum of the function H(B) are the stability limits with respect to an infinitesimally small homogeneous perturbation. In principle it can be assumed that the region of instability relative to the inhomogeneous perturbations is broader; however, it will be shown below that this is not the case, and that homogeneous phases with inductions B_1 and B_2 can exist, and the surface tension on the separation boundary is positive. Therefore, in a long cylinder situated in a longitudinal field, equal to H_k (see the figure), a first-order phase transition should take place, with an induction jump equal to $B_2 - B_1$, and in a sample having a nonzero demagnetization factor, a domain structure should exist.[3]



In a thin plate situated in a magnetic field H_0, the stratification into domains occurs in the interval $B_1 < H_0 < B_2$. The concentration of each of the phases is determined by the condition of conservation of the magnetic flux $c_1B_1 + c_2B_2 = H_0$. The period a of the domain structure is determined by the energy of the surface tension, which when calculated per unit area of the sample surface is proportional to l/a (l—thickness of the plate); it is also determined by the energy of emergence of the domains to the surface. The domain boundaries become bent near the surface, and the values of the induction B inside each domain differ from the values inside the sample both in magnitude and in direction. In this sense, the domain structure in our problem differs from the ferromagnetic structure, and is more readily similar to the intermediate state of superconductors. In connection with the fact that the connection between H and B is nonlinear, the corresponding magnetostatic problem does not reduce to a potential-theory problem. Therefore the calculation of the energy of emergence of the domains to the surface is a rather complicated problem. From dimensionality considerations we get only that this energy is proportional to a, so that $a \propto \sqrt{l}$.

2. In the transition region, the induction B changes from B_1 to B_2, and therefore the connection between H and B is not the same as in the homogeneous case. The inhomogeneity must be taken into account, for if the connection between H and B were to remain unchanged, then the thickness of the transition layer and the surface tension would be equal to zero.

Obviously, the value of the magnetization at any particular point is determined by the distribution of the induction in a region of dimensions on the order of the cyclotron radius r_0 of the electrons. We shall consider essentially the case when the thickness of the transition layer is large compared with r_0; this, as will be shown, takes place if the difference $B_2 - B_1$ is small compared with the period of the oscillations. In this case the derivative $\partial M_0/\partial B$[1] in the instability region differs little from $1/4\pi$, and $\partial H_0/\partial B \ll 1$. Putting for simplicity

$$4\pi M_0(B) = a \sin k\bar{B}, \tag{2}$$

where $\bar{B} = B - (B_1 + B_2)/2$ and $ak = 1 + \kappa^2$ with $\kappa^2 \ll 1$, we find that in the region of the instability

$$H_0(B) - H_k \approx -\kappa^2\bar{B} + k^2\bar{B}^3/6, \tag{3}$$

and the limits of this region are equal to

$$\bar{B}_{1,2} = \mp \kappa\sqrt{6}/k. \tag{4}$$

When $(\partial M_0/\partial B)_{\max}$ increases, the difference $B_2 - B_1$ approaches the value of the oscillation period.

The correction to the magnetization, which is connected with the inhomogeneity and which is proportional in the case of weak inhomogeneity to $\nabla^2 B$, will be calculated in the first approximation of perturbation theory, using a thermodynamic relation that generalizes (1) to include the inhomogeneous case:

$$\mathbf{M}(\mathbf{x}) = -\delta\Omega / \delta\mathbf{B}(\mathbf{x}). \tag{5}$$

To this end we put $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}'$, where $\mathbf{B}'$ is the inhomogeneous addition, and obtain Ω accurate to terms of second order in $\mathbf{B}'$. Such an approximation is perfectly sufficient for our purposes, and especially for an analysis of the stability against small perturbations.

We shall assume that $\mathbf{B}_0$ and $\mathbf{B}'$ are parallel to the z axis and that $\mathbf{B}$ depends only on y. The vector potential $\mathbf{A}$ is conveniently chosen as follows:

$$A_x = -B_0 y + A_1(y), \quad A_1(y) = -\int_0^y B'(y)\,dy, \qquad A_y = A_z = 0. \tag{6}$$

The single-electron Hamiltonian of the perturbation is equal to the sum of the terms of the first and second orders $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$:

$$\mathcal{H}^{(1)} = -\frac{e}{mc} A_1(y)\hat{p}_x - \frac{e^2 B_0}{mc^2} y A_1(y), \quad \mathcal{H}^{(2)} = \frac{e^2}{2mc^2} A_1^2(y), \tag{7}$$

where e and m are the charge and mass of the electron, c the velocity of light, and $\hat{p}_x$ is the operator of the x-component of the generalized momentum. We confine ourselves to the free-electron model and disregard the electron spin, the influence of which reduces only to a change of the phase of the oscillations.

