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CONTRIBUTION TO THE THEORY OF FLUCTUATIONS AND SCATTERING OF SLOW NEUTRONS IN FERROMAGNETS

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The fluctuations and scattering of slow neutrons in ferromagnets are investigated. Expressions for correlators of fluctuations of quantities characterizing the ferromagnet (e.g., magnetic field strength, magnetic induction, magnetic moment, density, displacement vector) are derived near the ferroacoustic resonance point as well as far from it. It is shown that the correlation functions possess sharp maxima near the frequencies of the natural oscillations of the crystal, i.e., of the spin and sound waves. Far from ferroacoustic resonance the fluctuations of the magnetic quantities (magnetic field strength and induction, magnetic moment) are large on the spin waves whereas fluctuations of the non-magnetic quantities (density, displacement vector) are large on the sound waves. Fluctuations of magnetic as well as nonmagnetic quantities are great near ferroacoustic resonance. Scattering of slow neutrons in a ferromagnet is examined, taking into account the interaction between the magnetic-moment oscillations and elastic waves. It is shown that on approaching the ferroacoustic-resonance point two closely spaced maxima appear in the neutron scattering differential cross section, instead of a single pronounced maximum due to neutron scattering by the spin wave. The coupling between elastic and spin waves manifests itself also in the scattering of slow neutrons by acoustic vibrations. The scattering cross section in this case may be several times greater than for an ordinary (non-ferromagnetic) crystal.

INTRODUCTION

As is known, when slow neutrons pass through condensed media sharp maxima appear in the spectrum of the scattered neutrons as a result of the possibility of propagation of weakly-attenuated oscillations in these media. Because of this the scattering of slow neutrons is one of the most important methods for the study of the properties of condensed media and, in particular, of their energy spectra.

In this paper we consider the scattering of slow neutrons in ferromagnetic crystals. In such crystals weakly-attenuated oscillations of two types are possible: acoustic and spin waves. A voluminous literature (see the monograph by A. Akhiezer and Pomeranchuk [1]) is devoted to the interaction of neutrons with acoustic vibrations (in ordinary, magnetically-ordered crystals). A large number of papers have been concerned with the interaction of neutrons with spin waves (see the review by Izyumov [2]). In these papers, however, the coupling between the spin and acoustic waves is not taken into account. In many cases the neglect of this coupling is actually justifiable, since the dimensionless parameter $\zeta = f^2\rho\mu_0^2 s^{-2}$ is small (ρ is the density of the ferromagnet, μ_0 is the equilibrium value of the magnetic moment per unit mass, f is the magnetostriction constant, and s is the speed of sound); in order of magnitude, $\zeta \approx 10^{-4}$—10^{-6}.

Nevertheless, in spite of the smallness of the parameter ζ, both branches of the oscillations turn out to be strongly coupled near ferroacoustic resonance (A. Akhiezer, Bar'yakhtar, and Peletminskiĭ [3]). This, as is shown in this paper, can significantly change the character of the neutron scattering in the resonance region. Namely, upon approaching the point of ferroacoustic resonance there will appear in the differential neutron scattering cross-section two very closely spaced maxima, instead of a single sharp maximum due to the scattering of neutrons by spin waves.

In the non-resonant region the coupling between spin and acoustic waves is particularly well manifested when neutrons are scattered with excitation or absorption of acoustic vibrations. We shall show that the magnetic-moment oscillations that accompany the acoustic wave give a significant contribution to the cross section for scattering of slow neutrons by sound vibrations; hence the cross section for neutron scattering by acoustic vibrations may be several times larger in the case of a ferromagnet than in an ordinary crystal.

The intensity of scattering of slow neutrons in crystals is determined, as is well known, by the level of fluctuations in them. Hence, in this paper we study, along with neutron scattering, the fluctuations in the quantities that characterize ferromagnets, taking into account the coupling between elastic waves and magnetic-moment oscillations.

1. SCATTERING CROSS-SECTION OF SLOW NEUTRONS

We first obtain the general expression for the scattering cross section of slow neutrons in a magnetically ordered crystal, taking into account both crystal-density oscillations and magnetic-moment oscillations. The cross section, per nucleus of the crystal, of a process in which a neutron makes a transition from a state with momentum $\mathbf{p}$ and spin projection σ to a state with momentum $\mathbf{p}' = \mathbf{p} - \hbar\mathbf{k}$ and spin projection σ', while the crystal goes from state f to state f′, has the form

$$d\sigma = \frac{2\pi}{\hbar}\frac{m_N}{vV\rho_0}\int d\mathbf{r}\,d\mathbf{r}'\exp\{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')\}(f',\sigma'|\mathscr{H}(\mathbf{r})|f,\sigma)$$
$$\times(f,\sigma|\mathscr{H}^+(\mathbf{r}')|f',\sigma')\,\delta\left(\frac{p^2-p'^2}{2m}+\mathscr{E}_f-\mathscr{E}_{f'}\right)\frac{d\mathbf{p}'}{(2\pi\hbar)^3}. \tag{1}$$

