AMPLIFICATION OF ULTRASONIC AND DRIFT WAVES IN SEMICONDUCTORS IN ELECTRIC AND MAGNETIC FIELDS

A. A. DEMIDENKO, S. I. PEKAR, V. N. PISKOVOĬ, and B. E. TSEKVAVA

Institute of Semiconductors, Academy of Sciences, Ukrainian S.S.R.

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A theory of the amplification of ultrasonic and drift waves in semiconductors is developed for different mechanisms of electron-phonon interaction. It is sssumed that the carriers have the same sign and drift in homogeneous electric and non-quantized magnetic fields. When the wave vectors of the drift and sound waves approach one another, the expressions for the amplification factors etc. differ considerably from the familiar forms for the isolated roots of the dispersion equation. Electron-phonon interaction due to the dependence of the dielectric constant on the deformation is considered in greater detail. In this case the amplification factor goes through a maximum and a minimum and then increases with the applied electric field.

THE theory of ultrasonic amplification was developed in 1956 on the basis of the electron-phonon interaction brought about by the deformation potential[1] and the piezoelectric effect.[2] Generalization of the theory to the case of an applied magnetic field has been made in a number of references.[3]

In a paper[4] on the theory of the amplification and generation of ultrasound by the carrier drift, an electron-phonon interaction was introduced that was proportional to the external applied electric field. In crystals with a large dielectric constant (several thousand), this interaction can dominate the other known mechanisms of electron-phonon coupling. The interaction mentioned is due to the dependence of the dielectric constant of the crystal on the deformation which, in the case of an isotropic medium, is expressed in the linear approximation by the formula

$$\varepsilon_{ik} = \varepsilon_0(\delta_{ik} - g_1\delta_{ik}\operatorname{div}\mathbf{u} - g_2 u_{ik}). \qquad (1)$$

Here ϵ_0 is the isotropic dielectric constant of the undeformed crystal, ϵ_{ik} the anisotropic dielectric tensor of the deformed crystal, $\mathbf{u}(\mathbf{r}, t)$ the displacement vector for elastic deformation of the medium, $u_{ik} = \frac{1}{2}(\partial u_i/\partial x_k + \partial u_k/\partial x_i)$ the deformation tensor, g_1 and g_2 are constants.

In[4], moreover, a drift wave was considered whose amplification was possible under favorable conditions at various mechanisms of electron-phonon coupling.

In the present work, the results of[4] are generalized to the case of an external magnetic field and are supplemented by consideration of closely-lying roots of the dispersion equation, one of which corresponds to the sound wave and the other to the drift wave. In passing, the contribution to the space charge from the change in the concentration of ionized donors in the deformation of the crystal is also considered. In calculating the forces acting on a unit volume of the crystal, the internal friction between the conduction and the lattice electrons is also taken into account. In the expression for the current, a term is taken into account which is due to the translational motion of the medium in the deformation. The difference between the constants of the deformed potential of the band and localized electrons is also considered.

1. FORCES ACTING ON THE DEFORMED CRYSTAL AND ON THE CURRENT CARRIERS

We consider an isotropic nonpiezoelectric crystal. Its potential energy can be expressed in terms of the following independent and continuous degrees of freedom: the elastic displacement vector of the medium $\mathbf{u}(\mathbf{r}, t)$, the concentration of current carriers in the conduction band $n(\mathbf{r}, t)$, and the concentration of carriers at the local levels $n_s(\mathbf{r}, t)$. The potential energy of the crystal can be written in the form

$$W = \int \left\{ \frac{\lambda}{2}(\operatorname{div}\mathbf{u})^2 + \mu u_{ik}^2 + \Big(\sum_s b_s n_s + b_0 n\Big)\operatorname{div}\mathbf{u} + \frac{1}{8\pi}\mathbf{D}\varepsilon^{-1}\mathbf{D}\right\} d\tau. \qquad (2)$$

Here λ and μ are the Lamé coefficients, b_0 and b_s the constants of the deformation potential of the band and of the localized electrons, **D** the electric displacement, and ϵ^{-1} the inverse of the dielectric tensor of the deformed crystal. Its magnetic permeability is assumed to be unity.

If the local levels of the electrons can be divided into two sharply delimited groups—one with levels whose thermoionization time is much less than the period of vibration of the wave—and the other whose levels having a thermoionization time much greater than vibration, then the population of the levels of the second group will remain constant upon passage of the wave. In (2), in the sum over s, the components corresponding to the levels of the second group form a term of the form (const · div **u**), which can be eliminated by a change in the point from which the quantities u_{ik} are measured. In what follows, we shall assume that this has been done, and by s we shall mean the number of the levels of the first group only.

For the calculation of the macroscopic potential $V(\mathbf{r}, t)$ acting on the conduction electron, it is necessary to take the vocational derivative of W with respect to en for fixed n_s and **u**. As a result, the effective field acting on the electron conduction is shown to be[1)]

$$\mathbf{E}^* = -\nabla V = -\frac{b_0}{e}\nabla(\nabla\mathbf{u}) + \mathbf{E} \tag{3}$$

(e is the charge of the current carrier), where **E** is the electric field intensity.

The force **F** acting on the lattice, per unit volume of the undeformed crystal, can be expressed in terms of the variation of W with respect to **u** for fixed n: $\delta_u W = -\int \mathbf{F}\delta\mathbf{u}d\tau$. It must be taken into account in this variation that n_s varies with **u** according to the law $\delta n_s = -\text{div}\, n_s\delta\mathbf{u}$. The quantity $\rho_L \equiv \rho - en$ also varies according to such a law; this quantity is the part of the space charge formed by the local charged electron centers, which move along with the lattice; ρ is the total density of the space charge;

$$\mathbf{F} = (\lambda+\mu)\nabla(\nabla\mathbf{u}) + \mu\Delta\mathbf{u} - \sum_s b_s n_s \nabla(\nabla\mathbf{u}) + \nabla\Big(\sum_s b_s n_s + b_0 n\Big) + \mathbf{E}\rho_L + \frac{\varepsilon_0}{8\pi}\{g_1\nabla\mathbf{E}^2 + g_2[\mathbf{E}(\nabla\mathbf{E}) + (\mathbf{E}\nabla)\mathbf{E}]\}. \tag{4}$$

Here the terms of the last line represent a force of the electrostrictive type. Some of the deviations from the known expressions are explained by the fact that part of the charge (of the current carriers) is not connected with the lattice.

