GALVANOMAGNETIC EFFECTS IN STRONG ELECTRIC FIELDS DURING NONELASTIC ELECTRON SCATTERING

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A theory of galvanomagnetic effects is considered for the case when the predominant mechanism of electron scattering is spontaneous emission of optical phonons (of frequency ω_0). The elastic scattering mechanisms (by impurity and acoustic phonons) are assumed to be weak. A situation of this type may be encountered at low temperatures $T \ll \omega_0$. The electric field strengths are bounded by the condition $E^- \ll E \ll E^+$, where the characteristic field strengths E^+ and E^- are related to the predominant and elastic scattering, respectively. Owing to the condition $E \ll E^+$, an electron which has acquired an energy $\epsilon = \omega_0$ during the acceleration emits almost immediately an optical phonon and stops moving. Together with the condition $T \ll \omega_0$ this means that almost all electrons are in a "passive" momentum-space region as defined by the condition $\epsilon(\mathbf{p}) < \omega_0$. Since $E \gg E^-$, acceleration of an electron from $\epsilon = 0$ to $\epsilon = \omega_0$ is not interrupted by elastic scattering and hence the dynamics of motion of an electron along the trajectory inside the passive region is not masked by collisions with the lattice. For those values of E/H for which the topology of the trajectories in the passive region changes, the dependence of the current on E and H possesses singularities (function and derivative discontinuities).

INTRODUCTION

THE most effective scattering mechanism in pure semiconductors in which a strong interaction takes place with optical phonons (of frequency ω_0) at low temperatures $T < \omega_0$ may be the spontaneous emission of these phonons (described by a relaxation time $\tau^+ \equiv \tau^0$). The remaining scattering mechanisms (by impurities or by acoustic phonons, and also compound scattering by optical phonons [1,2]) are almost elastic (corresponding to a relaxation time $\tau^- \equiv \tau$). Assuming that $\tau^+ \ll \tau^-$, let us determine the characteristic fields $E_0^{\pm}$ from the conditions $eE_0^{\pm}\tau^{\pm} = p_0$, where $p_0 = \sqrt{2m\omega_0}$; here, obviously, $E_0^- \ll E_0^+$. We confine ourselves to the field interval $E_0^- \ll E \ll E_0^+$ [3]. Owing to the condition $E \ll E_0^+$, an electron which has acquired under the influence of the field an energy $\epsilon = \omega_0$ emits an optical phonon almost instantaneously and therefore comes practically to rest. For this reason, a small fraction of the electrons, $(E/E_0^+)^{2/3}$, is located in the active region $\epsilon > \omega_0$. On the other hand, owing to the condition $E \gg E_0^-$, the stopped electron acquires an energy $\epsilon = \omega_0$ after a time $\tau_E \sim p_0/eE \ll \tau^-$, that is, without having time to experience elastic scattering. Therefore the dynamics of the electron-motion over the trajectories in the passive region $\epsilon(\mathbf{p}) < \omega_0$ is strongly pronounced, and the distribution acquires a "needle-like" directivity [4,5], being concentrated near the principal trajectory passing through the point $\mathbf{p} = 0$ [3]. An increased distribution anisotropy was observed also experimentally [6,7].

Under the foregoing conditions, galvanomagnetic phenomena in crossed fields have a singularity at a magnetic field value $H_0 = (2mc/p_0)E$, when the main trajectory becomes closed [3]. The meaning of this singularity lies in the fact that when $H > H_0$ the electron that is stopped after emission of an optical phonon cannot acquire an energy $\epsilon = \omega_0$ when it becomes accelerated in the electric field E. This hinders the emission of optical phonons and results in a "cutoff" of the dissipative effect.

We develop in this paper a theory of galvanomagnetic phenomena under these conditions for an arbitrary electron dispersion law (with one minimum). In addition, unlike [3], we consider the influence of elastic scattering within the passive region, a scattering important when $H > H_0$.

If the dispersion is anisotropic, H_0 depends on

the orientation of the fields **E** and **H** relative to the equal-energy surfaces. Therefore, in the presence of several anisotropic minima, the singularities of the galvanomagnetic phenomena should be observed at several values of the magnetic fields. These effects, however, are made complicated by the redistribution of the electrons between the minima, which takes place in strong electric fields [8]. This question will be considered by us in a later paper.

We note that if we orient the fields **E** and **H** in the same manner relative to all the minima, then they can be combined into one. In n-Ge this takes place when **H** ∥ [001] and **E** ∥ [100], but in n-Si this is impossible.

Galvanomagnetic effects in strong electric fields were considered, for an anisotropic dispersion law, by a number of workers [9-13], but only for the case when elastic scattering is the dominating mechanism.