Here $\mathscr{H} \equiv \mathscr{H}_N + \mathscr{H}_M$ is the Hamiltonian of the interaction of a slow neutron with the crystal,[4]

$$\mathscr{H}_N(\mathbf{r}) = -\frac{2\pi\hbar^2}{m'}\sum_l a\delta(\mathbf{r}-\mathbf{r}_l) \tag{2}$$

is the Hamiltonian of the interaction of the neutron with the crystal nuclei and is a sum of Fermi pseudopotentials (a is the scattering length of a neutron by a free nucleus, m' is the reduced mass of the neutron and nucleus, $\mathbf{r}_l$ is the radius vector of the l-th nucleus; for simplicity we do not consider the interaction of the neutron with the spins of the crystal nuclei); m_N is the nuclear mass;

$$\mathscr{H}_M(\mathbf{r}) = -g_0\hbar\mathbf{s}\mathbf{h}(\mathbf{r}) \tag{3}$$

is the Hamiltonian of the interaction of the neutron with the (microscopic) magnetic field of the crystal $\mathbf{h}$; m, v, and $\mathbf{s}$ are the mass, initial velocity, and the neutron spin operator, g_0 is the neutron gyromagnetic ratio, $\mathscr{E}_f$ is the crystal energy level, ρ_0 is the equilibrium value of the crystal density, and V is the volume of the system.

We now sum Eq. (1) for the scattering cross-section over the final states of the crystal and average over the initial states with the Gibbs factor $P_f \sim \exp[-\mathscr{E}_f/T]$ (T is the temperature of the crystal), and we also sum over the polarizations of the scattered neutrons. Assuming that the momentum change of the neutron is not too large ($bk \ll 1$, where b is the lattice constant) and, for simplicity, that the incident neutron beam is unpolarized, we obtain

$$d\sigma = \frac{m_N}{v\rho_0}\left\{\left(\frac{2\pi\hbar}{m'm_N}\right)^2|a|^2\langle\delta\rho^2\rangle_{\mathbf{k}\omega}\right.$$
$$\left.+\frac{g_0^2}{4}F^2(\mathbf{k})\langle\delta B^2\rangle_{\mathbf{k}\omega}\right\}\frac{d\mathbf{p}'}{(2\pi\hbar)^3} \tag{4}$$

where $\hbar\omega = (2m)^{-1}(p^2 - p'^2)$; $\langle\delta\rho^2\rangle$ and $\langle\delta B^2\rangle$ are the Fourier components of the correlators of the density and magnetic induction fluctuations, e.g.,

$$\langle\delta B^2\rangle_{\mathbf{k}\omega} = \int d\mathbf{r}\,dt\exp\{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')+i\omega(t-t')\} \tag{5}$$
$$\times\langle\mathbf{B}(\mathbf{r},t)\mathbf{B}(\mathbf{r}',t')\rangle,$$

and $F(\mathbf{k})$ is the so-called magnetic form factor (the angle brackets indicate thermodynamic and quantum-mechanical averages).

We note that in the case of spinless nuclei the matrix elements of the Hamiltonian (2) between states with different neutron spin projections are zero, so that the scattering of a neutron with a change in its spin orientation is completely described by the Hamiltonian (3). The differential scattering cross section of polarized neutrons with a change in spin orientation has in this case the form

$$d\sigma_{\uparrow\downarrow} = \frac{m_N g_0^2}{4v\rho_0}F^2(\mathbf{k})\{\langle\delta B^2\rangle_{\mathbf{k}\omega} - \langle(\delta\mathbf{B}\mathbf{n})^2\rangle_{\mathbf{k}\omega}\}\frac{d\mathbf{p}'}{(2\pi\hbar)^3} \tag{6}$$

($\mathbf{n}$ is a unit vector on the axis along which the spins of the incident neutrons are oriented).

2. DETERMINATION OF THE CORRELATION FUNCTIONS

Equation (4) expresses the scattering cross-section of slow neutrons in a magnetically ordered crystal in terms of the correlators of the density and magnetic-induction fluctuations. We proceed therefore to a calculation of the correlators of the fluctuations of the quantities that characterize a ferromagnet. According to the general method of fluctuation theory, which is based on the fluctuation-dissipation theorem, in order to do this we must introduce into the equations describing the system under consideration additional external quantities—the so-called "random forces."[5,6] Introducing random forces w and y into the equation of motion of the magnetic moment and the equation of elasticity, we obtain

$$\frac{\partial \boldsymbol{\mu}}{\partial t} = g[\boldsymbol{\mu}\mathbf{H}^e] - \frac{\lambda}{\rho_0\mu_0^2}[\boldsymbol{\mu}[\boldsymbol{\mu}\mathbf{H}^e]] + \mathbf{w},$$

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mathbf{f} - \frac{\dot{\mathbf{u}}}{\tau} + \mathbf{y} \qquad \left(\dot{\mathbf{u}} \equiv \frac{\partial \mathbf{u}}{\partial t}\right), \tag{7*}$$

where μ is the magnetic moment per unit mass, u is a displacement vector, $\mathbf{H}^e$ is the effective field, f is the force acting per unit mass, g is the gyromagnetic ratio, and λ and τ^{-1} are relaxation constants (a more precise form for the relaxation terms is not essential, since the limiting transition $\lambda \to \tau^{-1} \to 0$ will be made in the final results). Considering that the square of the magnetic moment of unit mass is an integral of the motion, $\mu^2 \equiv \mu_0^2$, we may express the random force w in terms of the transverse random force η: $\mathbf{w} = \mu_0^{-1}\boldsymbol{\mu}\times\boldsymbol{\eta}$.