If the levels of the first group mentioned above exceed the chemical potential by much more than kT, the concentration of electrons on each of them will remain proportional to the concentration of the electrons in the conduction band: $n_s = \nu_s(T)n$. The coefficient of proportionality ν_s depends on the deformation, inasmuch as the distance between the local levels, the effective mass of the electron in the conduction band, etc., depend on it. In an isotropic medium, $\nu_s = \nu_s^0 + \nu_s'\,\text{div}\,\mathbf{u}$. Furthermore, although the filling of the levels of the second group is not changed by the passage of the wave, their concentration n changes according to the law $n_c = n_c^0(1 - \text{div}\,\mathbf{u})$. As a result, it is shown that ρ can be expressed linearly in terms of the change in the carrier concentration and in terms of the deformation in the following way:

$$\frac{q}{e}\rho = n - n_0 + n_0 q_1 \,\text{div}\,\mathbf{u},\quad \frac{1}{q} = 1 + \sum_s \nu_s^0,$$
$$q_1 = 1 + q\sum_s \nu_s^1. \tag{5}$$

The summation is carried out over all levels of the first group. The quantities $n - n_0$ and ρ, as well as **u**, are of the first order of smallness. The equilibrium concentration of the carriers n_0 is of zeroth order.

By using the proportionality between n_s and n, we get, taking (5) into account, and keeping only terms of first order of smallness,

$$\nabla\Big(\sum_s b_s n_s + b_0 n\Big) = \frac{b}{e}\nabla\rho - bn_0\frac{q_1}{q}\nabla(\nabla\mathbf{u}),\quad b \equiv q\Big(\sum_s b_s\nu_s^0 + b_0\Big). \tag{6}$$

The last component of the right side of (6), just as any terms that are linear in **u**, can be combined with the first two components of (4), which leads to a simple renormalization of the elastic moduli λ and μ. In the third component of (4), in the linear approximation, n_s is replaced by the concentration in the undeformed crystal n_s^0; then this term is also removed by renormalization.

The electric field **E** entering into (3) is composed of the homogeneous field $\mathbf{E}_0$ averaged over the volume (zeroth order of smallness), the field of the space charge $\mathbf{E}_\rho$, and the contribution (oscillating in space) to the field brought about by the change in ϵ resulting from the deformation, $\mathbf{E}_u$ ($\mathbf{E}_\rho$ and $\mathbf{E}_u$ are of the first order of smallness):

[1)] In this formula the quantity b has erroneously been put in place of b_0 in [4].

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_\rho + \mathbf{E}_u, \quad \nabla \mathbf{E}_\rho = \frac{4\pi\rho}{\varepsilon_0},$$

$$\nabla \mathbf{E}_u = \left(g_1 + \frac{g_2}{2}\right)(\mathbf{E}_0\nabla)(\nabla \mathbf{u}) + \frac{g_2}{2}\mathbf{E}_0 \Delta \mathbf{u}. \qquad (7)$$

In the last term of (4), the components with the product $\mathbf{E}_0 \cdot \mathbf{E}_u$ are linear in $\mathbf{u}$ and vanish in the above-mentioned renormalization of the elastic moduli. The electron-phonon interaction, mentioned above, which is proportional to the external field, is represented by the terms $\mathbf{E}_0 \cdot \mathbf{E}_p$.

We now turn to the calculation of the current density in the crystal. We shall assume that the free path time of the carrier is much less than the period of the wave: $\omega \tau_f \ll 1$, where $\mathbf{k}$ is the wave vector and l is the free path length of the carriers. In this case one can use for the calculation of the current the mobility tensor of the carriers in the presence of the magnetic field and the tensor of diffusion coefficients D_H in the form which is obtained from the kinetic equation for static and spatially homogeneous fields. These tensors do not depend on the frequency of the wave ω or on $\mathbf{k}$ in this case. Then the current density is equal to

$$\mathbf{J} = en\mu_H \mathbf{E}^* - eD_H \nabla n + R\dot{\mathbf{u}}. \qquad (8)$$

Here the last term represents the contribution to the current due to the translational motion of the medium; R is a tensor of second rank. In the absence of a magnetic field, $R_{ik} = \rho\delta_{ik}$, where ρ is the total density of the space charge. The quantity $\rho\dot{\mathbf{u}}$ in Eq. (8) is a small quantity of second order and can be discarded. In the presence of the magnetic field $R\dot{\mathbf{u}}$ is of first order of smallness.

Besides the potential forces considered above, other non-potential forces act on the lattice which cannot be obtained from variation of W. To these belong the force of internal friction between the conduction electrons and the lattice and the force of the magnetic field on the charge moving along with the lattice, equal to $c^{-1}\rho_L \mathbf{u} \times \mathbf{H}$.

The mean force of the dragging of conduction electrons by the lattice can be obtained from the following considerations. Let $w(\mathbf{v}, \mathbf{v}')d^3v'$ be the probability of collision per unit time of an electron with a phonon or with an impurity, in which the initial velocity of the electron $\mathbf{v}$ goes over to a final velocity $\mathbf{v}'$ lying within an element of volume of velocity space d^3v'. The change in the momentum of the electron in such a collision is equal to $m_0(\mathbf{v}' - \mathbf{v})$, where m_0 is the mass of the free electron. Further, let the velocity distribution function of the electrons be $f(\mathbf{v}, \mathbf{r}, t)$. Then the change in the momentum of the electrons per unit volume of the crystal per second, as the result of collisions, is equal to

$$\mathbf{F} = m_0 \int\int (\mathbf{v}' - \mathbf{v}) w(\mathbf{v}, \mathbf{v}') f d^3v d^3v'.$$

The force of dragging the lattice by the electrons, per unit volume of the crystal, is equal to $-\mathbf{F}$.