1. KINETIC EQUATION

We shall assume that the optical phonon is emitted instantaneously ($\tau^0 = 0$), once the electron acquires an energy $\epsilon = \omega_0$. Then all the electrons are in the passive region Ω defined by the condition $\epsilon(\mathbf{p}) < \omega_0$.

The stationary distribution function $f(\mathbf{p})$ (normalized to the concentration n) in the region Ω is governed by the following kinetic equation:

$$\frac{\partial}{\partial t} f(\mathbf{p}) = F(f|\mathbf{p}) + S(f|\mathbf{p}) + \delta(\mathbf{p}) I(f) = 0. \quad (1.1)$$

Here the field term F describes the motion under the influence of the electric and magnetic fields:

$$F(f|\mathbf{p}) = -\operatorname{div} \mathbf{J}(f|\mathbf{p}),$$

$$\mathbf{J}(f|\mathbf{p}) = f(\mathbf{p}) \dot{\mathbf{p}}(\mathbf{p}), \quad \dot{\mathbf{p}}(\mathbf{p}) = e\mathbf{E} + \frac{e}{c}[\mathbf{v}(\mathbf{p})\mathbf{H}], \quad (1.2)$$

where **J** is the "current" in momentum space, connected with the motion over the trajectories in this space under the influence of the fields with "velocity" **p**. The scattering inside Ω is described by the term

$$S(f|\mathbf{p}) = \int_\Omega (d\mathbf{p}') f(\mathbf{p}') W(\mathbf{p}', \mathbf{p}) - \frac{f(\mathbf{p})}{\tau(\mathbf{p})}, \quad (1.3)$$

where the lifetime for this scattering is

$$\frac{1}{\tau(\mathbf{p})} = \int_\Omega (d\mathbf{p}') W(\mathbf{p}, \mathbf{p}'). \quad (1.4)$$

The last term describes optical-phonon emission such that the electrons are stopped, that is, they end up at the point p = 0. In this term

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*[v(p)H] ≡ v(p) × **H**.

$$I(f) = \int_{\Sigma_+} d\sigma \, J_n(f|\mathbf{p}) \quad (1.5)$$

there is a flux of electrons that reach, under the influence of the fields, the boundaries Σ of the passive region Ω, that is, they acquire an energy $\epsilon = \omega_0$ and become capable of emitting the optical phonon. Σ_+ is that part of the surface Σ where the projection of **J** on the outward normal is $J_n > 0$. Integrating (1.1) over Ω and noting that the scattering S conserves the number of particles inside Ω, we find, using (1.5), that

$$f(\mathbf{p}) = 0, \quad \mathbf{p} \in \Sigma_-, \quad (1.6)$$

where Σ_- is that part of Σ on which $J_n < 0$.

Since scattering inside Ω is rare in the field interval under consideration, we put, assuming S to be small,

$$f(\mathbf{p}) = f^0(\mathbf{p}) + f'(\mathbf{p}), \quad (1.7)$$

where f' is a small correction. Substituting (1.7) in (1.1) and discarding small terms of second order, we get

$$F(f^0|\mathbf{p}) + \delta(\mathbf{p}) I(f^0) = 0, \quad (1.8a)$$

$$F(f'|\mathbf{p}) + \delta(\mathbf{p}) I(f') = -S(f^0|\mathbf{p}). \quad (1.8b)$$

These equations are conveniently solved by introducing a coordinate system connected with the trajectories in momentum space [14], namely s—path length along the trajectory, and α—parameters defining the trajectory (integrals of motion). In the new variables $(d\mathbf{p}) = g(\alpha s)(d\alpha) ds$ is the volume element, $g(\alpha) = \int ds \, g(\alpha s)$ is the density of states near the trajectory, and $n(\alpha s) = g(\alpha s) f(\alpha s)$ is the distribution function. The velocity of motion along the trajectory is $\dot{s}(\alpha s) = |\dot{\mathbf{p}}(\mathbf{p})|$. Henceforth $\hat{\alpha}$ will correspond to the principal trajectory passing through the point p = 0; this point corresponds to s = ŝ. The other trajectories are called secondary. The values s = s$_+$ and s = s$_-$ correspond to the points of intersection of the trajectories wit the surfaces Σ_+ and Σ_-. In this notation we have

$$F(f|\mathbf{p}) = -\frac{1}{g(\alpha s)} \frac{\partial}{\partial s}[n(\alpha s)\dot{s}(\alpha s)], \quad (1.9)$$

$$\delta(\mathbf{p}) = \frac{1}{g(\alpha s)} \delta(\alpha - \hat{\alpha}) \delta(s - \hat{s}). \quad (1.10)$$

The Heaviside step function will be denoted $\theta(s)$.