The expressions for the effective field and force f, considering the coupling between the magnetic moment and elastic oscillations, have the form [7]

$$\mathbf{H}^e = \mathbf{H} + \alpha\rho_0\Delta\boldsymbol{\mu} - \beta\rho_0[\boldsymbol{\mu} - \mathbf{n}(\boldsymbol{\mu}\mathbf{n})] - (\beta+f)\rho_0\mu_0\nabla(\mathbf{n}\mathbf{u}) - f\rho_0\mu_0(\mathbf{n}\nabla)\mathbf{u},$$

$$f_i = \lambda_{ik,i'k'}\frac{\partial^2 u_{i'}}{\partial x_k\,\partial x_{k'}} + \rho_0\mu_0^2\left[(\beta+f)n_in_k\Delta u_k - fn_kn_{k'}\frac{\partial^2 u_i}{\partial x_k\,\partial x_{k'}}\right] + \mu_0(\mathbf{n}\nabla)H_i + (\beta+f)\rho_0\mu_0 n_i\,\mathrm{div}\,\boldsymbol{\mu} + f\rho_0\mu_0(\mathbf{n}\nabla)\mu_i,$$

$$\mathrm{div}(\mathbf{H} + 4\pi\rho_0\boldsymbol{\mu}) + 4\pi\mu_0\nabla\rho = 0, \tag{8}$$

where α is the exchange-interaction constant, β is the magnetic-anisotropy constant, n is a unit vector along the axis of easy magnetization (z axis), f is the magnetostriction constant, and $\lambda_{\dots}$ is the tensor of the elastic constants, which we henceforth express in its simplest form:

$$\lambda_{ik,i'k'} = (s_l^2 - 2s_t^2)\delta_{ik}\delta_{i'k'} + s_t^2(\delta_{ik'}\delta_{i'k} + \delta_{ii'}\delta_{kk'}) \tag{9}$$

(s_l, s_t are the velocities of longitudinal and transverse sound).

Following the general method of fluctuation theory, we must now, in determining the time derivative of the internal energy of the system, represent it in the form

$$\dot{U}(t) = \int \{\dot{\mathbf{x}}_m(\mathbf{r},t)\mathbf{X}_m(\mathbf{r},t) + \dot{\mathbf{x}}_e(\mathbf{r},t)\mathbf{X}_e(\mathbf{r},t)\}\,d\mathbf{r}, \tag{10}$$

where the $\dot{\mathbf{x}}$ are the "generalized thermodynamic velocities" and the X are the "generalized thermodynamic forces" corresponding to them (the subscripts m and e refer respectively to magnetic and elastic quantities). Differentiating the expression for the internal energy of a ferromagnet with respect to time [7]:

$$U = \int\Big\{\frac{H^2}{8\pi} + \frac{1}{2}\rho_0\dot{u}^2 + \frac{1}{2}\rho_0\lambda_{ik,i'k'}\frac{\partial u_i}{\partial x_k}\frac{\partial u_{i'}}{\partial x_{k'}} + \frac{1}{2}\alpha\rho_0^2\frac{\partial\mu_k}{\partial x_i}\frac{\partial\mu_k}{\partial x_i} + \frac{1}{2}f\rho_0^2\mu_0^2(n_in_{i'}\delta_{kk'} - n_kn_{k'}\delta_{ii'})\frac{\partial u_i}{\partial x_k}\frac{\partial u_{i'}}{\partial x_{k'}}$$
$$+ \rho_0^2\mu_0[f(\delta\mu_in_k + \delta\mu_kn_i) + \beta n_k\delta\mu_i]\frac{\partial u_k}{\partial x_i} + \frac{1}{2}\beta\rho_0^2\delta\mu^2 + \frac{1}{2}\beta\rho_0^2\mu_0^2[\nabla(\mathbf{n}\mathbf{u})]^2\Big\}d\mathbf{r} \qquad (\delta\boldsymbol{\mu} = \boldsymbol{\mu} - \boldsymbol{\mu}_0), \tag{11}$$

using Eqs. (7) and (8) and taking into account that since the system is closed there is no flux of energy through its boundaries, we obtain

$$\dot{U} = -\int\Big\{\rho_0\mu_0^{-1}[\boldsymbol{\mu}\mathbf{H}^e]\Big(\frac{\lambda}{\rho_0\mu_0}[\boldsymbol{\mu}\mathbf{H}^e] - \boldsymbol{\eta}\Big) + \rho_0\dot{\mathbf{u}}\Big(\frac{\dot{\mathbf{u}}}{\tau} - \mathbf{y}\Big)\Big\}\,d\mathbf{r}. \tag{12}$$