In the case of elastic collisions, and if the distribution function can be represented here in the usual form,

$$f = f_0(|v|, \mathbf{r}, t) + (\mathbf{v}, \mathbf{f}_1(|v|, \mathbf{r}, t)),$$

we get

$$\mathbf{F} = -m_0 \int \mathbf{v} \frac{f - f_0}{\tau(v)} d^3v.$$

In the case in which the time of free flight τ does not depend on the velocity, we get $\mathbf{F} = -\mathbf{J}_e m_0/\mu_0 m$, where J_e is the electron current density relative to the lattice: $\mathbf{J}_e = \mathbf{J} - \rho\dot{\mathbf{u}} \approx \mathbf{J}$; μ_0 is the mobility of the carriers in the absence of a magnetic field (although F is defined above for an arbitrary magnetic field); m is the effective mass of the conduction electron.

If τ depends on v, then the value of F ordinarily differs little from $-\mathbf{J}m_0/\mu_0 m$. We have used only the latter expression for the estimates below.

One can now write down the total force acting on a unit volume of the lattice by adding the nonpotential forces to (4) and taking (5)—(8) into account. Keeping only terms of first order of smallness, eliminating the constant terms (of zero order) by a change in the origin of u_{ik}, and eliminating terms linear in u by the above-mentioned renormalization of the elastic moduli, we can write down the total force in the form

$$\begin{aligned}\mathbf{F}_1 = {} & (\lambda + \mu)\nabla(\nabla \mathbf{u}) + \mu \Delta \mathbf{u} + \frac{b}{e}\nabla\rho \\ & + \frac{\varepsilon_0}{4\pi}\left\{\left(g_1 + \frac{g_2}{2}\right)(\mathbf{E}_0\nabla)\mathbf{E}_\rho + \frac{g_2}{2}\mathbf{E}_0(\nabla \mathbf{E}_\rho)\right\} \\ & + \left[q\left(\frac{m_0}{m}\frac{\mu_H}{\mu_0} - 1\right) + 1\right]\rho\mathbf{E}_0 - \frac{k_0 T}{e} q \frac{m_0}{m}\frac{\mu_H}{\mu_0}\nabla\rho \\ & + en_0\left(\frac{m_0}{m}\frac{\mu_H}{\mu_0} - 1\right)\mathbf{E}_\rho. \end{aligned} \qquad (9)$$

Here we have used the Einstein relation $D_H = k_0 T \mu_H/e$, which is also valid in the presence of a magnetic field. In the renormalization of the elastic moduli, the latter receive imaginary contributions in addition to the real ones. These are connected with the dissipative character of the force of internal friction. This corresponds to weak damping of the sound waves which, together with the damping from other reasons, can be taken into account phenomenologically.

In conclusion to this section, we shall consider the tensors of second rank μ_H and R. One can use the well-known expressions given in the literature for the mobility μ_H in the presence of a magnetic field. In particular, in the case of a strong but non-

quantizing magnetic field, when the cyclotron frequency of the carriers is much greater than the reciprocal of the free flight time, or $\mu_0 H/c \gg 1$, the tensor μ_H is easily determined from the equation[5]

$$\mu_H \mathbf{E} = \mu_0 \mathbf{h}(\mathbf{h}, \mathbf{E}) - \frac{c}{H}[\mathbf{hE}] + \frac{Q}{H^2}\{\mathbf{E} - \mathbf{h}(\mathbf{hE})\} + \ldots, \quad \mathbf{h} = \frac{\mathbf{H}}{|\mathbf{H}|}, \tag{10}*$$

in which $\mathbf{E}$ is an arbitrary vector. If the free flight time of the current carrier is $\tau_f = a v_T^{\nu}$ where v_T is the thermal velocity of the carrier, then we get for μ_0 and Q:

$$\mu_0 = \frac{4ea}{3\sqrt{\pi m}}\left(\frac{2k_0T}{m}\right)^{\nu/2}\Gamma\left(\frac{5+\nu}{2}\right)$$

$$Q = \frac{4c^2m}{3\sqrt{\pi}ea}\left(\frac{m}{2k_0T}\right)^{\nu/2}\Gamma\left(\frac{5-\nu}{2}\right),$$

$$\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}\,dx, \tag{11}$$

where c is the velocity of light and m the effective mass of the carrier.

For calculation of the tensor R, we shall treat (8) as a linear expansion of $\mathbf{J}$ in $\mathbf{E}^*$, ∇n, and $\dot{\mathbf{u}}$. Then the coefficients of this expansion should not depend on the quantities in which the expansion is carried out. Therefore R can be computed by determining the current for $\mathbf{E}^* = 0$, $\nabla n = 0$, and $\dot{\mathbf{u}} \neq 0$.

By assuming the waves to be sufficiently long and $\mathbf{u}(\mathbf{r}, t)$ to depend slowly on t, we transform to a set of coordinates moving with a velocity $\dot{\mathbf{u}}$ and accompanying a small region of the medium. In this system of coordinates the medium is at rest, the magnetic induction will be equal to (with a correction of the order $\dot{u}^2/c^2$) $\mathbf{B}' = \mathbf{B} - c^{-1}[\dot{\mathbf{u}} \times \mathbf{E}] = \mathbf{B} = \mathbf{H}$, inasmuch as we consider the case in which $\mathbf{E} = 0$ in the laboratory system of coordinates. We denote the field and current in the accompanying system of coordinates by a prime. In the latter there appears the electric field

$$\mathbf{E}' = \mathbf{E} + \frac{1}{c}[\mathbf{uB}] = \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]$$

and a current $\mathbf{J}' = en\mu_H\mathbf{E}'$. In the laboratory set of coordinates, this corresponds to the current $\mathbf{J} = \mathbf{J}' + \rho\dot{\mathbf{u}}$. By comparing this with (8) with $\mathbf{E}^* = 0$ and $\nabla n = 0$, we get

$$R\dot{\mathbf{u}} = -c^{-1}en\mu_H[\mathbf{H}\dot{\mathbf{u}}] + \rho\dot{\mathbf{u}}. \tag{12}$$

This relation determines the tensor R, inasmuch as the velocity $\dot{\mathbf{u}}$ is arbitrary. In the linear approximation, the term $\rho\dot{\mathbf{u}}$ should be discarded.

