DYNAMICS OF GENERATION OF GIANT COHERENT LIGHT PULSES. II.

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An investigation is made of the basic phases of development of giant pulses of coherent light: the formation of the field distribution in the linear domain of generation and the dependence of the distribution on the random initial amplitudes of the electromagnetic field, the transverse development of the generation in the nonlinear domain. The effect of the inhomogeneity in the index of refraction of the medium inside the resonator on the dynamics of generation of giant pulses is investigated.

1. INTRODUCTION

IN [1] (in future denoted by I) a theoretical investigation was given of the dynamics of generation of giant pulses of coherent light by an instantaneously Q-switched laser. Reference I was based on two nonlinear partial differential equations - an equation for the complex amplitude of the electromagnetic field in a resonator filled by an active medium, and an equation for the imaginary part of the dielectric permittivity of an active medium. The principal results of I were obtained by a numerical integration of these equations. The electrodynamic approach made it possible to investigate the space-time development of the pulse, the role played by the inhomogeneity of the distribution of the population inversion, the fine structure of the pulse and the divergence of the radiation. In particular, it was shown that the generation of the giant pulse proper begins in the central region of the crystal and in a time of the order of the duration of the pulse develops in a transverse direction encompassing the whole crystal. This result was verified by experimental investigations [2,3] 1). In [2] it was noted that the inhomogeneities in the index of refraction of the crystal exert a considerable influence on the space-time development of the generation and that it is necessary to take them into account.

The object of the present paper, which is a continuation of I, is, firstly, to investigate analytically the principal phases of the development of a giant pulse (the formation of a "jet" in the linear domain of generation, the dependence on the initial amplitudes of the electromagnetic field, the speed of transverse development of generation in the nonlinear domain) and, secondly, to investigate the effect of the inhomogeneities of the index of refraction of the medium inside the resonator on the dynamics of generation of a giant pulse.

The investigation is based on the equation for the complex amplitude of the electromagnetic field $\mathcal{E}(x, t)$ taking into account the inhomogeneities of the real part of the dielectric permittivity $\delta\epsilon'(x)$ [5]:

$$\frac{\partial \mathcal{E}(x,t)}{\partial t} = i\frac{c^2}{2\varepsilon_0\omega_0}\frac{\partial^2 \mathcal{E}(x,t)}{\partial x^2} - \frac{\omega_0}{2\varepsilon_0}[\varepsilon_0'' - \varepsilon''(x,t) + i\delta\varepsilon'(x)]\,\mathcal{E}(x,t), \tag{1}$$

where ϵ_0'' and $\epsilon''(x, t)$ describe respectively the losses and the amplification of the radiation, and on the equation for the imaginary part of the dielectric permittivity $\epsilon''(x, t)$:

$$\frac{\partial \varepsilon''(x,t)}{\partial t} = -\frac{2\sigma}{\hbar\omega_0}\varepsilon''(x,t)I(x,t), \qquad I(x,t) = \frac{c\varepsilon_0}{8\pi}\frac{\mathcal{E}\mathcal{E}^*}{2}, \tag{2}$$

where $I(x, t)$ is the flux density of the radiation, while the geometry of the problem and the notation is the same as in I.

The generation of a giant pulse can be divided into two phases. During the first phase after the Q switching, an exponential increase of the amplitude of the electromagnetic field occurs in the modes, from the spontaneous noise level up to a level almost sufficient for saturation of the amplification of the medium, but no appreciable saturation occurs as yet. Therefore, the first phase in the development of the pulse, the duration of which has been named the delay time, [6] can be called

1) It should be noted that observation of the transverse development of a giant pulse made it possible to understand why a giant pulse does not contract in the course of propagation in a *nonlinearly* amplifying and absorbing medium, and to propose a method of combatting this phenomenon [4].

the linear range of generation. During this time a "jet" of generation becomes established in the central region of the crystal. During the second phase saturation of the amplification of the medium occurs, i.e., radiation by the majority of the active particles. The generation of the giant pulse proper of radiation occurs during this time. The second phase may be called the domain of the nonlinear development of the generation. During this time the development of the initial "jet" occurs in the transverse direction and the generation encompasses the whole crystal.