Choosing now as "generalized thermodynamic forces and velocities" the quantities

$$\dot{\mathbf{x}}_m = -\frac{\lambda}{\rho_0\mu_0}[\boldsymbol{\mu}\mathbf{H}^e] + \boldsymbol{\eta}, \qquad \dot{\mathbf{x}}_e = -\frac{\dot{\mathbf{u}}}{\tau} + \mathbf{y},$$

$$\mathbf{X}_m = \rho_0\mu_0^{-1}[\boldsymbol{\mu}\mathbf{H}^e], \qquad \mathbf{X}_e = \rho_0\dot{\mathbf{u}}, \tag{13}$$

we represent $\dot{\mathbf{x}}$ in the form

$$\dot{\mathbf{x}}_m = -\gamma_m\mathbf{X}_m + \boldsymbol{\eta}, \qquad \dot{\mathbf{x}}_e = -\gamma_e\mathbf{X}_e + \mathbf{y},$$
$$\gamma_m = \lambda/\rho_0^2, \qquad \gamma_e = (\rho_0\tau)^{-1}.$$

The "kinetic coefficients" $\gamma_{m,e}$ are directly determined, as is known, by normalization of the random forces. Considering the connection between the quantities w and η, we obtain

$$\langle w_iw_j^*\rangle_{\mathbf{k}\omega} = 2(\delta_{ij} - n_in_j)\hbar\omega(N_\omega + 1)\lambda/\rho_0^2,$$
$$\langle y_iy_j^*\rangle_{\mathbf{k}\omega} = 2\delta_{ij}\hbar\omega(N_\omega + 1)(\rho_0\tau)^{-1},$$
$$\langle w_iy_j^*\rangle_{\mathbf{k}\omega} = \langle w_i^*y_j\rangle_{\mathbf{k}\omega} = 0, \tag{14}$$

where $N_\omega = [\exp(\hbar\omega/T) - 1]^{-1}$ is the Planck distribution function.

To find the correlation functions we must now express the magnetic moment per unit mass, the displacement vector, the density, and the other characteristic ferromagnetic quantities in terms of the random forces and then average over the random forces with the help of Eq. (14). We avoid the cumbersome general expressions for the correlation functions and consider only the most interesting cases of fluctuations near ferroacoustic resonance and fluctuations in the non-

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*$[\mathbf{u}\mathbf{H}^e] \equiv \mathbf{u}\times\mathbf{H}^e$.

resonant region. In doing this, we shall retain in the equation for the vector **u**, from among those terms proportional to the small parameter ζ, only those which lead to a coupling between the elastic and spin waves, disregarding those small terms which lead only to a redetermination of the elastic constants of the crystal and do not change the character of the magneto-elastic waves.

3. FLUCTUATIONS AND NEUTRON SCATTERING FAR FROM FERROACOUSTIC RESONANCE

As is known, the correlators of the fluctuations of the quantities characterizing a system have sharp maxima at frequency and wave-vector values that satisfy the dispersion equation of the characteristic vibrations of the system. Because of this, sharp maxima associated with the possibility of propagation of the characteristic vibrations in the system also arise in the scattering cross-section of slow neutrons.

The dispersion equation for the bound magneto-elastic vibrations in a ferromagnet have the form (neglecting dissipative terms, $\lambda \to \tau^{-1} \to 0$)

$$\left(\frac{\omega^2}{k^2 s_t^2}-1\right)^2\left(\frac{\omega^2}{k^2 s_l^2}-1\right)\left(\frac{\omega^2}{\omega_k^2}-1\right) - \zeta\left(\frac{\omega^2}{k^2 s_t^2}-1\right)f_1-\zeta^2 f_2=0, \quad (15)$$

where $\omega_k = (\Omega\Omega_1)^{1/2}$ is the frequency of the spin wave,

$$\Omega = g\rho_0\mu_0\left(\alpha k^2+\beta+\frac{H_0}{\rho_0\mu_0}+4\pi\sin^2\theta\right),$$

$$\Omega_1 = g\rho_0\mu_0\left(\alpha k^2+\beta+\frac{H_0}{\rho_0\mu_0}\right), \quad (16)$$

H_0 is the applied magnetic field (directed along the axis of easy magnetization), θ is the angle between the vectors **k** and **n**, and $f_{1,2}$ are certain functions (equal in order of magnitude to unity) whose explicit form we shall not give. As already indicated, the quantity ζ, which is of the order 10^{-4} to 10^{-6}, characterizes the coupling between the spin and sound waves.

In the nonresonant region ($|\omega_k^2 - s^2k^2| \gg \zeta^{1/2}\omega_k^2$), the vibrations in the ferromagnet divide into a spin branch, which has the dispersion law $\omega = \omega_k$, and acoustic branches with dispersion law $\omega = s_l k$ and $\omega = s_t k$ (we do not consider the effect of "entanglement" of transverse and longitudinal sound due to crystal anisotropy). We shall consider first the fluctuations in the acoustic vibrations and those features which the magnetic-moment oscillations accompanying the sound waves introduce into the scattering of slow neutrons by these waves.