The accompanying set of coordinates introduced above, strictly speaking, is not inertial, since $\ddot{\mathbf{u}} \neq 0$. Therefore, an additional d'Alembert force $-m_0\ddot{\mathbf{u}}$ acts on the current carrier in them, where m_0 is the mass of the free electron. For the sound frequencies considered, this force is negligible; the contribution from it to the current is of the order $m_0c\omega Ru/eH \ll R\dot{u}$. For weak magnetic fields, the latter inequality can be violated but then one can also neglect the term $R\dot{u}$.

2. WAVE EQUATIONS AND THEIR SOLUTION

The equation of continuity $\dot{\rho} + \operatorname{div}\mathbf{J} = 0$, after substituting in it (5), (7) and (8), and under the assumption that $\mathbf{u}$ and ρ are proportional to $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, takes the form

$$\left\{\frac{1}{\tau_1'} - i[\omega - q(\mathbf{kv})]\right\}\rho = \frac{\varepsilon_0 k^2\sqrt{\gamma}}{4\pi\tau}\{s_1M_1u_{\parallel} + s_2M_2u_{\perp} + s_2M_3u_p\}. \tag{13}$$

Here we have introduced the notation

$$\frac{1}{\tau} = \frac{4\pi en_0}{\varepsilon_0}(\mathbf{e}_{\parallel}, \mu_H\mathbf{e}_{\parallel}), \quad \frac{1}{\tau_1'} = \frac{1}{\tau} + qk^2(\mathbf{e}_{\parallel}, D_H\mathbf{e}_{\parallel}),$$

$$\frac{1}{\tau_0} = \frac{4\pi en_0\mu_0}{\varepsilon_0}; \quad M_1 = \frac{1}{s_1\sqrt{\gamma}}\left\{(g_1+g_2)E_{0\parallel} - \frac{i|\mathbf{k}|b_0}{e} - \frac{q_1}{q}\frac{\tau}{\mu_0\tau_0}\left[qv_{\parallel} - \frac{i}{|\mathbf{k}|}\left(\frac{1}{\tau_1'} - \frac{1}{\tau}\right)\right] - \frac{4\pi\omega\tau}{\varepsilon_0|\mathbf{k}|}(\mathbf{e}_{\parallel}, R\mathbf{e}_{\parallel})\right\},$$

$$M_2 = \frac{1}{s_2\sqrt{\gamma}}\left\{\frac{g_2}{2}E_{0\perp} - \frac{4\pi\omega\tau}{\varepsilon_0|\mathbf{k}|}(\mathbf{e}_{\parallel}, R\mathbf{e}_{\perp})\right\},$$

$$M_3 = -\frac{4\pi\omega\tau}{\varepsilon_0|\mathbf{k}|s_2\sqrt{\gamma}}(\mathbf{e}_{\parallel}R\mathbf{e}_p); \tag{14}$$

$\mathbf{v} = \mu_H\mathbf{E}_0$ is the drift velocity of the current carriers, s_1 and s_2 are the velocities of the longitudinal and transverse sound waves, γ is the density of the material; $\mathbf{e}_{\parallel}$, $\mathbf{e}_{\perp}$ and $\mathbf{e}_p$ are respectively the unit vectors of direction of $\mathbf{k}$, the direction perpendicular to $\mathbf{k}$ and lying in the plane $\mathbf{k}$, $\mathbf{E}_0$, and the direction perpendicular to $\mathbf{k}$ and $\mathbf{E}_0$; $u_{\parallel}$, $u_{\perp}$ and u_p are the projections of the vector $\mathbf{u}$ on these directions.

We now proceed to consider the equation of motion of the crystal lattice $\gamma\ddot{\mathbf{u}} = \mathbf{F}_1$. By substituting $\mathbf{u}$ and ρ in the exponential form and projecting this equation on the three directions, we get the following relation for the amplitudes:

$$(k^2 - k_1^2)u_{\parallel} = \frac{L_1}{s_1\sqrt{\gamma}}\rho; \quad (k^2 - k_2^2)u_{\perp} = \frac{L_2}{s_2\sqrt{\gamma}}\rho;$$

*$[\mathbf{hE}] \equiv \mathbf{h} \times \mathbf{E}$.

$$(k^2 - k_2^2)u_p = \frac{L_3}{s_2\sqrt{\gamma}}\rho. \tag{15}$$

Here we have introduced the notation

$$L_1 = \frac{1}{s_1\sqrt{\gamma}}\left\{(g_1 + g_2 + 1 - q)E_{0\parallel} + \frac{i|\mathbf{k}|}{e}b + \frac{m_0}{m\mu_0}\left[qv_\parallel - \frac{i}{|\mathbf{k}|}\left(\frac{1}{\tau_1'} - \frac{m}{m_0\tau_0}\right)\right]\right\},$$

$$L_2 = \frac{1}{s_2\sqrt{\gamma}}\left\{\left(\frac{g_2}{2} + 1 - q\right)E_{0\perp} + \frac{m_0}{m\mu_0}\left(qv_\perp - \frac{i}{|\mathbf{k}|\tau_2'}\right)\right\},$$

$$L_3 = \frac{qv_p - i/|\mathbf{k}|\tau_3'}{s_2\sqrt{\gamma}\,\mu_0}\frac{m_0}{m};$$

$$\frac{1}{\tau_2'} = \frac{4\pi e n_0}{\varepsilon_0}(\mathbf{e}_\perp, \mu_H\mathbf{e}_\parallel) + qk^2(\mathbf{e}_\perp, D_H\mathbf{e}_\parallel),$$

$$\frac{1}{\tau_3'} = \frac{4\pi e n_0}{\varepsilon_0}(\mathbf{e}_p, \mu_H\mathbf{e}_\parallel) + qk^2(\mathbf{e}_p, D_H\mathbf{e}_\parallel);$$

$$k_1 = \frac{\omega}{s_1},\quad k_2 = \frac{\omega}{s_2}. \tag{16}$$

The system of linear homogeneous equations (13) and (15) determines the amplitudes of the quantities $\mathbf{u}$ and ρ. One of these solutions has the form

$$k = k_2,\ \rho = 0,\ u_\parallel = 0,\ M_2u_\perp + M_3u_p = 0. \tag{17}$$

This wave does not interact with ρ and E_0 and is not amplified ($\mathbf{k}$ real). It is the usual transverse sound wave and in what follows we shall not be interested in it.