The character of the solution of the system (1.8), as will be shown below, depends on the topology of the trajectories inside the region Ω. When H = 0 all the trajectories are straight lines starting at Σ_- and ending at Σ_+. With increasing H, they become curved, and for sufficiently large H some trajectories may close inside Ω. From the point of view of the topology, the following two

circumstances are of importance: (i) the principal trajectory closed in Ω, and (ii) there are secondary trajectories closed in Ω. The point is that closed trajectories are "traps" of a sort, and an electron falling on such a trajectory is incapable of emitting an optical phonon. The special position of the principal trajectory is connected with the fact that all electrons that have emitted optical phonons end up subsequently on the principal trajectory. The part of the region Ω occupied by the closed trajectories will be denoted Ω_c, and that occupied by open trajectories Ω_a. The three resultant topological situations are illustrated in the figure (using for simplicity the usual dispersion law).
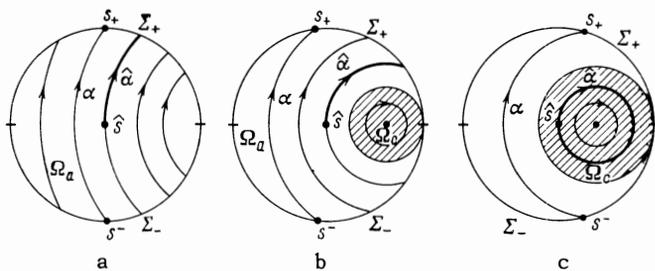
Topology of trajectories in the passive region in the presence of a magnetic field: a—no closed trajectory; b—there are closed trajectories but the principal trajectory is open; c—principal trajectory closed.

Before we proceed to solve (1.8), it is convenient to introduce beforehand the quantities that will be encountered. The period of revolution on the closed trajectory α is

$$\tau_F(\alpha) = \oint \frac{ds}{\dot{s}(\alpha s)} \qquad (1.11)$$

The period of the acceleration cycle on half of the principal trajectory (if the latter is not closed) is

$$\tau_F(\hat{\alpha}) = \int_{\hat{s}}^{s_+} \frac{ds}{\dot{s}(\hat{\alpha}s)}. \qquad (1.12)$$

The probability of transition between trajectories, averaged over the points of the initial trajectory α and summed over the points of the final trajectory α', is

$$W(\alpha, \alpha') = \int \frac{ds}{\tau_F(\alpha)\dot{s}(\alpha s)} \int ds' g(\alpha's') W(\alpha s, \alpha's'). \qquad (1.13)$$

If the trajectory α is closed, then the integration with respect to s must be understood in the sense of (1.11), and τ_F has also the same sense. If α pertains to a principal unclosed trajectory, then integration with respect to s and τ_F must be understood in the sense of (1.12). Integration with

respect to s' must be understood as $\oint_{s_+} ds' \ldots$ if the trajectory α' is closed and as $\int_{s_-}^{s_+} ds' \ldots$ if it is open.

The lifetimes on the trajectory α with respect to going off to closed or unclosed trajectories, that is, into the region Ω_c or Ω_a, are as follows:

$$\frac{1}{\tau_c(\alpha)} = \int_{\Omega_c} (d\alpha') W(\alpha, \alpha'), \qquad (1.14)$$

$$\frac{1}{\tau_a(\alpha)} = \int_{\Omega_a} (d\alpha') W(\alpha, \alpha'). \qquad (1.15)$$

The total lifetime on the trajectory α is

$$\frac{1}{\tau(\alpha)} = \frac{1}{\tau_c(\alpha)} + \frac{1}{\tau_a(\alpha)}. \qquad (1.16)$$

The number of electrons on the trajectory α is

$$n(\alpha) = \int ds\, n(\alpha s). \qquad (1.17)$$

The integration with respect to s in the last formula must be understood in the same way as the integration with respect to s' in (1.13). We denote, in addition, by n_a and n_c the number of electrons in Ω_a and Ω_c, and by $\hat{n}$ and $\tilde{n}$ the number of electrons on the principal trajectory and on all the secondary trajectories.

2. SOLUTION OF KINETIC EQUATION

We now proceed to solve (1.18), considering separately the different topological cases.

