INVESTIGATION OF COUPLED OSCILLATIONS OF THE MAGNETIZATION IN FERRITE SINGLE CRYSTALS IN THE PRESENCE OF DOMAIN STRUCTURE

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A theoretical and experimental investigation has been carried out on coupled oscillations of the magnetization in cubic single crystals with a negative anisotropy constant in the presence of domain structure. An external constant magnetic field **H** was applied along the [011] direction of the crystal. The domains are considered to be of the form of plane layers, perpendicular to the (100) plane and making an angle α with the direction of the field **H**. The calculation shows that three types of oscillation exist; two involve precession of the magnetization within the domains, and the third involves motion of the domain boundaries. The demagnetizing fields of the domains and of the specimen determine the coupling between these types of oscillation. An analysis is made of the conditions for excitation of each of these types of oscillation by an external microwave field, and the relations that determine the resonance frequencies are derived. All three types of oscillation, and the coupling between them, have been observed experimentally (on specimens of magnesium-manganese ferrites). For specimens of yttrium-iron garnet, two types of oscillation have been detected. For ferrites with cubic structure, and in the presence of a domain structure, resonance absorption under longitudinal excitation ($\mathbf{h} \parallel \mathbf{H}$) has been detected. The good agreement between theoretical and experimental data supports the idea that the domain structure in magnesium-manganese ferrites is approximately parallel, whereas that in yttrium-iron garnet is approximately perpendicular.

INTRODUCTION

IN reference [1] the authors considered the influence of domain structure on the condition for ferromagnetic resonance (FMR) in ferrite single crystals with cubic symmetry and with a negative anisotropy constant K_1. An external constant magnetic field H is directed along the [011] axis. Experimental results obtained on single crystals of MgMn ferrites could not be fully explained within the framework of the theory under consideration. In the course of further investigations, the range of working frequencies was extended (on the low-frequency side), and the excitation conditions were changed. As a result of these measurements, additional absorption peaks were obtained, for which the resonance frequencies form a new branch in the ω'—H' plane. Analysis of the excitation conditions (see below) leads to the supposition that the additional resonance absorption detected is due to motion of the domain boundaries. In this connection we have carried out a calculation of the conditions for FMR in the presence of a domain structure, with allowance for displacement of the domain boundaries.

1. THEORY

Vlasov and Onoprienko [2] carried out a calculation of the FMR frequencies for a uniaxial single crystal in the presence of domain structure, with allowance for displacement of the domain walls. An analogous calculation can be made also for a specimen with cubic symmetry.

We consider the case of a single crystal with a negative first anisotropy constant K_1. We shall consider domains in the form of thin plates, perpendicular to the (100) plane and making an angle α with the external field H, which is directed along the [011] axis (Fig. 1). Following [2], we write the Lagrangian function

$$L = T - G,$$

where $T = T_M + T_\gamma$ is the "kinetic" energy density, T_M is the "kinetic" energy density of the domains, T_γ is the "kinetic" energy density

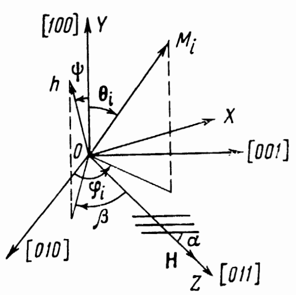

FIG. 1. Coordinate system and domain-boundary arrangement in relation to the constant magnetic field.

of the domain walls, and G is the density of the thermodynamic potential.

For a specimen in the form of an ellipsoid of revolution, with axis coinciding with the [011] direction (Fig. 1), the Lagrangian function has the form