For all the other solutions, k is different from k_2. For them, elimination of $u_\perp$ and u_p from (13) and (15) leads to the following condition of solvability of these equations:

$$P(k, a_1, a_2) \equiv (k^2 - k_1^2)(k^2 - k_2^2)(k - k_3) + ik^2[a_1(k^2 - k_2^2) + a_2(k^2 - k_1^2)] = 0, \tag{18}$$

where

$$k_3 = \frac{\omega}{qv_\parallel} + \frac{i}{qv_\parallel\tau_1'},\qquad a_1 = \frac{M_1L_1\varepsilon_0}{4\pi q\tau v_\parallel},$$
$$a_2 = \frac{(M_2L_2 + M_3L_3)\varepsilon_0}{4\pi q\tau v_\parallel}. \tag{19}$$

The formula (18) corresponds to Eq. (20) in[4].

From (18) one can determine the dispersion law $\mathbf{k}(\omega)$ for the remaining waves. In the general case, this can be done numerically if the parameters which enter into (8) are known. If we assume that the constants of the electron-phonon coupling a_1 and a_2 are sufficiently small, we can obtain an approximate analytic solution of Eq. (18).

In the zeroth approximation, we set $a_1 = a_2 = 0$ in Eq. (18). Then the five roots of Eq. (18) are $\pm k_1$, $\pm k_2$, and k_3. For definiteness, we assume the real parts of k_1, k_2 and k_3 to be positive. We then introduce the terms with a_1 and a_2 as small perturbations and find the corrections to the values of the roots given above. With this, as is well known, we obtain significantly different results for roots that are isolated in the zeroth approximation and for neighboring roots. Usually the numbers k_1 and k_2 are not close together (they differ by a factor of one and a half or two). Therefore, the roots $-k_1$ and $-k_2$ are always isolated. If k_3 is close to neither k_1 nor k_2, then all five roots are isolated.

For an arbitrary isolated root, the first-order correction is equal to

$$\Delta(k_i) \equiv k_i - k_i^0 = -P(k_i^0, a_1, a_2)\Big/\left[\frac{\partial P}{\partial k}(k, 0, 0)\right]_{k=k_i^0}. \tag{20}$$

Here k_i^0 is the value of the i-th isolated root in the zeroth approximation. Substituting $P(k, a_1, a_2)$ in (20) from (18), we get (j = 1, 2).

$$\Delta(\pm k_j) = -\frac{i}{2}\frac{k_ja_j}{k_j \mp k_3} = \frac{k_ja_jqv_\parallel\tau_1'}{2}\frac{\pm 1 - i\omega\tau_1'[qv_\parallel/s_j \mp 1]}{1 + (\omega\tau_1')^2[qv_\parallel/s_j \mp 1]^2}, \tag{21}$$

$$\Delta(k_3) = -i\left[\frac{a_1}{1 - k_1^2/k_3^2} + \frac{a_2}{1 - k_2^2/k_3^2}\right]. \tag{22}$$

Formulas (21) have the usual form and correspond to results for the amplification of sound waves given in the literature. In particular, they correspond to formulas (22) and (24) in[4]. Formula (22) describes the space-charge wave (the drift wave) and corresponds to Eq. (26) in[4].

In the amplification of hypersound, however, the most interesting case is the one in which the drift velocity of the carriers is close to one of the sound velocities. If the imaginary part of k_3 in this case is sufficiently small (see below), then the root k_3 is close to k_1 or to k_2. In this case Eqs. (20)—(22) are not applicable for neighboring roots. Then the correction for the isolated roots $-k_1$ and $\pm k_2$ is found from Eq. (21). Knowledge of these roots permits us to lower the power of (18) and to obtain for the remaining roots a quadratic equation that is valid also when the root k_3 is far from k_1. The solution of the quadratic equation, with accuracy up to terms of second order in a_1 and a_2, has the form[2)]

[2)]The coefficients a_1, a_2, and k_3 depend on k through the parameter τ_1' introduced in Eqs. (4), (16), and also through the deformation potential. This dependence is assumed to be weak and must be replaced in these coefficients by the corresponding root (18) of the zeroth approximation.

$$k=\frac{1}{2}\Big\{k_1+k_3-\frac{i}{2}a_1\frac{k_1+2k_3}{k_1+k_3}+ia_2\frac{k_3^2}{k_2^2-k_3^2}$$
$$\pm\Big[(k_1-k_3)^2-2ia_1k_3-i(k_1-k_3)$$
$$\times\Big(\frac{a_1k_1}{k_1+k_3}+\frac{2a_2k_3^2}{k_2^2-k_3^2}\Big)\Big]^{1/2}\Big\}\ . \qquad (23)$$

The expansion of the root in the powers of a_1 and a_2 leads to Eqs. (21) and (22). If the roots k_1 and k_3 are close together ($|k_1-k_3|\ll|k_3|$), then Eq. (23) is greatly simplified:

$$k=1/2\{k_1+k_3+[(k_1-k_3)^2-2ia_1k_3]^{1/2}\}. \qquad (24)$$

To obtain Eqs. (21)—(24), we have assumed that $|a_1|,\ |a_2|\ll|k_j|$.