2. LINEAR DEVELOPMENT OF GENERATION SOLUTION OF EQUATIONS

In the domain of the linear development of generation one can neglect the saturation of amplication $\epsilon''(x, t)$ and take $\epsilon''(x, t) \equiv \epsilon'(x)$. Then the investigation reduces to the solution of the single equation (1), which is analytically possible only for certain special forms of the functions $\delta\epsilon'(x)$ and $\epsilon''(x)$. Of practical interest are $\epsilon'(x)$ and $\epsilon''(x)$ which have the following form:

$$\varepsilon'(x) = \varepsilon_0 + \delta\varepsilon_0' / \mathrm{ch}^2 px, \qquad \varepsilon''(x) = \varepsilon_m'' / \mathrm{ch}^2 px. \quad (3)^*$$

Here $\epsilon'(x)$ represents, depending on the sign of $\delta\epsilon_0'$, a positive or a negative lens [2] while $\epsilon''(x)$ describes the distribution of the population inversion with a maximum at the center of the crystal.

We seek the solution of (1) in the form

$$\mathscr{E}(x, t) = U(x) e^{-i\Omega t}. \quad (4)$$

We obtain the following equation for the eigenfunctions and eigenvalues:

$$\frac{d^2U}{dx^2} + k^2\left(i\varepsilon_0'' - \frac{i\varepsilon_m'' + \delta\varepsilon_0'}{\mathrm{ch}^2 px} + \frac{2\Omega\varepsilon_0}{\omega_0}\right)U = 0, \quad (5)$$

where $k = \omega_0/c$. We consider only the discrete spectrum. The solution of (5) finite at $x = \infty$, is [8]

$$U = (1 - \xi^2)^{\mu_\pm/2} F(\mu_\pm - q, \mu_\pm + q + 1, \mu_\pm + 1, {}^1\!/_2(1 - \xi)), \quad (6)$$

where F is the hypergeometric function; $\xi = \tanh x$;

$$\mu_\pm = \pm i\frac{k}{p}\left[i\varepsilon_0'' + \frac{2\Omega\varepsilon_0}{\omega_0}\right]^{1/2},$$

$$2q + 1 = \left[1 - \frac{4k^2}{p^2}(\delta\varepsilon_0' + i\varepsilon_m'')\right]^{1/2};$$

the + sign corresponds to a positive lens ($\delta\epsilon_0' > 0$) and vice versa. In order that the solution should remain finite at $x = -\infty$ it is necessary to have (then F is a polynomial of the n-th degree in ξ)

$$\mu_+ - q = -n, \quad \mu_- - q = n + 1, \quad n = 0, 1, 2, \ldots \quad (7)$$

Relation (7) determines the complex eigenfrequencies $\Omega_n = \Omega_n' + i\Omega_n''$ of the different types of oscillations. In lasers utilizing luminescent crystals and glasses we have

$$\varepsilon_m'' \approx 10^{-6}, \quad \lambda \approx 10^{-4}\ \mathrm{cm}, \quad p \approx 1/a \approx 1\ \mathrm{cm}^{-1},$$
$$\delta\varepsilon_0' \gtrsim 10^{-5}.$$

Therefore, one can use the approximation

$$4\frac{k^2}{p^2}|\delta\varepsilon_0'|, \quad 4\frac{k^2}{p^2}\varepsilon_m'' \gg 1. \quad (8)$$

Then for $|\delta\epsilon_0'| \gg \epsilon_m''$ the expressions for the eigenfrequencies have the form

$$\frac{2\Omega_n'\varepsilon_0}{\omega_0} = -\delta\varepsilon_0' + \left(n + \frac{1}{2}\right)\frac{p}{k}\left\{\begin{matrix} 2\sqrt{\delta\varepsilon_0'} \\ \varepsilon_m''/\sqrt{|\delta\varepsilon_0'|} \end{matrix}\right\}$$
$$- \left(n + \frac{1}{2}\right)^2\frac{p^2}{k^2}, \quad (9)$$

$$\frac{2\Omega_n''\varepsilon_0}{\omega_0} = \varepsilon_m'' - \varepsilon_0'' - \left(n + \frac{1}{2}\right)\frac{p}{k}\left\{\begin{matrix} \varepsilon_m''/\sqrt{\delta\varepsilon_0'} \\ 2\sqrt{|\delta\varepsilon_0'|} \end{matrix}\right\}, \quad (10)$$