$$
\begin{aligned}
L = {} & \nu M\gamma^{-1}\cos\theta_1\dot{\varphi}_1 + (1-\nu)M\gamma^{-1}\cos\theta_2\dot{\varphi}_2 + {}^1\!/_2 m_\gamma d\dot{\nu}^2 \\
& - {}^1\!/_4 K_1\nu(\sin^2 2\theta_1 + \sin^4\theta_1\sin^2 2\varphi_1) \\
& - {}^1\!/_4 K_1(1-\nu)(\sin^2 2\theta_2 + \sin^4\theta_2\sin^2 2\varphi_2) \\
& + \nu MH\sin\theta_1\cos(\pi/4-\varphi_1) \\
& + (1-\nu)MH\sin\theta_2\cos(\pi/4-\varphi_2) \\
& - {}^1\!/_2 M^2\{N_t[\nu\sin\theta_1\cos(\pi/4+\varphi_1) \\
& + (1-\nu)\sin\theta_2\cos(\pi/4+\varphi_2)]^2 \\
& + N_t[\nu\cos\theta_1 + (1-\nu)\cos\theta_2]^2 \\
& + N_z[\nu\sin\theta_1\cos(\pi/4-\varphi_1) + (1-\nu)\sin\theta_2\cos(\pi/4-\varphi_2)]\} \\
& - 2\pi M^2\nu(1-\nu)[\sin\theta_1\cos(\alpha-\varphi_1-\pi/4) \\
& - \sin\theta_2\cos(\alpha-\varphi_2-\pi/4)]^2 \\
& + Mh\{\sin\psi\cos\beta[\nu\sin\theta_1\cos(\pi/4-\varphi_1) \\
& + (1-\nu)\sin\theta_2\cos(\pi/4-\varphi_2)] \\
& + \cos\psi[\nu\cos\theta_1 + (1-\nu)\cos\theta_2] \\
& + \sin\psi\sin\beta[\nu\sin\theta_1\sin(\pi/4-\varphi_1) \\
& + (1-\nu)\sin\theta_2\sin(\pi/4-\varphi_2)]\}.
\end{aligned}
\tag{1}
$$

Here M is the saturation magnetization, m_γ is the effective mass of a boundary, d is the mean width of a domain, ν is the relative volume of one of the types of domain, N_t and N_z are the transverse and longitudinal demagnetizing factors of the specimen, θ_i and φ_i are the polar and azimuthal angles of the magnetization vectors in the domains, and ψ and β are the polar and azimuthal angles of the external alternating magnetic field vector (Fig. 1). The demagnetizing factor of the domains is taken equal to 4π. Included in the thermodynamic potential are: the anisotropy energy, the energy of interaction with the external constant magnetic field, the demagnetizing-field energy of the specimen, the energy of the demagnetizing fields of the domains, and the energy of interaction with the alternating magnetic field.

The equilibrium state of the system is determined by the equations

$$
\frac{\partial G}{\partial\theta_i} = \frac{\partial G}{\partial\varphi_i} = \frac{\partial G}{\partial\nu} = h = 0, \quad i = 1, 2. \tag{2}
$$

By solution of this system of equations we find

$$
\begin{aligned}
\nu = {}^1\!/_2, \quad \theta = \theta_1 = \pi - \theta_2, \\
\varphi_1 = \varphi_2 = \pi/4.
\end{aligned}
\tag{3}
$$

From (2) we get the expression for the magnetization curve,

$$
H' = \sin\theta\,[N_z M' - 2 + 3\sin^2\theta], \tag{4}
$$

where $M' = M/|K_1/M|$ is the reduced magnetization and $H' = H/|K_1/M|$ is the reduced magnetic field.

To find the characteristic frequencies of the system, we consider small oscillations near the equilibrium state; here we suppose that $q_i = q_{i0} + \Delta q_i e^{i\omega t}$, where $q_i = \nu$, θ_i, or φ_i. Following Vlasov and Onoprienko [2], we expand the Lagrangian function in a Taylor's series about the equilibrium state and take account only of terms of the second order of smallness with respect to h and Δq_i. In the expansion of T_M we suppose that $M_i = \nu M$ is constant in magnitude. On inserting the Lagrangian function, expanded as a series, into the Euler-Lagrange system of equations

$$
\frac{d}{dt}\frac{\partial L}{\partial(\Delta\dot{q}_i)} = \frac{\partial L}{\partial\Delta q_i}, \tag{5}
$$

we get

$$
\begin{aligned}
& A\Delta\theta^+ - iz\Delta\varphi^+ + E\Delta\varphi^- + n\Delta\nu = -h_y'\sin\theta, \\
& B\Delta\theta^- - iz\Delta\varphi^- = h_z'\cos\theta, \\
& iz\Delta\theta^+ + C\Delta\varphi^+ = h_x'\sin\theta, \\
& E\Delta\theta^+ + iz\Delta\theta^- + D\Delta\varphi^- = 0, \\
& n\Delta\theta^+ + [G_{\nu\nu} - z^2 m_\gamma d\gamma^2/M'\sin^2\theta]\Delta\nu = 2h_y'\cos\theta.
\end{aligned}
\tag{6}
$$