Sound waves. The correlators of the fluctuations of the displacement vector and of the fluctuations of density in the sound waves are determined by the formulas

$$\langle\delta u_i\delta u_j^*\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega+1|\rho_0^{-1}\left\{\frac{k_ik_j}{k^2}\delta(\omega^2-s_l^2k^2) + \left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\delta(\omega^2-s_t^2k^2)\right\},$$

$$\langle\delta\rho^2\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega+1|\rho_0k^2\delta(\omega^2-s_l^2k^2). \quad (17)$$

We give expressions for the correlators of the fluctuations of the magnetic field, magnetic induction, and projection of the magnetic-induction vector on the axis of easiest magnetization:

$$\langle\delta H^2\rangle_{\mathbf{k}\omega} = 4(2\pi)^3\hbar|N_\omega+1|\rho_0\mu_0^2\{(\xi_l\sin\theta+\omega\cos\theta)^2 s_l^{-2} \times\delta(\omega^2-s_l^2k^2)+\xi_t^2\sin^2\theta s_t^{-2}\delta(\omega^2-s_t^2k^2)\},$$

$$\langle\delta B^2\rangle_{\mathbf{k}\omega} = 4(2\pi)^3\hbar|N_\omega+1|\rho_0\mu_0^2\{[(\xi_l\cos\theta+\omega\sin\theta)^2 + \xi_l^2\omega^2\Omega_1^{-2}]s_l^{-2}\delta(\omega^2-s_l^2k^2) + (\xi_t^2\cos^2\theta+\xi^2)s_t^{-2}\delta(\omega^2-s_t^2k^2)\},$$

$$\langle\delta B_z^2\rangle_{\mathbf{k}\omega} = 4(2\pi)^3\hbar|N_\omega+1|\rho_0\mu_0^2\sin^2\theta\{(\xi_l\cos\theta+\omega\sin\theta)^2 \times s_l^{-2}\delta(\omega^2-s_l^2k^2)+\xi_t^2\cos^2\theta s_t^{-2}\delta(\omega^2-s_t^2k^2)\}, \quad (18)$$

where

$$\xi_l = \omega\Omega_1 g\rho_0\mu_0(4\pi-\beta-2f)\sin\theta\cos\theta(\omega_k^2-s_l^2k^2)^{-1},$$

$$\xi_t = \omega g\rho_0\mu_0\{\omega^2\cos^2\theta f^2+\Omega_1^2[(\beta+f)\sin^2\theta-f\cos^2\theta]^2\}^{1/2} \times(\omega_k^2-s_t^2k^2)^{-1},$$

$$\xi = \omega g\rho_0\mu_0\{\Omega^2\cos^2\theta f^2+\omega^2[(\beta+f)\sin^2\theta-f\cos^2\theta]^2\}^{1/2} \times(\omega_k^2-s_t^2k^2)^{-1}.$$

In the case of longitudinal sound the correlator of the fluctuations of the magnetic moment per unit mass has the relatively simple form

$$\langle\delta\mu_i\delta\mu_j\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega+1|\frac{\mu_0^2}{\rho_0 s_l^2}\xi_l^2\tau_{ij}\delta(\omega^2-s_l^2k^2), \quad (19)$$

where $\tau_{xx} = 1$, $\tau_{xy} = \tau^*_{yx} = i\omega\Omega_1^{-1}$, $\tau_{yy} = \omega^2\Omega_1^{-2}$ (the y axis is chosen in the $\mathbf{n}\times\mathbf{k}$ direction), and the remaining components of the tensor τ are zero. (We shall not give the much more complicated expression for the correlator of the fluctuations of the magnetic moment in a transverse sound wave.)

We now determine the scattering cross-section of slow neutrons by sound vibrations. According to (4) this quantity is the sum of two terms:

$$d\sigma = d\sigma_N + d\sigma_M,$$

where $d\sigma_N$ is the cross section for scattering due to interaction of the neutron with crystal nuclei, and $d\sigma_M$ is the cross section for scattering of the neutron by magnetic moment fluctuations. Substituting (17) and (18) into (4), we find

$$d\sigma_N = |N_\omega + 1|\,(m'^2 m_N v)^{-1}|ak|^2 \delta(\omega^2 - s_l^2 k^2)\,d\mathbf{p}',$$

$$d\sigma_M = |N_\omega + 1|\,(g_0\mu_0\hbar^{-1})^2 \{[(\xi_l \cos\theta + \omega\sin\theta)^2 + \xi_l^2\omega^2\Omega_1^{-2}]\,s_l^{-2}\,\delta(\omega^2 - s_l^2k^2) + (\xi_t^2\cos^2\theta + \xi^2)\,s_t^{-2}\delta(\omega^2 - s_t^2k^2)\}\,\frac{m_N}{v}\,d\mathbf{p}'. \qquad (20)$$

Comparing these expressions, we see that the cross section $d\sigma_M$ can equal $d\sigma_N$ in order of magnitude or even exceed it.