<u>All trajectories open.</u> Integrating (1.8a) from s_- to s with allowance for (1.6), and determining $I(f^0)$ from the normalization condition, we get

$$n^0(\alpha s) = n\delta(\alpha - \hat{\alpha})\theta(s - \hat{s}) \frac{1}{\tau_F(\hat{\alpha})\dot{s}(\alpha s)}. \qquad (2.1)$$

<u>Principal trajectory open.</u> Integrating (1.8a) along a closed trajectory, we have

$$n^0(\alpha s) = \tilde{n}^0(\alpha) \frac{1}{\tau_F(\alpha)\dot{s}(\alpha s)}, \qquad \alpha \in \Omega_c. \qquad (2.2)$$

The tilde denotes here that the distribution pertains to the secondary trajectories. Integrating the same equation along an open trajectory, we obtain

$$n^0(\alpha s) = \hat{n}^0\delta(\alpha - \hat{\alpha})\theta(s - \hat{s}) \frac{1}{\tau_F(\hat{\alpha})\dot{s}(\hat{\alpha}s)}, \qquad \alpha \in \Omega_a. \qquad (2.3)$$

The quantities $\tilde{n}^0(\alpha)$ and $\hat{n}^0$ are not determined by (1.8a), since they are invariants of the operator in the left side of (1.8). To determine them we must consider the conditions under which the next approximation, (1.8b), can be solved. Integrating (1.8b) along a closed trajectory, we get

$$\oint ds\, g(\alpha s) S(f^0|\alpha s) = 0, \qquad \alpha \in \Omega_c. \qquad (2.4)$$

Integrating the sum equation over the entire region Ω_a we get

$$\int_{\Omega_a} (d\alpha) \int ds\, g(\alpha s) S(f^0 | \alpha s) = 0. \qquad (2.5)$$

Substituting (1.3) in the last two formulas and using the notation introduced above, we obtain a system of equations with which to determine $\tilde{n}^0(\alpha)$ and $\hat{n}^0$:

$$\int_{\Omega_c} (d\alpha') \tilde{n}^0(\alpha') W(\alpha', \alpha) - \frac{\tilde{n}^0(\alpha)}{\tau(\alpha)} + \hat{n}^0 W(\hat{\alpha}, \alpha) = 0, \quad (2.6a)$$

$$\int_{\Omega_c} (d\alpha') \frac{\tilde{n}^0(\alpha')}{\tau_a(\alpha')} - \frac{\hat{n}^0}{\tau_c(\alpha)} = 0. \qquad (2.6b)$$

Principal trajectory closed. Integrating (1.8a) along an unclosed trajectory with allowance for (1.6) we get

$$n^0(\alpha s) = 0, \quad \alpha \in \Omega_a. \qquad (2.7)$$

Integrating the same equation in the entire region Ω_c we find that $I(f^0) = 0$. From this we get

$$n^0(\alpha s) = n^0(\alpha) \frac{1}{\tau_F(\alpha) \dot{s}(\alpha s)}, \quad \alpha \in \Omega_c. \qquad (2.8)$$

Integrating (1.8b) along a closed trajectory, we obtain

$$\oint ds\, g(\alpha s) S(f^0 | \alpha s) = -\delta(\alpha - \hat{\alpha}) I(f'). \qquad (2.9)$$

Integrating the same equation over the entire region of Ω_a with allowance for (1.6) we get

$$I(f') = \int_{\Omega_a} (d\alpha) \int ds\, g(\alpha s) S(f^0 | \alpha s). \qquad (2.10)$$

Eliminating $I(f')$ from the last two equations and substituting in the resultant equation the expression for S as given by (1.3), we obtain an equation for the determination of $n^0(\alpha)$:

$$\int_{\Omega_c} (d\alpha') n^0(\alpha') W(\alpha', \alpha) - \frac{n^0(\alpha)}{\tau(\alpha)}$$

$$+ \delta(\alpha - \hat{\alpha}) \int_{\Omega_c} (d\alpha') \frac{n^0(\alpha')}{\tau_a(\alpha')} = 0. \qquad (2.11)$$

It is natural to seek a solution of this equation in the form

$$n^0(\alpha) = \hat{n}^0 \delta(\alpha - \hat{\alpha}) + \tilde{n}^0(\alpha), \qquad (2.12)$$

where $\tilde{n}^0$ is the distribution over the secondary closed trajectories. Substituting (2.12) in (2.11) and separating the singular and regular parts, we obtain two equations that coincide formally with (2.6).

Equations (2.1)–(2.3), (2.7), and (2.8) show that

in the zeroth approximation (that is, when $\tau_F = 0$), the distribution $n(\alpha s)$ differs from zero only on the closed trajectories (if they exist) and on half of the principal trajectory (if it is unclosed). This is physically obvious, for only such trajectories are "stable" relative to the emission of optical phonons, no emission takes place at all on closed trajectories, and on the principal trajectory emission leads to a return to the middle of the same trajectory. Such trajectories (or their parts) will be called invariant. The number of electrons on them is an invariant of the fast process comprising motion under the influence of the field and emission of optical phonons. An electron which is on an invariant trajectory (for example, as a result of elastic scattering) is "carried away" instantaneously (within a time τ_F) by the field to the surface Σ_+, emits an optical phonon, and ends up on the principal trajectory.