Here the following notation has been introduced:

$$
\begin{aligned}
A &= 2(G_{\theta_1\theta_1} + G_{\theta_1\theta_2}) = (1-\sin^2\theta)(9\sin^2\theta - 2) \\
&\quad + M'N_t\sin^2\theta + 4\pi M'\sin^2\alpha(1-\sin^2\theta), \\
B &= 2(G_{\theta_1\theta_1} - G_{\theta_1\theta_2}) = (1-\sin^2\theta)(9\sin^2\theta - 2 + M'N_z), \\
C &= 2(G_{\varphi_1\varphi_1} + G_{\varphi_1\varphi_2}) = \sin^2\theta(5\sin^2\theta - 2 + M'N_t), \\
D &= 2(G_{\varphi_1\varphi_1} - G_{\varphi_1\varphi_2}) = \sin^2\theta(5\sin^2\theta - 2 + 4\pi M'\cos^2\alpha), \\
E &= 4G_{\theta_1\varphi_2} = \pi M'\sin 2\alpha\sin 2\theta, \\
n &= -M'N_t\sin 2\theta, \qquad G_{\nu\nu} = 4M'N_t\cos^2\theta,
\end{aligned}
$$

$$z = \frac{\omega}{\gamma|K_1/M|}\sin\theta, \quad h' = \frac{h}{|K_1/M|},$$

$$\Delta\theta^{\pm} = \frac{\Delta\theta_1 \pm \Delta\theta_2}{2}, \quad \Delta\varphi^{\pm} = \frac{\Delta\varphi_1 \pm \Delta\varphi_2}{2}.$$

All derivatives ($G_{\theta\varphi}$ etc.) are evaluated in the equilibrium state. As is evident from the system (6) of equations of motion, there are three types of oscillation: $\Delta\theta^+$, $\Delta\varphi^+$; $\Delta\theta^-$, $\Delta\varphi^-$; and $\Delta\nu$. The first two types of oscillation are precessions of the magnetization within the domains; the third ($\Delta\nu$) is a displacement of domain boundaries. In general, all three types of oscillation are coupled. The coupling between oscillations of types (+) and (-) is present because of the demagnetizing fields of the domains and is determined by the coefficient E. The coupling between the displacement of the walls and the oscillations of the magnetization in domains is present because of the high-frequency demagnetizing field of the specimen and is determined by the coefficient n. Since displacement of the domain boundaries can occur only under the action of an asymmetric transverse field, with respect to which the domains are not equivalent, there is possible a direct coupling of the displacement of the domain boundaries with oscillations of type (+), which produce an unavoidable transverse demagnetizing field. According to this principle, coupling between $\Delta\nu$ and oscillations of type (-) is absent. The coupling between oscillations of types (+) and (-) is absent for parallel ($\alpha = 0°$) and perpendicular ($\alpha = 90°$) domain structures. The coupling between oscillations of types (+) and $\Delta\nu$ is greatest for a specimen in the form of a cylinder ($N_t = 2\pi$) and is absent for a disk ($N_t = 0$). In the absence of the couplings ($E = n = 0$), oscillations of type (+) are excited by a transverse high-frequency field (either symmetric, h_x, or asymmetric, h_y), oscillations of type (-) only by a longitudinal field h_z, oscillations $\Delta\nu$ only by an asymmetric field h_y. In the presence of the couplings, the oscillations are excited principally by these same high-frequency fields, but excitation of them by other components is also possible; the intensity of the oscillations, however, will be less under such excitation than under the principal excitation.