The wave functions of the unperturbed problem are characterized by the quantum numbers n, p_x, and p_z. It is very important that the perturbation is diagonal in p_x and p_z. In particular,

$$\langle n, p_x, p_z | \mathcal{H}^{(1)} | n', p_x, p_z \rangle = \frac{|e|\omega_0}{c} \frac{1}{L_y} \sum_q A_1(q) e^{iqy_0} \langle n | y e^{iqy} | n' \rangle, \tag{8}$$

where $\omega_0 = |e|B_0/mc$ is the oscillation frequency, $y_0 = cp_x/|e|B_0$ its center, and $|n\rangle$ and $|n'\rangle$ are the wave functions of the oscillator with center at the point $y = 0$, while $A_1(y)$ is written in the Fourier representation (L_y is the y-dimension of the normalization volume).

To calculate the thermodynamic potential Ω, we use the formula of the thermodynamic perturbation theory:[4]

$$\Omega - \Omega_0 = \bar{V}_{\lambda\lambda} + \sum_{\lambda \neq \lambda'} \frac{|V_{\lambda\lambda'}|^2 w_\lambda}{E_\lambda^{(0)} - E_{\lambda'}^{(0)}} - \frac{1}{2T} (\overline{V_{\lambda\lambda}^2} - \bar{V}_{\lambda\lambda}^2), \tag{9}$$

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[1] Here and henceforth the quantities calculated for the homogeneous case are designated by the index zero.

where $E_\lambda^{(0)}$ and $E_{\lambda'}^{(0)}$ are the unperturbed energy levels of the system, $V_{\lambda\lambda'}$ are the matrix elements of the perturbation operator, w_λ is the probability of finding the system in the state λ, and the superior bar denotes averaging over the Gibbs ensemble.

The correction to Ω can be readily expressed in terms of the single-particle matrix elements. The first-order correction is of no interest, since it does not contain the derivatives of the inductions:

$$\Omega^{(1)} = -M_0 L_x L_z \int dy\, B'(y).$$

The second-order correction is equal to

$$\begin{aligned}\Omega^{(2)} = {} & \frac{N_0 e^2}{2mc^2} L_x L_z \int A_1^2(y)\, dy \\ & - \frac{e^2 m \omega_0^2}{c^2} \frac{L_x L_z}{(2\pi)^3} \int dp_z \int dq\, |A_1(q)|^2 \cdot \\ & \times \sum_n \Big\{ 2w_{n,\,p_z} \sum_{k=1}^{\infty} \frac{1}{k} [\,|\langle n|ye^{iqy}|n+k\rangle|^2 \\ & - |\langle n|ye^{iqy}|n-k\rangle|^2] \\ & + \frac{\omega_0}{T} w_{n,\,p_z} (1 - w_{n,\,p_z})\, |\langle n|ye^{iqy}|n\rangle|^2 \Big\}.\end{aligned} \tag{10}$$

The first term in this formula is connected with the diagonal matrix elements of the quantity $A_1^2(y)$ and N_0 is the number of electrons per unit volume. In the second term, $w_{n,\,p_z}$ are the single-particle occupation numbers.