The distribution of the electrons along the invariant trajectories is determined by the quantity $\dot{s}(\alpha s)$: The probability of finding the electron at point s is inversely proportional to the velocity with which it "jumps through" this point while moving along the trajectory.

The distribution of the electrons among the invariant trajectories is determined by Eqs. (2.6) and (2.11), which are (as always in such cases) balance equations for the invariants of the fast process relative to the slow processes. For this reason, they contain the probabilities (1.13) averaged over the fast process of motion along the invariant trajectory. From this point of view there is an elementary interpretation for Eq. (2.6a), which is the balance equation for the particles on the secondary closed trajectory α. In the interpretation of (2.6b), which is the balance equation for the principal trajectory α, attention should be called to the following: The terms representing departure contain not the total departure probability, but only the probability of departure to the closed trajectories in Ω_c, since an electron entering into Ω_a returns immediately to the principal trajectory. The arrival terms contain the probability of departure from the closed trajectory in Ω_a, since an electron that enters Ω_a ends up immediately on the principal trajectory.

We note also that (2.6) and (2.11) are homogeneous with respect to W, and therefore the distribution of the trajectories does not depend on the intensity of elastic scattering. This is connected with the fact that we are considering a stationary mode.

After f^0 has been obtained, f' is determined from (1.8b):

$$n'(as) = \frac{1}{\dot{s}(as)} \int_{s_-}^{s} ds' \, g(as') S(f^0|as'), \quad a \in \Omega_a, \quad a \neq \hat{a},$$

$$\dot{n}'(as) = Cn^0(as), \quad a \in \Omega_c, \quad a = \hat{a}. \quad (2.13)$$

The constant C is determined from the normalization condition and it is obvious that $C < 0$. From (2.13) we get the following estimate for the "background" on the secondary trajectories:

$$n' \sim \frac{1}{\dot{s}} p_0 \frac{f_0}{\tau} \sim \frac{\tau_F}{\tau} n^0 \ll n^0. \quad (2.14)$$

We see from this that $|C| \sim \tau_F/\tau \ll 1$, that is, scattering inside Ω causes the distribution on the invariant trajectories to decrease somewhat in amplitude but without change in form.

3. GALVANOMAGNETIC PHENOMENA

To calculate the current, it is convenient to represent it in the form

$$\mathbf{j} = e \int (da) n(a) \mathbf{v}(a), \quad (3.1)$$

where we have introduced the average velocity on the trajectory

$$\mathbf{v}(a) = \frac{1}{n(a)} \int ds \, n(as) \mathbf{v}(as). \quad (3.2)$$

If we consider the distribution $n(as)$ in the zeroth approximation then, as seen from Sec. 2, averaging over the distribution along the trajectory can be replaced by averaging over the time for motion along the trajectory. Because of this, the quantities $\mathbf{v}(a)$ can be calculated in pure dynamic fashion. The average velocity for the principal trajectory (if it is unclosed) will be denoted $\mathbf{v}(\hat{a}) \equiv \mathbf{v}_a$; it contains the components $\mathbf{v}_{aL}$ and $\mathbf{v}_{aT}$ parallel and perpendicular to $\mathbf{H}$. The average velocity for the closed trajectory, as shown in [14], is

$$\mathbf{v}(a) = \mathbf{v}_L(a) + \mathbf{v}_T(a), \quad a \in \Omega_c,$$

$$\mathbf{v}_T(a) = v_D \mathbf{d}, \quad v_D = c\frac{E}{H}, \quad \mathbf{d} = [\mathbf{eh}], \quad (3.3)$$

where the unit vectors $\mathbf{e}$ and $\mathbf{h}$ determine the directions of $\mathbf{E}$ and $\mathbf{H}$. The independence of $\mathbf{v}_T$ of the form of the trajectory makes it possible to calculate $\mathbf{j}_T$ without knowing the detailed distribution of the electrons over the closed trajectories, and knowing only their total number:

$$\mathbf{j}_T = en_a \mathbf{v}_{aT} + en_c \mathbf{v}_D = en[(1-\eta)\mathbf{v}_{aT} + \eta \mathbf{v}_D], \quad \eta = n_c/n. \quad (3.4)$$

Knowledge of this distribution is essential for the calculation of $\mathbf{j}_L$. If all the trajectories are open, then $\eta \equiv 0$; if the principal trajectory is closed, $\eta \equiv 1$; if only the secondary trajectories are closed, then $0 < \eta < 1$. Therefore the function $\eta(\mathbf{E}, \mathbf{H})$ should have discontinuities (of either the functions or the derivatives) at those values of $\mathbf{E}$ and $\mathbf{H}$ for which the topology of the trajectories changes. Similar discontinuities are possessed by the $n(\alpha)$ distribution.