From the vanishing of the determinant of the system (6), the equation for the characteristic frequencies is obtained ($\omega' = \omega/\gamma|K_1/M|$ is the reduced frequency):

$$(\omega'^2 - \omega_{30}'^2)(\omega'^2 - \omega_1'^2)(\omega'^2 - \omega_2'^2) - \Lambda^4(\omega'^2 - \omega_{20}'^2) = 0;$$

$$\omega_{1,2}'^2 = \frac{1}{2\sin^2\theta}\{AC + BD \pm [(AC - BD)^2 + 4E^2BC]^{1/2}\},$$

$$\omega_{20}'^2 = \frac{BD}{\sin^2\theta}, \quad \omega_{30}'^2 = \frac{G_{\nu\nu}M'}{\gamma^2 dm_\gamma}, \quad \Lambda^4 = \frac{Cn^2M'}{\gamma^2 dm_\gamma \sin^2\theta}, \tag{7}$$

where $\omega_{30}'^2$ is the characteristic frequency of domain-boundary displacement, and $\omega_{1,2}'^2$ is the frequency of coupled oscillations of the magnetization in the domains. From the relations obtained it is evident that in order to calculate the dependence of all the frequencies on the field, it is necessary to know the value of the parameter $l^2 = \gamma^2 dm_\gamma$. Since this value is unknown, the relations $\omega' = f(H')$ will be discussed during analysis of the experimental results.

2. EXPERIMENT

An experimental investigation of FMR in the presence of domain structure was made on spherical specimens of single crystals of yttrium-iron garnet (YIG) and MgMn ferrite. The MgMn specimens used in the measurements were the same as in [1]. The YIG specimen had diameter 1.2 mm, $4\pi M = 1750$ G, $|K_1/M| = 43$ Oe, $\Delta H = 0.9$ Oe. The measurement method was like that used in [1], with the sole difference that the range was extended on the low-frequency side (to 560 MHz). In the range 1200 to 2200 MHz, the resonator used was Π-shaped, insuring both longitudinal and transverse (symmetric and asymmetric) excitation.

A. Yttrium-iron garnet. The results obtained for YIG are shown in Fig. 2a, in the form of a dependence of the reduced frequencies on the reduced field. Figure 3 gives the corresponding records of the absorption curves. As is evident from Fig. 2a, several branches of resonance frequencies were obtained; the resonance peaks for branches AK, AB, and AC are excited only by a transverse high-frequency field, either symmetric or asymmetric. Resonance absorption on branch IJ is observed only under transverse asymmetric excitation. Resonance absorption under longitudinal excitation was not detected in the frequency range used. The excitation conditions and the course of the dependence of resonance frequencies on the size of the field H′ lead to the supposition that in YIG there is a purely perpendicular domain structure ($\alpha = 90°$). The branch AK corresponds to FMR in a specimen magnetized to saturation, the branch AB to oscillations of type (+), and the branch IJ to oscillations of the domain boundaries, $\Delta\nu$. A resonance-frequency dependence similar to branch AB was obtained by Manuĭlova and Bogdanova [3]. The origin of the resonance absorption corresponding to branch AC

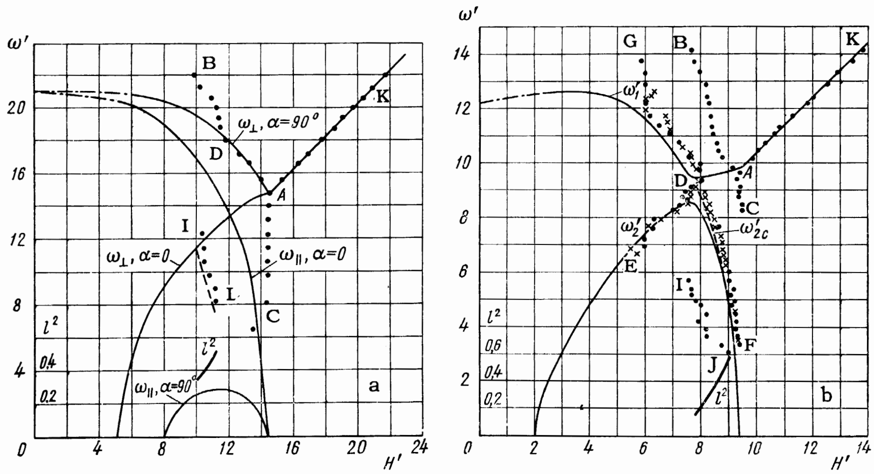

FIG. 2. Dependence of reduced resonance frequencies ω' and parameter l^2 on reduced constant magnetic field for spherical specimens: a, YIG; b, MgMn ferrite, $M' = 2.03$. Curves, theoretical; points, experimental: ●, $h \perp H$; X, $h \parallel H$.