If the roots are so close that $|k_1-k_3|^2\ll|a_1k_3|$, Eq. (24) is further simplified:

$$k=1/2\{k_1+k_3\mp(1-i)\sqrt{a_1k_1}\}. \qquad (25)$$

In all cases, the ratio of the amplitudes is determined by Eqs. (15), in which it is necessary to substitute the corresponding values of **k**. From these formulas it is seen that the solution of (21) with $j=1$ corresponds to a wave in which $u_{\parallel}\gg u_{\perp},\ u_p$, while for $j=2$, we have $u_{\parallel}\ll u_{\perp},\ u_p$, and these waves are predominantly acoustic. Solution (22) corresponds predominantly to a drift wave. The solution (23) is a mixed acoustic-charge wave.

3. DISCUSSION OF THE RESULTS. ESTIMATES

The intensity of the wave in space varies according to the law

$$I=I_0\exp[-2(\mathbf{e}_{\parallel},\mathbf{r})\operatorname{Im}k].$$

Thus amplification takes place if the imaginary part of the wave vector $\operatorname{Im}k<0$. Here **k** is determined by Eqs. (21)—(25), but to the imaginary parts of **k** there must still be added quantities which take into account phenomenologically the different wave damping mechanisms not treated in the given theory.

In Eqs. (14) and (16), the expressions M_i and L_i contain terms resulting from completely different mechanisms of electron-phonon interaction. These terms can be of different orders of magnitude; therefore, it is convenient to estimate them and to discard those of higher order. For the estimates, we shall assume the quantities q, q_1, and b_0/b to be of the order of unity. In (14), in the expression for M_1, the ratio of the terms $i|\mathbf{k}|b_0/e$ to the term

$$\frac{iq_1\tau}{|\mathbf{k}|q\mu_0\tau_0}\Big(\frac{1}{\tau_1'}-\frac{1}{\tau}\Big)$$

is equal to b_0/q_1k_0T. If b_0 is of the order of 5—10 eV, then the second term can be neglected. In (16) in the expression for L_1, one can discard the term with $1/\tau_1'$ in comparison with the term $i|\mathbf{k}|b/e$, but if H is large, one must keep the term with $1/\tau_0$.

If $g_1,\ g_2\gg1$, one can discard the terms with $v_{\parallel}$ and $v_{\perp}$ in L_1 and L_2 in comparison with $g_1E_{0\parallel}$ and $g_2E_{0\perp}$, respectively, for arbitrary magnetic fields. In M_1 this can be done only in the absence of a magnetic field.

Thus, for H = 0, if we neglect the difference between b_0 and b and add the piezoelectric interaction, we obtain

$$L_1=M_1^*=\frac{1}{s_1\sqrt{\gamma}}\Big\{(g_1+g_2)E_{0\parallel}-\frac{d_{\parallel,\parallel,\parallel}}{\varepsilon_0}+ib\frac{|\mathbf{k}|}{e}\Big\},$$
$$a_1=\frac{\varepsilon_0|L_1|^2}{4\pi q\tau v_{\parallel}},\quad L_2=M_2=\frac{1}{s_2\sqrt{\gamma}}\Big\{\frac{g_2}{2}E_{0\perp}-\frac{d_{\parallel,\parallel,\perp}}{\varepsilon_0}\Big\}\ a_2$$
$$=\frac{\varepsilon_0(L_2^2+L_3^2)}{4\pi q\tau v_{\parallel}},\quad L_3=M_3=-\frac{d_{\parallel,\parallel,p}}{s_2\sqrt{\gamma}\,\varepsilon_0}. \qquad (26)$$

Here d_{ikl} is the piezoelectric tensor entering in the formula

$$D_i=D_{0i}+\varepsilon_{ik}E_k+d_{ikl}u_{kl}, \qquad (27)$$

where **D** is the electric induction, **E** the field intensity, and u_{kl} the deformation.

We now consider a strong magnetic field $\mathbf{H}\perp\mathbf{k},\ \mathbf{E}_0$. In the estimates, we shall assume $v_{\parallel}$ to be of the order of the phase velocity of the wave. Then in the expression for M_2 in (14) the ratio of the first component to the second is of the order $g_2/2$, i.e., the second component can be neglected. The corresponding component in M_1, connected with the friction force, must be retained, since its ratio to the first component is of the order of

$$\frac{\mu_0H}{c}\frac{E_{0\perp}}{(g_1+g_2)E_{0\parallel}}.$$

Thus, in the case considered, after the appropriate terms have been neglected, we get

$$M_1=\frac{1}{s_1\sqrt{\gamma}}\Big\{(g_1+g_2)E_{0\parallel}-\frac{q_1cH}{Q}E_{0\perp}$$
$$+\frac{H^2s_1}{Q}-\frac{d_{\parallel,\parallel,\parallel}}{\varepsilon_0}+ib_0\frac{|\mathbf{k}|}{e}\Big\},$$
$$L_1=\frac{1}{s_1\sqrt{\gamma}}\Big\{(g_1+g_2)E_{0\parallel}-\frac{d_{\parallel,\parallel,\parallel}}{\varepsilon_0}+i\frac{|\mathbf{k}|}{e}\Big(b+\frac{4\pi e^2n_0}{\varepsilon_0k^2}\Big)\Big\},$$
$$L_2=\frac{1}{s_2\sqrt{\gamma}}\Big\{\frac{g_2}{2}E_{0\perp}-\frac{d_{\parallel,\parallel,\perp}}{\varepsilon_0}+i\frac{4\pi cen_0m_0}{\mu_0H\varepsilon_0|\mathbf{k}|m}\Big\}, \qquad (28)$$

M_2, M_3, and L_2 have the same form as in (26).