where the upper line inside the curly brackets corresponds to $\delta\epsilon_0' > 0$ while the lower line corresponds to $\delta\epsilon_0 < 0$. For $\delta\epsilon_0' = 0$ we have correspondingly

$$\frac{2\Omega_n'\varepsilon_0}{\omega_0} = \left(n + \frac{1}{2}\right)\frac{p}{k}\sqrt{2\varepsilon_m''} - \left(n + \frac{1}{2}\right)^2\frac{p^2}{k^2},$$
$$\frac{2\Omega_n''\varepsilon_0}{\omega_0} = \varepsilon_m'' - \varepsilon_0'' - \left(n + \frac{1}{2}\right)\frac{p}{k}\sqrt{2\varepsilon_m''}. \quad (11)$$

The expressions for the eigenfrequencies Ω_n have the following physical meaning. The term $(\omega_0/2\epsilon_0)\delta\epsilon_0'$ in (9) is a frequency shift which is the same for all the transverse modes by an amount determined by the change in the optical length at the center of the crystal. This term is of no interest to us. The second term linear in n is much larger than the third term which is quadratic in n since for reasonable values of n we have $(n + 1/2)p/k \ll 1$. Thus, there exists a set of practically equidistant modes, and this is in agreement with the theory of confocal resonators [9,10]. The term $(\omega_0/2\epsilon_0)(\epsilon_m'' - \epsilon_0'')$ in (10) describes the rate of increase in amplitude which is the same for all modes and which is determined by the amplification at the center of the crystal. The second term, which is usually much smaller than the first, characterizes the decrease in the

*ch ≡ cosh.

2)The case $\delta\epsilon'_0 = 0$ has been investigated in [7].

amplification coefficient for a particular mode as the number of this mode increases.

Substituting the values of μ into (6) we obtain expressions for the distribution of the square of the amplitude of the field of the principal mode over the end of the crystal:

$$|U_0|^2 = (1/\operatorname{ch} px)^{bk/p+1},$$

$$b = \begin{cases} 2\sqrt{\delta\varepsilon_0'} & \text{for } \delta\varepsilon_0' \gg \varepsilon_m'', \quad \delta\varepsilon_0' > 0 \\ \sqrt{2\varepsilon_m''} & \text{for } \delta\varepsilon_0' = 0 \\ \varepsilon_m''/\sqrt{|\delta\varepsilon_0'|} & \text{for } |\delta\varepsilon_0'| \gg \varepsilon_m'', \quad \delta\varepsilon_0' < 0 \end{cases} \quad (12)$$

where usually $bk \gg p$.

3. FIELD DISTRIBUTION AND DEPENDENCE ON INITIAL CONDITIONS

The distribution of the field in a laser at the end of the linear development of the pulse is, generally speaking, determined by a superposition of different types of oscillations

$$\sum_n E_{0n} U_n(x) \exp\{-i\Omega_n \tau_d\},$$

where E_{0n} are the initial random complex amplitudes of the field in different modes, and τ_d is the duration of the linear development of the pulse (the delay time). The delay time is obtained from the condition of a definite, say 20%, saturation of the amplification of the medium. From (2) it follows that for this it is necessary that the following condition be satisfied

$$\frac{c}{8\pi}\langle E_0^2\rangle \int_0^{\tau_d} \exp\{2\Omega_0'' t\}\, dt \approx \frac{\hbar\omega}{10\sigma}, \quad (13)$$

where $\langle E_0^2 \rangle$ is the average square of the intensity of spontaneous radiation in the given oscillation mode which was evaluated in I, and Ω_0'' is determined by expression (10). As a result of this we obtain a general expression for the time of linear development of generation:

$$\tau_d \approx \frac{1}{2\Omega_0''} \ln\left(\frac{5\Omega_0''\hbar\omega}{\sigma c\langle E_0^2\rangle}\right), \quad (14)$$

which is valid for a laser with an inhomogeneous distribution of the population inversion and of the index of refraction.