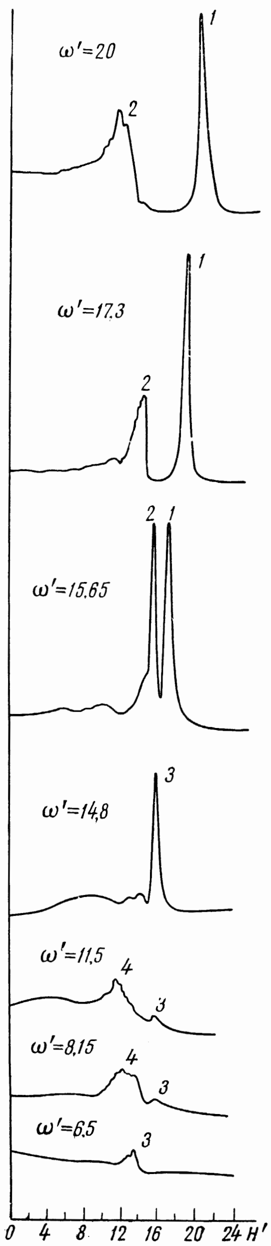

FIG. 3. Absorption curves for YIG specimen under transverse excitation, symmetric (frequencies 20 and 17.3) and asymmetric (frequencies 15.65 to 6.5): 1, resonance peak of branch AK; 2, branch AB; 3, branch AC; 4, branch IJ.

remains unclear to us and requires further investigation.

Drawn in solid lines in Fig. 2a are the theoretical dependences of the resonance frequencies ω_1' and ω_2' for a perpendicular domain structure, without allowance for coupling with boundary displacement. From a comparison of the experimental dependences with the theoretical it is apparent that on the section AD there is good agreement of the frequency values obtained with the theoretical. In the range of constant fields in which comparatively intense resonance of domain boundaries is observed (branch IJ), coupling of oscillations of type (+) with domain-boundary oscillations occurs, and this leads to an abrupt departure of the experimental values (section BD) from the theoretical curve, which was obtained without allowance for this coupling. The frequency range for longitudinal resonance lies appreciably below that used in the research (Fig. 2a), and therefore longitudinal resonance absorption was not detected. It should be remarked that the intensity of the absorption peaks for the branch IJ is greatest in the middle of the branch ($\omega' = 10$); for the branch AB, the intensity of the peaks increases on approach to the point A, and reaches the intensity of a resonance peak in the saturated ferrite ($\omega' = 15.65$ in Fig. 3). On further diminution of the frequency, the two peaks fuse and form a single peak, belonging to branch AC. The intensity of the absorption peaks of this branch decreases abruptly from A to C.

For comparison of the experimental values of the frequencies of branches IJ and BD with the theoretical dependences of the frequencies of coupled oscillations of magnetization precession and boundary displacement, it is necessary to determine the value of the parameter $l^2 = \gamma^2 \mathrm{d} m_\gamma$.

In the case of YIG, in which $\alpha = 90°$, one can, for example, determine this parameter according to the experimental value of frequencies of branch BD, and then use the theoretical formula to calculate frequencies corresponding to branch IJ.