As already mentioned, in the case of spinless nuclei the spin orientation of a neutron does not change when it is scattered in magnetically unordered crystals. But if the crystal is ferromagnetic, the scattering cross section of neutrons with a change in spin orientation has, according to (6) and (18), the form

$$d\sigma_{\uparrow\downarrow} = |N_\omega + 1|\left(\frac{g_0\mu_0}{\hbar}F(\mathbf{k})\right)^2 \frac{m_N}{v} \times \{[(\xi_l\cos\theta + \omega\sin\theta)^2\cos^2\theta + \xi_l^2\omega^2\Omega_1^{-2}]\,s_l^{-2}\delta(\omega^2 - s_l^2k^2) + (\xi_t^2\cos^4\theta + \xi^2)\,s_t^{-2}\delta(\omega^2 - s_t^2k^2)\}\,d\mathbf{p}'. \qquad (21)$$

Spin waves. The correlator of the fluctuations of the magnetic moment per unit mass in a spin wave is determined by the expression

$$\langle\delta\mu_i\,\delta\mu_j^*\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega + 1|\,g\mu_0\rho_0^{-1}\Omega_{ij}\delta(\omega^2 - \omega_k^2), \qquad (22)$$

where

$$\Omega_{xx} = \Omega_1, \quad \Omega_{xy} = \Omega_{yx}^* = i\omega, \quad \Omega_{yy} = \Omega,$$
$$\Omega_{xz} = \Omega_{zx} = \Omega_{yz} = \Omega_{zy} = \Omega_{zz} = 0. \qquad (23)$$

For the fluctuations of magnetic induction and of the magnetic field we obtain from this

$$\langle\delta B^2\rangle_{\mathbf{k}\omega} = 4(2\pi)^3\hbar|N_\omega + 1|\,g\rho_0\mu_0(\Omega_1\cos^2\theta + \Omega) \times \delta(\omega^2 - \omega_k^2),$$

$$\langle\delta H^2\rangle_{\mathbf{k}\omega} = 4(2\pi)^3\hbar|N_\omega + 1|\,g\rho_0\mu_0\Omega_1\sin^2\theta\,\delta(\omega^2 - \omega_k^2). \qquad (24)$$

The correlators of the fluctuations in the density and in the longitudinal (δu_l) and transverse (δu_t) components of the displacement vector accompanying the spin wave have the form

$$\langle\delta\rho^2\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega + 1|\,k^4\xi_l^2\rho_0\mu_0\omega_k^{-2}(g\Omega_1)^{-1}\delta(\omega^2 - \omega_k^2),$$

$$\langle\delta u_{l,t}^2\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega + 1|\,k^2\xi_{l,t}^2\,\mu_0\omega_k^{-2}(g\rho_0\Omega_1)^{-1}\delta(\omega^2 - \omega_k^2). \qquad (25)$$

Substituting (24) and (25) into (4), we obtain the well-known expression for the cross section for scattering of slow neutrons by a spin wave:

$$d\sigma = |N_\omega + 1|\left(\frac{g_0}{\hbar}F(\mathbf{k})\right)^2 g\mu_0(\Omega_1\cos^2\theta + \Omega) \times \frac{m_N}{v}\,\delta(\omega^2 - \omega_k^2)\,d\mathbf{p}' \qquad (26)$$

(the contribution of density fluctuations to the scattering cross section is very small in this case).

We note in concluding this section that at frequencies far from the frequencies of the characteristic vibrations of a ferromagnet, the correlation functions are small (proportional to the relaxation constants λ and τ^{-1}); hence these frequency regions make a very small contribution to the scattering cross section of slow neutrons.

4. FLUCTUATIONS AND SCATTERING OF NEUTRONS NEAR FERROACOUSTIC RESONANCE

As was shown in the preceding section, far from ferroacoustic resonance the fluctuations of the magnetic quantities (magnetic field, induction, and magnetic-moment density) are large in one of the vibrational branches of a ferromagnet—the spin wave branch—and the fluctuations of the density and the displacement vector are small; in the other branches, on the other hand, i.e., in the longitudinal and transverse acoustic wave branches, fluctuations in the displacement vector are large and in the magnetic quantities relatively small. The situation is otherwise near ferroacoustic resonance, when $\omega_k = s_l k$ (longitudinal resonance) or $\omega_k = s_t k$ (transverse resonance). In this case two branches of magnetoelastic waves arise in a ferromagnet in which the fluctuations in all the quantities characterizing the ferromagnet (both magnetic and non-magnetic) are large. (In the case of transverse resonance there arises still a third branch, in which fluctuations of the displacement vector are large.) Because of this, two very closely-spaced maxima close to resonance can arise in the differential cross section for the scattering of slow neutrons instead of a single sharp maximum due to scattering of neutrons by a spin wave.

Longitudinal ferroacoustic resonance. The dispersion equation for the coupled spin and transverse sound waves has close to resonance, the form

$$(\omega^2 - \omega_+^2)^2(\omega^2 - \omega_-^2)^2 + (\omega_+^2 - \omega_-^2)^2(2\gamma\omega)^2 = 0; \qquad (27)$$

$$\omega_\pm^2 = {}^1/_2(\omega_k^2 + s_l^2k^2) \pm {}^1/_2\{(\omega_k^2 - s_l^2k^2)^2 + 4\Omega_1 g\rho_0^2\mu_0^3k^2 \times \sin^2\theta\cos^2\theta(\beta + 2f - 4\pi)^2\}^{1/2},$$

$$\gamma = \frac{1}{2}\left\{\tau^{-1} + \frac{\lambda}{g\rho_0\mu_0}(\Omega + \Omega_1)\right\}. \qquad (27')$$

Equation (27) has two solutions, corresponding to the two branches of magnetoelastic waves with frequencies ω_+ and ω_- and the attenuation decrement γ.