For the magnetic field selected above, according to (10),

$$\mathbf{v}=-\frac{c}{H}[\mathbf{hE}_0]+\frac{Q}{H^2}\mathbf{E}_0+\ldots,\quad \frac{\mu_0H}{c}\gg1. \qquad (29)$$

We are interested in the case in which this drift velocity is of the order of the sound velocity s_i (i = 1, 2). It then follows from (29) that $E_0/H \sim s_i/c$, i.e., $E_0 \ll H$. It then follows that both the components that contain H in the expression for M_1 in (28) are always of the same order. If we set $Q = c^2/\mu_0$ for the estimate (which is obtained if we set $\nu = 0$ in (11)), then the ratio of each of these terms to the term $g_1 E_{0\|}$ is of the order

$$\frac{\mu_0 H}{c g_1} \approx \frac{eH\tau_f}{mcg_1} = \frac{\omega_c \tau_f}{g_1},$$

where ω_c is the cyclotron frequency. For the strong magnetic fields under consideration $\omega_c \tau_f \gg 1$. This inequality, as is well known, is the condition for the possibility of observing cyclotron resonance in the crystal. This is satisfied comparatively rarely (low temperatures, high mobilities). In order of magnitude, $\omega_c \tau_f \approx \mu_0 H/c$ does not usually exceed 10. As to the value of g_1, in crystals with a large dielectric constant, it is much greater than 10. Therefore, in such crystals, one can neglect the terms in the expression for M_1 with H and H^2 even for the strong magnetic field.

In (28), the terms with n_0 can be shown to be important for large concentrations of carriers and long waves. Below, in the consideration of crystals with very large dielectric constants or piezomoduli, we shall neglect in (28) terms with n_0 and b.

For an estimate of the order of magnitude of the constant g_1, the approximate value $g_1 \approx (\epsilon_0 + 2) \times (\epsilon_0 - 1)/3\epsilon_0 \sim \epsilon_0/3$ was obtained in[4], starting out from the generalized Lorentz-Lorenz law, i.e., under the assumption that only the concentration of molecules changes with pressure, and not the molecular polarizability. Actually, the latter also changes with pressure, which makes a contribution to the change of ϵ of the same order as the change in the concentration of the molecules. There is no foundation for assuming that these contributions almost cancel each other. Therefore, one can hope that the estimate $g_1 \sim \epsilon_0/3$ gives the correct order of magnitude (but not necessarily the correct sign).

The dependence of ϵ on the hydrostatic pressure was studied experimentally in barium titanate ($BaTiO_3$). For this ferroelectric, the Curie temperature, without pressure is equal to 120°C; it falls with increase in pressure. For temperatures below the Curie temperature, when the barium titanate is tetragonal, we have experimentally $(\epsilon - \epsilon_0)/\epsilon_0 = 0.07$—$0.10$ for a single crystal at a pressure of 10^3 kg/cm^2.[6] If we disregard the anisotropy of this crystal and use for the estimate Eq. (1) and also the equation for an isotropic medium div $u = -3p/(3\lambda + 2\mu)$, and take the value 10^{13} dynes/cm^2 for $3\lambda + 2\mu$ (a certain mean value of the elastic moduli was taken in the great scatter of experimental data), then we obtain for the pressure $p = 10^3$ kg/cm^2, div $u = -3 \times 10^{-4}$. We then obtain $g_1 = 230$—330. The theoretical estimate of $g_i \sim \epsilon_0/3$ gives 330—1000, for ϵ_0 equal to 10^3[8] and 3×10^3[9] respectively.

At a temperature above the Curie point, at 123°C, when the barium titanate possesses cubic symmetry, we have experimentally[8] $\epsilon_0 = 6 \cdot 10^3$, $(\epsilon - \epsilon_0)/\epsilon_0 = 0.16$ for a single crystal at a pressure of 10^3 kg/cm^2. Then for the previous value of div u, the experimental value of g_1 is seen to be equal to $g_1 = -500$ (the closeness to the Curie temperature?). According to the theoretical estimate, we get $g_1 = +2000$. Thus the estimate gives the correct order of magnitude. There are still no data for the determination of the constant g_2. There is no reason for thinking that g_2 can exceed g_1 in order of magnitude.

As an example, we consider a crystal with a large dielectric constant, in which one can neglect all the mechanisms of electron-phonon interaction except the interaction which is proportional to the applied field. If we introduce the notation

$$a_j = a_j^0 \frac{k_j}{\omega\tau_1'} \frac{qv_\|}{s_j},$$

$$a_j^0 \equiv \frac{\varepsilon_0}{4\pi\gamma (\mathbf{e}_\|, \mu_H \mathbf{e}_0)^2 q^2} \frac{\tau_1'}{\tau} \begin{cases} (g_1 + g_2)^2 (\mathbf{e}_\|, \mathbf{e}_0)^2, & j = 1, \\ \left(\frac{g_2}{2}\right)^2 (\mathbf{e}_\perp, \mathbf{e}_0)^2, & j = 2, \end{cases}$$

$$\mathbf{e}_0 = \frac{\mathbf{E}_0}{|\mathbf{E}_0|}, \qquad (30)$$

then k/k_j [determined by Eq. (23)] will depend on three dimensionless parameters:

Figure 1 shows the dependence of the absorption coefficients $\alpha \equiv 2$ Im (k) on the electric field E_0. The continuous curves represent $\alpha_\pm/k_1$, determined by Eq. (23). For comparison, the dotted lines represent the values of $\alpha_{1,3}/k_1$ obtained from Eqs. (21) and (22), which are valid for isolated roots. All the roots correspond to values of the parameters $a_1^0 = 0.5$, $a_2^0 = 0$ and $\omega\tau_1' = 30$. Such a_j^0 are realized, for example, for $\mathbf{E}_0$ parallel to k (or $g_2 = 0$), $\epsilon_0 = 2000$, $g_1 + g_2 \sim \epsilon_0/3$, $\gamma = 4$ g/cm^2, q = 1, $(\mathbf{e}_\|, \mu_H \mathbf{e}_0) = 20$ cm^2/V-sec, $\tau_1' = \tau$, and H = 0. In the region of fields (drift velocities) when the dotted curves depart from the continuous curves, Eqs. (21) and (22) are inapplicable and it is necessary to use the more general formulas (23) and (24). This can occur for any type of electron-phonon coupling if it is sufficiently large, whether or not $\omega\tau_1'$ is large. Thus, if $|k_j - k_3|^2 \lesssim |2a_j k_3|$, Eqs. (21) and (22) do not follow from (23) and it is impossible