In calculating the matrix elements in formula (10), it is convenient to express them in terms of Laguerre polynomials. Using the generating functions for the Hermite and Laguerre polynomials, we can readily obtain the relation

$$\langle n|e^{iqy}|n+k\rangle = \sqrt{\frac{n!}{(n+k)!}}\, i^k z^{k/2} e^{-z/2} L_n^k(z),$$
$$z = q^2/2m\omega_0,$$

hence,

$$\langle n|ye^{iqy}|n+k\rangle = \frac{q}{m\omega_0} i^{k-1} \sqrt{\frac{n!}{(n+k)!}} \frac{d}{dz}\{z^{k/2} e^{-z/2} L_n^k(z)\}.$$

Expanding the matrix elements in powers of q, we get

$$\begin{aligned}\sum_{k=1}^{\infty} \ldots = {} & \frac{1}{2m\omega_0} - \frac{q^2 \tilde{n}}{m^2\omega_0^2} + \frac{q^4}{4m^3\omega_0^3}\left(3\tilde{n}^2 + \frac{1}{4}\right) \\ & - \frac{q^6}{24m^3\omega_0^3}(5\tilde{n}^3 + 2\tilde{n}) + \ldots,\end{aligned} \tag{11}$$

$$\begin{aligned}|\langle n|ye^{iqy}|n\rangle|^2 = {} & \frac{q^2\tilde{n}^2}{m^2\omega_0^2} - \frac{q^4}{2m^3\omega_0^3}\left(\tilde{n}^3 + \frac{\tilde{n}}{4}\right) \\ & + \frac{q^6}{4m^4\omega_0^4}\left(\frac{5}{12}\tilde{n}^4 + \frac{1}{3}\tilde{n}^2 + \frac{1}{56}\right) + \ldots,\end{aligned} \tag{12}$$

where $\tilde{n} = n + 1/2$.

The off-diagonal elements in formula (10) cannot be calculated in the quasiclassical approximation, since the differences

$$|\langle n|ye^{iqy}|n+k\rangle|^2 - |\langle n|ye^{iqy}|n-k\rangle|^2$$

vanish in this approximation. In calculating the sum over n, this is very important, since the total contribution from the off-diagonal and diagonal matrix elements turns out to be in the quasiclassical approximation, the total derivative with respect to n of a function that vanishes when n = 0 and $\tilde{n} = \infty$, i.e., the resultant expression oscillates when B_0 is varied. In order of magnitude, the result is smaller than that obtained if we were to retain in (10) only the diagonal matrix elements, by a factor $\sqrt{\mu_0/\omega_0} \gg 1$ (μ_0—chemical potential of the electrons).

The zeroth term of the expansion (11) exactly cancels the term proportional to $\int A_1^2(y)dy$ in formula (10). The expansion terms proportional to q^2 yield

$$\Omega_0^{(2)} = \frac{e^2}{mc^2} \frac{L_x L_z}{(2\pi)^3} \int dp_z \int dq\, |B'(q)|^2 \sum_n \frac{\partial}{\partial n}(\tilde{n}^2 w_{n,p_z}).$$

We have used here the relations $|qA_1(q)| = |B'(q)|$ and

$$\frac{\omega_0}{T} w_{n,\,p_z}(1 - w_{n,\,p_z}) = -\frac{\partial w_{n,\,p_z}}{\partial n}.$$

Starting from the formula for the thermodynamic potential in the homogeneous case

$$\Omega_0 = -2Tm \frac{L_x L_y L_z}{(2\pi)^2} \int dp_z \sum_n \omega_0 \ln\left(1 + \exp\left[\frac{\mu_0 - \varepsilon_{n,\,p_z}}{T}\right]\right), \tag{13}$$

where $\epsilon_{n,\,p_z} = \tilde{n}\omega_0 + p_z^2/2m$ are the energy levels of the electrons, we can readily show that

$$\frac{\partial M_0}{\partial B} = -\frac{e^2}{m^2c^2} \frac{2m}{(2\pi)^2} \int dp_z \sum_n \frac{\partial}{\partial n}(\tilde{n}^2 w_{n,p_z}). \tag{14}$$

Therefore

$$\Omega_0^{(2)} = -\frac{L_x L_z}{2} \frac{\partial M_0}{\partial B} \int dy\, B'^2(y),$$

which is an addition to the thermodynamic potential neglecting the energy of the inhomogeneity. The latter is given by the next term of the expansion in q (actually the expansion is carried out over the parameter qr_0, $r_0 = cp_0/|e|B_0$, p_0 is the Fermi momentum):

$$\begin{aligned}\Omega_{\text{inhom}} = {} & -\frac{e^2}{2m^2c^2\omega_0} \frac{L_x L_z}{(2\pi)^3} \int dp_z \int dq\, q^2 |B'(q)|^2 \\ & \times \sum_n \frac{\partial}{\partial n}\left[\left(\tilde{n}^3 + \frac{\tilde{n}}{4}\right) w_{n,p_z}\right].\end{aligned} \tag{15}$$