The discontinuities of η lead to discontinuities in the dependence of $\mathbf{j}$ on $\mathbf{E}$ and $\mathbf{H}$. The character of these discontinuities depends on the details of the dispersion law and on the singularities of scattering within Ω. Unfortunately, the integral equations of the preceding sections cannot be solved for any realistic scattering law. Therefore it is instructive to consider the special case when

$$W(\mathbf{p}, \mathbf{p}') = \frac{1}{\tau\Omega} = \text{const}, \quad \mathbf{p}, \mathbf{p}' \in \Omega. \quad (3.5)$$

Such a scattering, of course, is not elastic.

We then get from (1.13)

$$W(a, a') = \frac{1}{\tau} \frac{g(a')}{\Omega}, \quad (3.6)$$

and from (1.14)—(1.16)

$$\frac{1}{\tau_c(a)} = \frac{1}{\tau}\frac{\Omega_c}{\Omega} \quad \frac{1}{\tau_a(a)} = \frac{1}{\tau}\frac{\Omega_a}{\Omega}, \quad \frac{1}{\tau(a)} = \frac{1}{\tau}. \quad (3.7)$$

Under these assumptions, the system (2.6) can be easily solved, and we obtain

$$\tilde{n}(a) = ng(a)/\Omega, \quad \hat{n} = n\Omega_a/\Omega. \quad (3.8)$$

From this we get

$$n_c = \tilde{n} = n\Omega_c/\Omega, \quad n_a = \hat{n}, \quad \eta = \Omega_c/\Omega. \quad (3.9)$$

In solving (2.11) in the form (2.12) we obtain similar expressions $\tilde{n}(\alpha)$, $\tilde{n}$, and $\hat{n}$. Now, however,

$$n_c = \hat{n} + \tilde{n} = n, \quad n_a = 0, \quad \eta = 1. \quad (3.10)$$

These results show that in all three topological cases the distribution of the electrons over the trajectories can be represented in the following manner: first, all the electrons are uniformly distributed over Ω, and then those of them which turn out to be in Ω_a fall on the principal trajectory.

Thus, for a scattering of type (3.5) the singularities of $\eta(\mathbf{E}, \mathbf{H})$ are determined by the ratio $\Omega_c(\mathbf{E}, \mathbf{H})/\Omega$. It is obvious that when closed secdary trajectories appear the function η is contin-

uous, for in this case Ω_c increases from zero, so that in such a change of the topology only a discontinuity of the derivatives of the function η is possible. When the principal trajectory is closed we have $\Omega_c|_0 < \Omega$, and in general $\Omega_a|_0 = \Omega - \Omega_c|_0$ is of the order of $\Omega_c|_0$; therefore, in this change of topology the function η has a discontinuity whose magnitude is the order of the value of the function.

The results are valid, in order of magnitude, also in the case when $W(\mathbf{p}, \mathbf{p}')$ is not constant but has no higher-order singularities. The elastic scattering considered by us has a singularity at $\epsilon \approx \epsilon'$. Actually, however, the absence of singularities of $W(\alpha, \alpha')$ is significant; when E is sufficiently strong the trajectories cross many equal-energy surfaces, and the elastic singularity becomes insignificant after averaging in (1.13).

An exception is soft scattering, which has a singularity at $\mathbf{p} \approx \mathbf{p}'$. In this case $W(\alpha, \alpha')$ can have a singularity at $\alpha \approx \alpha'$, that is, a noticeable transition probability exists only between closely-lying trajectories. When the principal trajectory is open and is far from the region Ω_c, the transition from the principal trajectory to Ω_c is difficult; at the same time, the transition from Ω_c to the principal trajectory is not difficult and can be effected via the trajectories lying on the boundary of the regions Ω_c and Ω_a, since falling into Ω_a is equivalent to a transfer to the principal trajectory. In this case we should have $n_c \ll \hat{n}$. Therefore for soft scattering $\eta = 0$ if the principal trajectory is not closed and $\eta = 1$ if it is closed.