If during the delay time τ_d the amplitude of the principal mode becomes much larger than the amplitude of the subsequent modes then, firstly, the distribution of the field over the end of the crystal at the instant of generation of the giant pulse proper will not depend on the random initial amplitudes of the field and, secondly, the form of the distribution will be determined by expression (12) for $|U_0(x)|^2$. This holds under the condition

$$\exp[(\Omega_0'' - \Omega_1'')\tau_d] \gg 1 \quad \text{or} \quad \tau_d(\Omega_0'' - \Omega_1'') \gtrsim 2. \quad (15)$$

We first consider the case $\delta\epsilon_0' = 0$. Then condition (15) assumes the form

$$\frac{\tau_d c p}{2\varepsilon_0}\sqrt{\frac{\varepsilon_m''}{2}} \gtrsim 1. \quad (16)$$

For Q-switched lasers using luminescent crystals and glasses $\tau_d \approx 50$ nsec, $\epsilon_0 \approx 3$, $\epsilon_m'' \approx 2 \times 10^{-6}$. Condition (16) is satisfied for $p \gtrsim 4\ \text{cm}^{-1}$. Consequently, an inhomogeneity of population inversion with a transverse dimension of the order of 0.5 cm leads to a lack of dependence of the giant pulse on random initial conditions for the field.

In the case of an inhomogeneity of the refractive index $|\delta\epsilon_0'| \gg \epsilon_m''$ the following condition must be satisfied instead of (16):

$$\frac{\tau_d c p}{2\varepsilon_0}\begin{Bmatrix} \varepsilon_m''/2\sqrt{\delta\varepsilon_0'} \\ 2\sqrt{|\delta\varepsilon_0'|} \end{Bmatrix} \gtrsim 1, \quad (17)$$

where the upper row inside the figure brackets corresponds to $\delta\epsilon_0' > 0$, while the lower row corresponds to $\delta\epsilon_0' < 0$. In the case of an inhomogeneity of the type of a positive lens, $\delta\epsilon_0'$ must be sufficiently small ($\lesssim 3 \times 10^{-6}$), otherwise a dependence on the initial conditions would be observed. But a small inhomogeneity of the type of a negative lens ($\gtrsim 10^{-6}$) is sufficient to produce a lack of dependence on initial conditions. Physically these results are explained by the fact that for $\delta\epsilon_0' > 0$ the adjacent transverse modes are grouped near the axis of the resonator and are therefore amplified in the case of an inhomogeneous distribution of $\epsilon''(x)$ in practically the same manner. For $\delta\epsilon_0' < 0$ the transverse modes conversely extend into regions with smaller amplification, and this leads to a difference in their amplification coefficient.

Thus, an inhomogeneity in the distribution of population inversion and an inhomogeneity in the index of refraction of the type of a negative lens lead to a lack of dependence of the field distribution over the end of the crystal $|\mathscr{E}(x)|^2$ on random initial conditions. In this case the field distribution towards the end of the linear build up of the pulse is $|\mathscr{E}(x)|^2 \approx |U_0(x)|^2$. The magnitude of the generation at that instant (the size of the "jet") $2x_0$ is determined by the condition $|U_0(x_0)|^2 = 1/2$.

The results obtained above for an infinite layer can be directly carried over to the case of a finite layer (crystal) if the dimension of the established field distribution $2x_0$ is smaller than the crystal diameter $2a$ so that diffraction phenomena can be neglected.

In the other limiting case, when there are no transverse inhomogeneities in the population inversion and in the index of refraction, conditions (16) and (17) are not satisfied. Only the diffraction losses at the edges of the mirrors can discriminate between various modes, but during the short time of linear build up[3] ($\tau_d \lesssim 10^{-7}$ sec) the role played by such discrimination for the lower modes is unimportant. Therefore, the field distribution towards the end of the linear buildup of generation is random. In multimode lasers which usually generate tens of axial modes the random distributions for the different axial modes are averaged out and the total distribution is homogeneous. In such a laser the transverse development of the generation and the stretching out of the giant pulse associated with it do not occur. In single-mode lasers with instantaneous Q-switching and with a high degree of homogeneity of the resonator and of the population inversion the field distribution at the beginning of the generation of the giant pulse proper must be random.

We now proceed to investigate the transverse development of the region of generation in the case when the size of the region of generation is considerably smaller than the crystal diameter towards the end of the linear buildup.