In calculating the sum over n we must use the Poisson formula

$$\sum_{n} f(n+{}^{1}/_{2}) = \sum_{-\infty}^{\infty} (-1)^{l} \int_{0}^{\infty} dx\, f(x)\, e^{2\pi i l x}.$$

The non-oscillating term with $l = 0$ is missing in our case. The main contribution to Ω_{inhom} is made by the term $\tilde{n}^{3}\partial w/\partial n$, i.e., the diagonal matrix element in (10). An important role in (15) is played by large $\tilde{n} \approx \mu_0/\omega_0$ and small p_z. Equating (15) with (14) we can, without calculating the integrals in the Poisson formula, verify that

$$\Omega_{\text{inhom}} = \frac{1}{8}\frac{\partial M_0}{\partial B} r_0^2 \frac{L_x L_z}{2\pi} \int dq\, q^2 |B'(q)|^2$$
$$= \frac{1}{8}\frac{\partial M_0}{\partial B} r_0^2 L_x L_z \int dy \left(\frac{\partial B}{\partial y}\right)^2, \qquad (16)$$

from which it follows that in the case of weak inhomogeneity ($qr_0 \ll 1$) we have

$$H = H_0(B) - \pi \frac{\partial M_0(B)}{\partial B} r_0^2 \frac{\partial^2 B}{\partial y^2}. \qquad (17)$$

The energy of the inhomogeneity, as can be readily seen, can be negative (owing to the presence of the oscillating factor $\partial M_0/\partial B$). In this connection, an analysis for stability against infinitesimally small inhomogeneous perturbations is essential. To this end, we must calculate $\Omega^{(2)}$ for arbitrary values of qr_0.

The diagonal matrix elements in (10), which make the principal contribution to the terms with $l \neq 0$ in the summation in accord with the Poisson formula, can be calculated in the quasiclassical approximation:

$$|\langle n|ye^{iqy}|n\rangle|^2 = r^2 J_1^2(qr); \quad r^2 = 2n/m\omega_0, \quad qr \ll \mu_0/\omega_0,$$

where J_1 is the Bessel function. Derivation similar to the preceding ones, lead to the following general expression for $\Omega^{(2)}$:

$$\Omega^{(2)} = -2\frac{\partial M_0}{\partial B}\frac{L_x L_z}{2\pi} \int \frac{dq}{q^2 r_0^2} J_1^2(qr_0)\, |B'(q)|^2. \qquad (18)$$

In the analysis of the stability it is necessary to consider the thermodynamic potential $\tilde{\Omega}$, which is equal to

$$\tilde{\Omega} = \Omega + \int d\mathbf{x}(B^2/8\pi - HB/4\pi), \qquad (19)$$

and to vary B for a specified H. The first-approximation addition to Ω vanishes, and the second-approximation addition is equal to

$$\tilde{\Omega}^{(2)} = \frac{L_x L_z}{2\pi} \int dq\, |B'(q)|^2 \left\{\frac{1}{8\pi} - 2\frac{\partial M_0}{\partial B}\frac{1}{q^2 r_0^2} J_1^2(qr_0)\right\}. \qquad (20)$$

It is positive if $\partial M_0/\partial B > 1/4\pi$, since $J_1(x)/x \leq {}^{1}/_{2}$ (the equal sign holds when $x = 0$). Thus, the metastability limits are the points at which $\partial M_0/\partial B = 1/4\pi$.

It follows from (18) that the density of the thermodynamic potential is determined by the value of B averaged over the area of the orbit:

$$\bar{B}(y) = \int dy'\, K(y-y')\, B(y'). \qquad (21)$$

The Fourier transform of the function $K(y - y')$ is

$$K(q) = 2J_1(qr_0)/qr_0.$$

In the coordinate representation we get

$$K(y) = \begin{cases} \dfrac{2}{\pi r_0}\sqrt{1 - y^2/r_0^2} & y < r_0 \\ 0 & y > r_0 \end{cases}. \qquad (22)$$