The foregoing can be corroborated by calculation is one chooses the probability of the soft scattering in the form

$$W(\mathbf{p}, \mathbf{p}') = \frac{1}{\tau} (2\pi\lambda^2)^{-3/2} \exp\left\{ -\frac{|\mathbf{p} - \mathbf{p}'|^2}{2\lambda^2} \right\}, \quad (3.11)$$

where the average change of momentum in the scattering is $\lambda \ll p_0$. In this case, for the ordinary dispersion law we can calculate the probability $W(\alpha, \alpha')$ and estimate the times $\tau_c(\hat{\alpha})$ and $\tau_a(\alpha')$ which enter in (2.6b). It turns out here that

$$\frac{1}{\tau_c(\hat{\alpha})} \sim \lambda \exp\left\{ -\left(\frac{\hat{p}}{\lambda}\right)^2 \right\}, \quad \left\langle \frac{1}{\tau_a(\alpha')} \right\rangle \sim \lambda. \quad (3.12)$$

We have indicated only the dependence on λ as $\lambda \to 0$; the angle brackets denote averaging over Ω_c; $\hat{p}$ is a certain momentum of the order of p_0, indicating the removal of the principal trajectory from the region Ω_c. It is clear from (3.12) and (2.6b) that as $\lambda \to 0$ we have $\tilde{n}^0/\hat{n}^0 \to 0$, that is, that all the electrons are on the principal trajectory.

4. PARABOLIC DISPERSION LAW

Let us consider a parabolic dispersion law with an effective-mass tensor $\hat{m}$; we introduce the following average masses: state-density m_d, cyclotron m_h, and ohmic m_e:

$$m_d{}^3 = m_1 m_2 m_3,$$

$$\frac{1}{m_h{}^2} = \frac{h_1{}^2}{m_2 m_3} + \frac{h_2{}^2}{m_3 m_1} + \frac{h_3{}^2}{m_1 m_2}, \quad \frac{1}{m_e} = \frac{e_1{}^2}{m_1} + \frac{e_2{}^2}{m_2} + \frac{e_3{}^2}{m_3}. \quad (4.1)$$

The indices 1, 2, and 3 denote here projections on the principal axes of the tensor m.

The electron trajectories in momentum space are best determined from the following conditions [14]:

$$\varepsilon^*(\mathbf{p}) \equiv \varepsilon(\mathbf{p}) - \mathbf{p v}_D = \text{const}, \quad p_h \equiv \mathbf{p h} = \text{const}. \quad (4.2)$$

For a parabolic dispersion law the surfaces $\epsilon^*(\mathbf{p})$ are ellipsoids which coincide with the equal-energy surfaces, but are displaced by a vector $\mathbf{p}_D = \hat{m}\mathbf{v}_D$. Therefore the trajectories constitute a two-parameter family of ellipses of equal eccentricity, the planes of which are perpendicular to $\mathbf{h}$; the centers of all the ellipses lie on a straight line passing through $\mathbf{p}_D$ in the direction of $\hat{m}\mathbf{h}$.

The topology of the trajectories inside Ω, as can be readily shown from elementary geometrical considerations, is determined by comparison of v_D with $v_0 = (2\omega_0/m_0)^{1/2}$, where $m_0 = m_h{}^2/m_e$. If $v_D > v_0$, then the line of centers passes outside the region Ω, and therefore there are no closed trajectories in the passive region. When $v_D = v_0$, the line of the centers is tangent to the surface Σ; when $v_D < v_0$, the line of centers crosses Ω, and there are closed trajectories in the passive region. The principal trajectory is closed if $v_D < v_0/2$.

The average velocity on the principal trajectory can be readily calculated from the equation of motion; this yields

$$\mathbf{v}_a = \mathbf{v}_a^{(+)} + \mathbf{v}_a^{(-)}, \quad (4.3)$$

where

$$\mathbf{v}_a^{(+)} = \tfrac{1}{2} v_0 m_h (\hat{m}^{-1} \mathbf{e}) L(\varkappa), \quad (4.4a)$$

$$\mathbf{v}_a^{(-)} = \tfrac{1}{2} v_0 \frac{m_h{}^2}{m_d{}^3} [\mathbf{e}, \hat{m}\mathbf{h}] T(\varkappa), \quad (4.4b)$$

and we have introduced the functions of the dimensionless parameter

$$L(\varkappa) = \frac{\varkappa}{\arcsin \varkappa},$$

$$T(\varkappa) = \frac{1}{\varkappa} - \frac{\sqrt{1-\varkappa^2}}{\text{arc sin } \varkappa}, \quad \varkappa = \frac{v_0}{2v_D} = \frac{v_0}{2c}\frac{H}{E}. \quad (4.5)$$