From (7) we have

$$l^2 = \frac{\xi^2}{\omega_{1e}'^2}\left[1+\frac{p^4}{4\xi^2}\frac{1}{\omega_{1e}'^2-\omega_{1t}'^2}\right], \quad \xi^2 = 4M'^2N_t\cos^2\theta,$$

$$\frac{p^4}{\xi^2} = \frac{4\Lambda^4}{\omega_{30}'^2} = 4M'N_t(5\sin^2\theta - 2 + M'N_t),$$

$$\omega_{1t}'^2 = \frac{1}{\sin^2\theta}AC,$$

where ω_{1e}' are experimental values of frequency (branch BD). For frequencies of branch IJ we get

$$\omega_3'^2 = {}^1/_2\{\omega_{1t}'^2 + \omega_{30}'^2 - [(\omega_{1t}'^2 - \omega_{30}'^2)^2 + 4\Lambda^4]^{1/2}\},$$

where

$$\omega_{30}'^2 = \xi^2/l^2.$$

The result of the calculation was the dependence of l^2 on reduced field that is shown in Fig. 2a. The parameter l^2 decreases with decrease of field, from 0.52 at $H' = 11.2$ to 0.35 at $H' = 10$. Since the parameter l^2 is proportional to the ratio of the domain width to the boundary width, decrease of it can be due to increase of the boundary width on increase of the angle between the magnetization vectors in neighboring domains. The frequencies ω_3' calculated with this value of the parameter l^2 are drawn in Fig. 2a with a dotted line. It is seen that they agree well with the experimental points of the branch IJ.

B. Magnesium-manganese ferrites. Two specimens, of different compositions, were used in the measurements. The first, of diameter 1.54 mm, had $4\pi M = 3180$ G, $|K_1/M| = 125$ Oe; for the second, of diameter 1 mm, $4\pi M = 3000$ G, $|K_1/M| = 150$ Oe. Figure 4 gives several records of absorption curves for the first specimen, for various modes of excitation. The experimental dependences of frequency on field are given in Fig. 2b. In contrast to similar data given in [1], there are detected in the low-frequency range additional absorption peaks, corresponding to branch IJ, which are excited only by an asymmetric transverse high-frequency field. It was shown in [1], on the basis of an analysis of the intensity of the peaks from excitation modes and of the dependence of the frequencies on the field, that in the case being considered there is a lamellar domain structure, in which the boundaries make a small angle ($\alpha \approx 10°$) with the direction of the constant field.

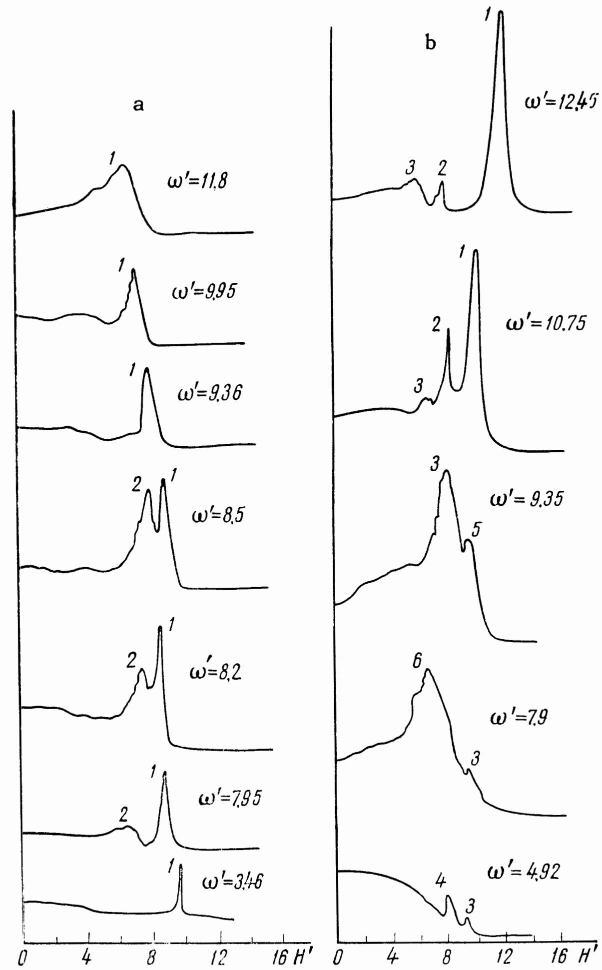

FIG. 4. Absorption curves for MgMn ferrite ($M' = 2.03$). a, longitudinal excitation: 1, resonance peak of curve FDG; 2, branch DE. b, transverse excitation, symmetric (frequencies 12.45 to 7.9) and symmetric (frequency 4.92): 1, resonance peak of branch AK; branch AB; 3, branch FDG; 4, branch IJ; 5, branch AC; 6, branch DE.