The analysis of the instability against infinitesimally small perturbations can be generalized to include the case in arbitrary electron dispersion. Calculating in the quasiclassical approximation the diagonal matrix elements of the perturbation operator $\mathbf{v}\mathbf{A}_1$ (v—electron-velocity operator), we can readily show that the kernel K(y) in relation (21) is equal to

$$K(y) = S^{-1} D(y). \qquad (23)$$

Here D(y) is the dimension of the extremal orbit with center at the point $y = 0$ along the x axis at a fixed value of the coordinate y, and S is the area of the orbit. The Fourier transform K(q) reaches in this case also the maximum value (equal to unity) at the point $q = 0$. Replacing in formula (20) the quantity $2J_1(qr_0)/qr_0$ by the kernel K(q), we find that the metastability limits are the same as in the free-electron model.

3. So far we have investigated the case of perturbation parallel to the field $\mathbf{B}_0$. Let us consider now the case of a perturbation $\mathbf{B}'$ parallel to the x axis and dependent only on y. This is necessary in order to ascertain whether the direction of the induction **B** varies in the transition layer (in general, the direction of **B** can vary along an axis perpendicular to the domain boundary). It will be shown below that actually the direction of **B** remains constant.

Choosing the addition $\mathbf{A}_1$ to the vector potential in the form

$$A_{1x} = A_{1y} = 0, \quad A_{1z} = A_1(y) = \int_0^y B'(y)\,dy, \qquad (24)$$

we find that

$$\langle n, p_x, p_z|\mathcal{H}^{(1)}|n', p_x, p_z\rangle = -\frac{ep_z}{mcL_y}\sum_q A_1(q)\, e^{iqy_0}\langle n|e^{iqy}|n'\rangle. \qquad (25)$$

The second-order addition to the thermodynamic potential is

$$\Omega^{(2)} = -\frac{2e^2}{mc^2}\frac{L_xL_z}{(2\pi)^3}\int dq\,|A_1(q)|^2\int dp_z\,p_z^2$$
$$\times\sum_n\Big\{\sum_{k=1}^{\infty}\frac{1}{k}[|\langle n|e^{iqy}|n+k\rangle|^2-|\langle n|e^{iqy}|n-k\rangle|^2]$$
$$\times w_{n,p_z}-\frac{1}{2}\frac{\partial w_{n,\,p_z}}{\partial n}\quad(|\langle n|e^{iqy}|n\rangle|^2-1)\Big\}, \tag{26}$$

where we took into account the fact that

$$\frac{e^2}{mc^2}\frac{L_xL_z}{(2\pi)^3}\int dq\,|A_1(q)|^2\int dp_zp_z^2\sum_n\frac{\partial w_{n,\,p_z}}{\partial n}$$
$$=\frac{N_0e^2}{2mc^2}L_xL_z\int dy\,A_1^2(y).$$

Expanding the matrix elements in formula (26) in powers of q, we get

$$\sum_{k=1}^{\infty}\ldots=\frac{q^2}{2m\omega_0}-\frac{3\tilde n q^4}{8m^2\omega_0^2}+\frac{q^6}{m^3\omega_0^3}\left(\frac{5}{48}\tilde n^2+\frac{7}{576}\right)+\ldots, \tag{27}$$

$$|\langle n|e^{iqy}|n\rangle|^2=1-\frac{\tilde n q^2}{m\omega_0}+\frac{3\tilde n^2+{}^1/_4}{8m^2\omega_0^2}q^4$$
$$-\left(\frac{5}{72}\tilde n^3+\frac{7}{288}\tilde n\right)\frac{q^6}{m^3\omega_0^3}+\ldots, \tag{28}$$

i.e., the expression in the curly brackets in formula (27) is, in the approximation under consideration, the total derivative with respect to n, just as in the preceding case. The energy of the inhomogeneity is equal to (when $qr_0 \ll 1$)

$$\Omega_{\text{inhom}}=\frac{3e^2}{8m^3c^2\omega_0^2}\frac{L_xL_z}{(2\pi)^3}\int dq\,q^2|B'(q)|^2$$
$$\times\int dp_z\,p_z^2\sum_n\frac{\partial}{\partial n}\left[\left(\tilde n^2+\frac{1}{12}\right)w_{n,p_z}\right]. \tag{29}$$