This is the expression for the correlator of the fluctuations of the magnetic moment per unit mass near the point of ferroacoustic resonance ($|\omega_k^2 - s_l^2 k^2| < \zeta^{1/2}\omega_k^2$):

$$\langle\delta\mu_i\,\delta\mu_j^*\rangle_{\mathbf{k}\omega} = \hbar|N_\omega + 1|g\mu_0\rho_0^{-1}\Omega_{ij}\{2\gamma\omega_-[(\omega^2-\omega_-^2)^2 + (2\gamma\omega_-)^2]^{-1} + 2\gamma\omega_+[(\omega^2-\omega_+^2)^2 + (2\gamma\omega_+)^2]^{-1}\}, \qquad (28)$$

where the tensor Ω_{ij} is given by Eq. (23). If the attenuation of the waves is small ($\gamma \ll \omega\zeta^{1/2}$), this expression takes the form

$$\langle\delta\mu_i\,\delta\mu_j^*\rangle_{\mathbf{k}\omega} = 2\pi\hbar|N_\omega + 1|g\mu_0\rho_0^{-1}\Omega_{ij}\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\}. \qquad (29)$$

From this we obtain for the fluctuations of the magnetic induction and magnetic field

$$\langle\delta B^2\rangle_{\mathbf{k}\omega} = 2(2\pi)^3\hbar|N_\omega + 1|g\rho_0\mu_0(\Omega_1\cos^2\theta + \Omega) \times\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\},$$

$$\langle\delta H^2\rangle_{\mathbf{k}\omega} = 2(2\pi)^3\hbar|N_\omega + 1|g\rho_0\mu_0\Omega_1\sin^2\theta\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\}. \qquad (30)$$

Near longitudinal ferroacoustic resonance the correlators of the fluctuations of density and the displacement vector have the form

$$\langle\delta\rho^2\rangle_{\mathbf{k}\omega} = \pi\hbar|N_\omega + 1|\rho_0 k^2\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\},$$

$$\langle\delta u_i\,\delta u_j^*\rangle_{\mathbf{k}\omega} = \pi\hbar|N_\omega + 1|\rho_0^{-1}k^{-2}k_ik_j \times\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\}. \qquad (31)$$

Comparing these formulas with Eqs. (17), (22), and (23), we see that on approaching the point of ferroacoustic resonance the sharp maximum in the expressions for the correlators of the fluctuations in the magnetic (nonmagnetic) quantities, which is due to the possibility of propagation of spin (sound) waves in the ferromagnet, is split into two maxima.

Substituting (30), (31) into (4), we obtain

$$d\sigma = \frac{1}{2}|N_\omega + 1|\left(\frac{g_0}{\hbar}F(\mathbf{k})\right)^2 g\mu_0(\Omega_1\cos^2\theta + \Omega)\frac{m_N}{v} \times\{\delta(\omega^2-\omega_-^2) + \delta(\omega^2-\omega_+^2)\}d\mathbf{p}'. \qquad (32)$$

Thus, near the point of ferroacoustic resonance the sharp maximum, due to the scattering of neutrons by spin waves of the slow-neutrons scattering cross section is split into two maxima.[1)]

We note that if the attenuation of the magnetoelastic waves is not small ($\omega \gg \gamma \gtrsim \omega\zeta^{1/2}$), then the two maxima arising in the expressions for the correlation functions and for the differential cross section of neutron scattering are superposed on one another. In this case the correlation functions and the scattering cross section near ferroacoustic resonance are determined by the same relations (17), (22)—(24), and (26) as they are far from resonance.

Transverse ferroacoustic resonance. The dispersion equation for the coupled spin and transverse acoustic waves has, near resonance, the form

$$\{(\omega^2-\omega_1^2)^2(\omega^2-\omega_2^2)^2 + (\omega_1^2-\omega_2^2)^2(2\gamma\omega)^2\} \times\{(\omega^2 - s_t^2k^2)^2 + (\tau^{-1}\omega)^2\} = 0;$$

$$\omega_{1,2}^2 = {}^1/_2(\omega_k^2 + s_t^2k^2) \pm {}^1/_2\{(\omega_k^2 - s_t^2k^2)^2 + 4g\rho_0^2\mu_0^3k^2G\}^{1/2},$$

$$G = \Omega_1[(\beta + f)\sin^2\theta - f\cos^2\theta]^2 + \Omega f^2\cos^2\theta \qquad (33)$$

and the quantity γ is determined from Eq. (27′). Equation (33) has two solutions, corresponding to the two branches of magnetoelastic waves with frequencies ω_1 and ω_2 and decrement γ, and a third solution, corresponding to an "almost pure acoustic" wave with frequency $s_t k$ and decrement $(2\tau)^{-1}$.