The resonance absorption corresponding to branch IJ, just as in YIG, corresponds to resonance of domain boundaries. The intensity of the absorption peaks is greatest in the middle of this branch ($\omega' = 4.25$). Calculation of the frequencies ω_2' and ω_3', by the method described above, for angles $\alpha = 5$ to $20°$, gives a large discrepancy with the experimental values, especially for branch IJ. In the theoretical calculation of frequencies in Section 1 it was supposed that the restoring force is determined by the demagnetized fields of the specimen surface. But MgMn ferrite, in contrast to YIG, is a material with considerable internal inhomogeneities, a fact which, in particular, results in a comparatively large coercive force for these ferrites. For this reason, the restoring force and the coupling between magnetization precession in the domains and displacement of the domain boundaries may, in MgMn ferrites, differ appreciably from that considered in the theory.

To a certain degree it is possible to take account of this peculiarity, by determining from the experimental values of the frequencies the two parameters ξ^2 and l^2, which characterize the frequency ω_{30}' and the coupling of the boundary displacement with the oscillations of the magnetization in the domains. Since the coupling between $\Delta\nu$ and the oscillations of type (+) expresses itself most strongly in ω_1 and ω_3, the experimental values of these frequencies were chosen for the calculation of the parameters indicated above. The parameters ξ^2 and l^2 were calculated with the following formulas, obtained from (7):

$$l^2 = \frac{p^4}{4}\frac{\eta_1 - \eta_3}{\omega_{1e}'^2 - \omega_{3e}'^2},$$

$$\xi^2 = \frac{p^4}{4}\frac{\omega_{3e}'^2\eta_1 - \omega_{1e}'^2\eta_3}{\omega_{1e}'^2 - \omega_{3e}'^2},$$

where

$$\eta_1 = \frac{\omega_{1e}'^2 - \omega_{20}'^2}{(\omega_{1e}'^2 - \omega_1'^2)(\omega_{1e}'^2 - \omega_2'^2)},$$

$$\eta_3 = \frac{\omega_{3e}'^2 - \omega_{20}'^2}{(\omega_{3e}'^2 - \omega_1'^2)(\omega_{3e}'^2 - \omega_2'^2)}$$

$$p^4 = 4M'^3N_t^2\sin^2 2\theta\,(5\sin^2\theta - 2 + M'N_t);$$

ω_{1e}' and ω_{3e}' are the experimental values of the frequencies of branches AB and IJ. Then by using the calculated parameters l^2 and ξ^2 and the value of the two roots ω_{1e}' and ω_{3e}' and solving equation (7), we find the value of the frequency ω_{2c}', corresponding to one of the types of coupled oscillations of the magnetization in the domains. The results of this calculation are given in Fig. 2b in the form of the dependence of l^2 and ω_{2c}' (dotted curve) on field. The parameter l^2 depends on the field in the same way as in YIG, and apparently that dependence of l^2 in MgMn ferrite is determined by the same mechanism as in YIG.

The frequency ω_{2c}' is slightly larger than the values ω_2' obtained by calculation according to formulas that disregard the influence of boundary motion ($n = 0$) but take account of coupling between the types of oscillation (+) and (-) ($\alpha = 10°$). But all the values of ω_{2c}' are smaller than the experimental values; this apparently means that there exist also other mechanisms of coupling between the oscillations under consideration. For fields $H' < 7.5$ Oe, the oscillations corresponding to resonance of domain boundaries (branch IJ) disappear, and there remain only the coupled oscillations (+) and (-). The character of the excitation of peaks of the branches DE and DG and the qualitative agreement of the experimental frequencies with the theoretical curves ($\alpha = 10°$) show that in this range of fields, coupling with boundary displacement is absent and the assumed model of domain structure exists.

Similar results were obtained also on another specimen of MgMn ferrite. In the case of MgMn, just as for YIG, resonance absorption corresponding to branch AC is observed.

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