Using the formula for the magnetization in the homogeneous case, which is obtained from (13) by differentiating with respect to B and integrating with respect to p_z by parts

$$M_0=\frac{|e|}{2\pi^2mc}\int dp_z\,p_z^2\sum_n\frac{\partial}{\partial n}(\tilde n w_{n,p_z}),$$

it is easy to show that

$$\Omega_{\text{inhom}}=\frac{3}{32}\bar\chi r_0^2L_xL_z\int dy\left(\frac{\partial B_x}{\partial y}\right)^2, \tag{30}$$

where $\bar\chi = M_0(B)/B$. Thus, the energy of the inhomogeneity in the case under consideration is much smaller (by a factor μ_0/ω_0) than in the case of a perturbation parallel to $\mathbf{B}_0$; this is natural, since the longitudinal susceptibility is much larger than the transverse one.

The field H_x in the presence of an inhomogeneous addition B_x should be equal to

$$H_x=H_0(B)\frac{B_x}{B}-\frac{3\pi}{4}\bar\chi r_0^2\frac{\partial^2B_x}{\partial y^2}. \tag{31}$$

In the transition layer $H_x = 0$ (this follows from Maxwell's equation curl $\mathbf{H} = 0$ under the assumption that all the quantities vary along the axis perpendicular to the separation boundary). The term with the derivative in (31) is small compared with B_x, i.e., the equation does not have non-zero slowly-varying solutions. Consequently, in calculating the domain wall it is necessary to use the formulas obtained in the preceding section.

4. If the jump of the induction is small compared with the period of the oscillations, then we can put in (17) $\partial M_0/\partial B = 1/4\pi$, and $H_0(B)$ can be replaced in accordance with formula (3). Recognizing that $H = H_k$, we obtain a simple differential equation for the determination of the function B(y):

$$-\varkappa^2\tilde B+\frac{k^2\tilde B^3}{6}=\frac{r_0^2}{4}\frac{d^2\tilde B}{dy^2}. \tag{32}$$

The boundary conditions are

$$\tilde B(\pm\infty)=\pm\varkappa\sqrt{6}/k. \tag{33}$$

Introducing the dimensionless variables u and ξ

$$u=\frac{k\tilde B}{\varkappa\sqrt{6}},\quad \xi=\frac{y}{d},\quad d^2=\frac{r_0^2}{8\varkappa^2}, \tag{34}$$

we rewrite relations (32) and (33) as follows:

$$-u+u^3=2u'',\quad u(\pm\infty)=\pm1, \tag{35}$$

after which we can readily obtain the solution

$$u=\operatorname{th}(\xi/2). \tag{36}$$

The surface tension Δ is determined by the relation

$$\Delta=\frac{1}{4\pi}\int_{-\infty}^{\infty}dy\left[\int_{B_1}^{B(y)}(H_0(B)-H_k)\,dB+\frac{r_0^2}{8}\left(\frac{\partial B}{\partial y}\right)^2\right], \tag{37}$$

which is conveniently transformed into[2]

$$\Delta=\frac{3r_0\varkappa^3}{4\sqrt{2}\,\pi k^2}\int_{-\infty}^{\infty}d\xi\left[\frac{1}{4}(1-u^2)^2+u'^2\right]. \tag{38}$$

After simple calculations we get

$$\Delta=\frac{r_0\varkappa^3}{\sqrt{2}\pi k^2}=\frac{r_0\varkappa(B_2-B_1)^2}{24\sqrt{2}\pi} \tag{39}$$

It is convenient to write the surface tension Δ in the form

$$\Delta=\frac{1}{24\pi}d\left(\frac{\partial H_0}{\partial B}\right)_{1,2}(B_2-B_1)^2. \tag{40}$$

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[2] The solution of (36) can be obtained by minimizing this functional under the condition $u(\pm\infty) = \pm1$.

The obtained result has a simple physical meaning, since the quantity $(\partial H_0/\partial B)_{1,2}(B_2 - B_1)^2$ is the characteristic difference of the energy densities of the unstable state and of the state with induction B_1 or B_2.

In the case when $B_2 - B_1$ is not small compared with the period of the oscillations, including in the limit when $(\partial M_0/\partial B)_{\max} \gg 1$, the dimension of the domain wall d is of the order of r_0. In this case formula (40) gives the correct order of magnitude of the surface energy.

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