Here $\mathbf{v}^+(a)$ is that part of the average velocity which is even relative to inversion of the magnetic field. It contains components that are longitudinal and transverse with respect to $\mathbf{e}$, but the transverse component is connected not with the Lorentz force but with the anisotropy of the crystal and determines the Sasaki effect [15]. Connected with the Lorentz force is the odd term $\mathbf{v}_a^{(-)}$. The average velocity on the closed trajectory with parameters $\alpha \equiv (\epsilon^* \mathbf{ph})$ is

$$\mathbf{v}(\alpha) = v_D\mathbf{d} + \frac{m_h^2}{m_d^3}(p_h - v_D(\mathbf{h}\hat{m}\mathbf{d}))\mathbf{h}. \quad (4.6)$$

To calculate the current $\mathbf{j}_T$ it is necessary to know $\eta(E, H)$, which at given values of $\mathbf{e}$ and $\mathbf{h}$ is a function of $\eta(\varkappa)$. The change of topology corresponds to the points $\varkappa = \frac{1}{2}$ (appearance of closed trajectories) and $\varkappa = 1$ (closing of the principal trajectory). Therefore the discontinuities of $\mathbf{j}_T$ for specified field directions correspond to definite E/H ratios.

To calculate the character of the discontinuities, and also the current $\mathbf{j}_L$, we assume the approximation (3.5). It is easy to verify that in this case (3.8) corresponds to

$$f(\mathbf{p}) = n/\Omega, \quad \mathbf{p} \in \Omega_c. \quad (4.7)$$

Therefore the average velocity of all the electrons in Ω_c corresponds simply to the "center of gravity" of this region and is calculated in the form

$$\mathbf{v}_c = \frac{1}{\Omega_c}\int_{\Omega_c} (d\alpha)\mathbf{v}(\alpha), \quad (4.8)$$

yielding

$$\mathbf{v}_c^{(+)} = 0, \quad (4.9a)$$

$$\mathbf{v}_c^{(-)} = \frac{1}{2}v_0\frac{m_h^2}{m_d^3}[\mathbf{e}, \hat{m}\mathbf{h}]\frac{1}{\varkappa}. \quad (4.9b)$$

Elementary calculation yields also

$$\Omega_c = \frac{3}{2}\left\{\left(1 - \frac{1}{4\varkappa^2}\right)^{1/2} - \frac{1}{3}\left(1 - \frac{1}{4\varkappa^2}\right)^{3/2}\right.$$

$$\left. - \frac{1}{2\varkappa}\arcsin\left(1 - \frac{1}{4\varkappa^2}\right)^{1/2}\right\}\Omega, \quad \varkappa > \frac{1}{2}. \quad (4.10)$$

It follows from (4.10) that

$$\Omega_c \approx \frac{2}{5}(\varkappa - \frac{1}{2})^{5/2}\Omega \quad (\varkappa \to \frac{1}{2}). \quad (4.11a)$$

$$\Omega_c \approx \frac{1}{4}\left\{\frac{9\sqrt{3}}{4} - \pi\right\}\Omega \approx 0,19\Omega \quad (\varkappa \to 1). \quad (4.11b)$$

We see therefore that $\eta(\kappa)$ has a discontinuity of the function when $\kappa = 1$, and a discontinuity of the third derivative when $\kappa = \frac{1}{2}$. Bearing this remark in mind, and also comparing (4.9) and (4.4), we find that when $\kappa = 1$ the component $\mathbf{j}^{(+)}$, which is even with respect with the magnetic-field inversion has a discontinuity, while $\mathbf{j}^{(-)}$ is continuous and has a maximum with a discontinuity of the first derivative. These singularities can be observed on the gauss-ampere characteristics (dependence of $\mathbf{j}$ on H at fixed E), where the discontinuity and the maximum correspond to a certain critical field

$$H_0 = \left(\frac{v_0}{2c}\right)^{-1}E, \quad (4.12)$$

which depends through v_0 on the orientations of H and E relative to the ellipsoid axes.

The qualitative character of the dependences of $\mathbf{j}^{(+)}$ and $\mathbf{j}^{(-)}$ on H coincides with the character of the dependence of the dissipative and Hall currents in the isotropic model shown in Fig. 3 of our earlier paper [3]. It is apparently difficult to observe experimentally the weak continuities at $\kappa = \frac{1}{2}$.

The physical cause of the discontinuities at $H = H_0$ is as follows: When $H > H_0$ the electron is under the conditions of a strong magnetic field after emitting the phonon, since it executes $\omega_H\tau \gg 1$ revolutions before it is scattered inside Ω. When $H < H_0$, the electron executes an incomplete revolution before the next scattering act. Thus, when $H = H_0$ a transition takes place from a weak magnetic field to a strong one, and this transition occurs jumpwise, unlike the ordinary situations.

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