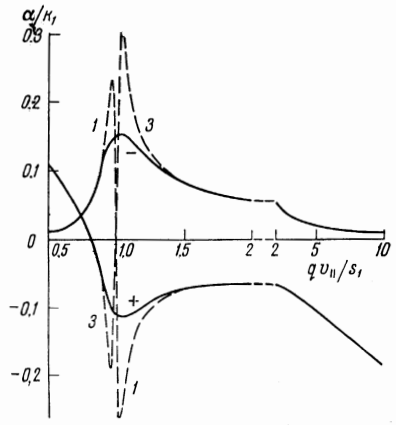

FIG. 1. Absorption coefficients of the acoustical-electric waves $\alpha = 2\,\mathrm{Im}\,(k)$. The curves + and − are computed from the general formula (23), curves 1 and 3 from Eqs. (21) and (22) for isolated roots;

$$a_1^0 = 0.5,\ a_2^0 = 0,\ \omega\tau_1' = 30$$

to use them (see[2e], p. 1296). In the region $qv_{||} \approx s_1$, this inequality can be rewritten in the form

$$\left|\frac{M_j L_j \varepsilon_0 \tau_1'}{2\pi\tau}(1 + i\omega\tau_1')\right| \gtrsim 1 \qquad (31)$$

For the piezoelectric electron-phonon interaction in CdS in the research of Hutson, McFee, and White[2], $ML\epsilon_0/2\pi = 0.07$ and $\omega\tau_1' = 5$ at 45 MHz. Here, the left side of (31) is equal to 0.35 and one can still use Eq. (21). But if $\omega\tau_1'$ increases by 5—10 times as a result of a change in the frequency or carrier density, or as a result of switching on of a magnetic field, then, according to (31), Eq. (21) cannot be used for the isolated roots.

It was noticed in[4] that in an isotropic medium, for a drift of the current carriers and for any type of electron-phonon coupling, there are not one but two longitudinal acoustic-charge waves. It was shown that under favorable conditions one of them is amplified when the drift velocity of the carriers is less than the velocity of sound. All this is illustrated in Fig. 1.

When E_0 or $v_{||}$ increases, the curve $\alpha_+ = \alpha_1$ approaches a straight line asymptotically; the amplification factor, passing through a minimum, begins to increase linearly with the field. This is a characteristic of an electron-phonon interaction proportional to the external field.

Figure 2 shows the real part of k, computed from Eq. (23) (solid curve) or computed from Eqs. (21) and (22) (dotted). Figure 2 corresponds to the same values of the parameters as in Fig. 1. It is seen from the drawing that wave 1 approaches sound in its properties with a phase velocity $\approx s_1$. The wave 3 approaches a drift wave, with a phase velocity $\approx qv_{||}$.

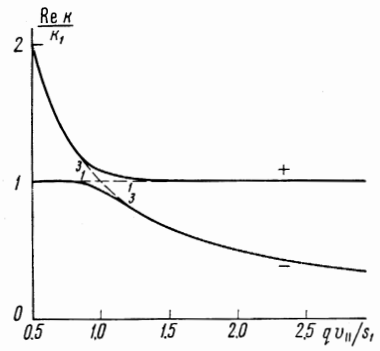

FIG. 2. Real parts of the wave vectors. The notation on the curves and the parameters are the same as in Fig. 1.

In the region of maximum amplification, $\alpha_+/k_1 = -0.113$. For a frequency of 45 MHz and for $s_1 = 2 \times 10^5$ cm/sec, this corresponds to 690 dB/cm.

In conclusion, we consider the dependence of the parameters a_j^0 and $\omega\tau_1'$ on the strength of the magnetic field H for two different relative orientations of the vectors k, E_0, and H. The magnetic field is assumed to be strong and for v one uses Eq. (10). Only an electron-phonon interaction which is proportional to the external applied field is considered, and in the expressions for M_j and L_j we neglect the terms containing H. We note that for $H \perp k$

$$\frac{1}{\tau} = \frac{4\pi e n_0}{\varepsilon_0}\frac{Q}{H^2}. \qquad (32)$$

Therefore, $\omega\tau_1'$ increases with the magnetic field proportionally to H^2. The ratio τ_1'/τ does not depend on the magnetic field:

$$\frac{\tau_1'}{\tau} = \frac{1}{1 + k^2 r^2}, \quad r = \left(\frac{qk_0 T\varepsilon_0}{4\pi e^2 n_0}\right)^{1/2}. \qquad (33)$$

Here k_0 is the Boltzmann constant and r the Debye screening radius.

A. For $k||E_0 \perp H$, according to (10), $v_{||} = QE_0/H^2$, $a_2^0 = 0$, and $(e_{||} \cdot \mu_H e_0) = Q/H^2$, i.e., a_1^0 (see (30)) is proportional to H^4. If it is desirable to keep $v_{||}$ unchanged as H increases (for example, to stay at maximum amplification if $\omega\tau_1' \gg 1$), then, simultaneously with the increase in H, one must increase E_0 proportionally to H^2. Here, in the case of roots that are close together, the maximum of $|\alpha_\pm|$ is proportional to H; in the case of isolated roots, the maximum of $|\alpha_1|$ is proportional to H^4.

B. For k, E_0, and H mutually perpendicular, $v_{||} = cE_0/H$, $a_1^0 = 0$, and $(e_{||} \cdot \mu_H e_0) = c/H$, i.e., a_2^0 is proportional to H^2. To keep $v_{||}$ constant, it is necessary to increase E_0 along with increase in H, in proportion with H.

As in case A, so also in case B, a_j^0 and $\omega\tau_1'$ increase with increase in H, i.e., the criterion of

neighboring roots (31) is confirmed. The simple power-law dependence of a_j^0 and τ_1' on H noted above is apparently valid only in the region of limiting large magnetic fields.

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