The correlator of the fluctuations in the magnetic moment per unit mass near the point of transverse resonance ($|\omega_k^2 - s_t^2k^2| < \zeta^{1/2}\omega_k^2$) has the form

$$\langle\delta\mu_i\delta\mu_j^*\rangle_{\mathbf{k}\omega} = \hbar|N_\omega + 1|\frac{g\mu_0}{\rho_0}\Omega_{ij} \times\sum_{n=1,2} 2\gamma\omega_n[(\omega^2-\omega_n^2)^2 + (2\gamma\omega_n)^2]^{-1}, \qquad (34)$$

where the tensor Ω_{ij} is given by Eq. (23). Comparing (34) and (28), we see that the expression for the correlator of the fluctuations of the magnetic moment near the point of transverse resonance differs from the corresponding expression for the case of longitudinal resonance only by the replacement of the resonance frequencies $\omega_\pm$ by $\omega_{1,2}$. It is easy to show that the expressions for the correlators of the fluctuations of the magnetic

[1)]Splitting of the maximum in the differential cross-section for the scattering of slow neutrons near ferroacoustic resonance was mentioned in [8], where, starting from the simplest model for magnon-phonon interactions, the case of cubic crystals was considered.

induction and the magnetic field and for the scattering cross-section of slow neutrons near the point of transverse resonance can also be obtained from the corresponding expressions for the case of longitudinal resonance (Eqs. (30) and (32)) by means of the replacement $\omega_{\pm} \rightarrow \omega_{1,2}$.

The correlator of the fluctuations of the displacement vector near the point of transverse resonance has the form

$$\langle \delta u_i \delta u_j^* \rangle_{\mathbf{k}\omega} = \frac{\pi}{2} \hbar |N_\omega + 1| \rho_0^{-1} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \times \{\delta(\omega^2 - \omega_1^2) + \delta(\omega^2 - \omega_2^2) + 2\delta(\omega^2 - s_t^2 k^2)\}. \quad (35)$$

Thus, in the expressions for the correlators of the fluctuations of the magnetic quantities and in the expression for the Newton scattering cross section near the point of transverse resonance, instead of a single sharp maximum, two arise, and in the expressions for the correlator of the fluctuations of the displacement vector, three.

We note that if the attenuation of the magnetoelastic waves is not small ($\omega \gg \gamma \gtrsim \omega \zeta^{1/2}$), then, as in the case of longitudinal resonance, the correlators of the fluctuations and the neutron scattering cross section near the point of transverse resonance are determined by the same formulas, (17), (22)—(24), and (26), as they are far from resonance.

We pause now briefly to discuss the angular and energy distribution of the scattered neutrons. The scattering of slow neutrons in a ferromagnet occurs, as is known, particularly intensely if the changes in the energy and momentum of the neutron $\hbar\omega$ and $\hbar k$ are connected by the relation $\omega^2 = \omega_k^2$, where ω_k is the spin-wave frequency. Assuming for simplicity that $\alpha k^2 \gg 1$ and ignoring the recoil of the neutron ($\hbar k \ll mv$), we see that the scattering angle ϑ (the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$) is uniquely determined by the energy change of the neutron $\Delta E = \hbar|\omega|$ and its initial energy E:

$$\vartheta^2 = \frac{m_s}{m} \frac{\Delta E}{E} - \left(\frac{\Delta E}{2E} \right)^2, \quad m_s = \hbar (2\alpha \rho_0 g \mu_0)^{-1}, \quad (36)$$

and $\vartheta \ll 1$. At very low temperatures ($T \ll \Delta E$) almost all the scattered neutrons have the energy $E' = E - \Delta E$; at $T \gtrsim \Delta E$ the number of scattered neutrons with energies $E + \Delta E$ and $E - \Delta E$ is of the same order of magnitude.

In order that the phenomenon of ferroacoustic resonance be manifested in the neutron scattering, it is necessary that in addition to condition (36) still another condition, $\omega_k \approx ks$, be fulfilled; here $s = s_l$ ($s = s_t$) in the case of longitudinal (transverse) resonance. This condition brings in still another connection between ΔE and ϑ:

$$\vartheta^2 = \left(\frac{\Delta E}{2E} \right)^2 \left(\frac{v^2}{s^2} - 1 \right).$$

Thus, for the observation of ferroacoustic resonance by the distribution of scattered neutrons, the scattering angle must be close to an angle ϑ_0 and the energy change close to an amount ΔE_0, where

$$\vartheta_0 = 2\frac{m_s}{m} \left(\frac{s}{v} \right)^2 \left(\frac{v^2}{s^2} - 1 \right)^{1/2}, \quad \Delta E_0 = 4E_0 \frac{m_s}{m} \left(\frac{s}{v} \right)^2. \quad (37)$$

The splitting of the maximum in the distribution of scattered neutrons occurs in the narrow range of angles $\vartheta = \vartheta_0 [1 + O(\zeta^{1/2})]$ and conveyed energies $\Delta E = \Delta E_0 [1 + O(\zeta^{1/2})]$; hence in order to observe this effect it is necessary to have an angular resolution of the order of $0.01\vartheta_0$ and an energy resolution of the order of $0.01\Delta E_0$.

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