4. NONLINEAR TRANSVERSE DEVELOPMENT OF GENERATION

The nonlinear phase of generation during which the giant pulse proper is generated begins from the moment when the field distribution formed during the linear buildup reaches an amplitude sufficient to saturate amplification. Saturation of amplification occurs first within the region of maximum amplitude of $|\mathscr{E}(x, t)|^2$, and then as the intensity is built up in the peripheral regions it extends in the transverse direction. Thus, the nonlinear transverse development of the region of generation is a result of the delay in the development of the amplitude of the field in the edge regions of the generator.

The transverse development of generation can be investigated with the aid of Eqs. (1) and (2). The term with $\partial^2\mathscr{E}/\partial x^2$ in (1) describes the linear (diffraction) diffusion of the field. For $x_0^2/L\lambda \gg 1$ the contribution of this term is negligibly small and it may be neglected. Results of exact calculations in I also show that the rate of diffraction diffusion of the field is small compared to the rate of the nonlinear transverse build up of generation. In this approximation going over to the intensity I(x, t) Eqs. (1) and (2) can be rewritten in the form

$$\frac{\partial I(x,\tau)}{\partial\tau}=\left[\varepsilon''(x)\exp\left\{-\frac{2\sigma}{\hbar\omega}\int_0^\tau I(x,\tau')\,d\tau'\right\}-\varepsilon_0''\right]I(x,\tau), \tag{18}$$

where $\tau = t - \tau_d$, τ_d is the time of the linear buildup (delay), $\epsilon''(x)$ describes the initial distribution of the population inversion. To determine the rate of transverse buildup it is sufficient to follow the transverse motion of the boundary of saturation of amplification.

The motion of a given level of saturation $\delta = \epsilon''(x, \tau)/\epsilon''(x)$ is determined by the condition

$$\int_0^{\tau_s} I(x,\tau)\,d\tau=\frac{1}{2\sigma}\ln\frac{1}{\delta}. \tag{19}$$

We follow the level of low saturation ($\delta \approx 0.8$) when $\epsilon''(x, \tau < \tau_s)$ can be considered as coincident with $\epsilon''(x)$. Then we have

$$I(x,\tau<\tau_s)=I_0(x)\exp\left\{\frac{\omega_0}{2\varepsilon_0}[\varepsilon''(x)-\varepsilon_0'']\tau\right\},$$

where $I_0(x) = I(x, 0)$ is the distribution of the field intensity towards the end of the linear buildup. Substituting the expression for $I(x, \tau < \tau_s)$ into (19) we obtain

$$I_0(x)\exp\left\{\frac{\omega_0}{\varepsilon_0}[\varepsilon''(x)-\varepsilon_0'']\tau_s\right\}\approx\frac{1}{2\sigma}\ln\frac{1}{\delta}\frac{\omega_0}{\varepsilon_0}[\varepsilon''(x)-\varepsilon_0'']. \tag{20}$$

Differentiating (20), and taking into account the fact that $\epsilon''(x)$ is a smoother function than $I_0(s)$

$$\frac{1}{I_0(x)}\frac{\partial I_0(x)}{\partial x}\gg\frac{1}{\varepsilon''(x)}\frac{\partial\varepsilon''(x)}{\partial x},$$

we determine the velocity of the motion of the saturation boundary $v = dx/d\tau_s$:

$$v=-\frac{\omega_0}{\varepsilon_0}[\varepsilon''(x)-\varepsilon_0'']\frac{I_0(x)}{\partial I_0(x)/\partial x}\bigg|_{x=x(\tau_s)}. \tag{21}$$

The quantity

$$s=I_0(x)\bigg/\frac{\partial I_0(x)}{\partial x}\bigg|_{x=x(\tau\ )}$$

determines the spatial steepness of the fronts of the initial distribution $I_0(x)$. For a bell-shaped distribution with an exponential dependence of $I_0(x)$ for $|x| \to a$ the spatial steepness s is constant ($I_0(x) \approx \exp(-|x|/s)$, $|x| \gtrsim x_0$) and is related to the half-width of the distribution by the approximate relation $s \approx 0.7x_0$. However, if the index of refraction is inhomogeneous in the resonator the quantity s can vary along the fronts $I_0(x)$. For example, in a confocal resonator the quantity s decreases as $|x|$ increases.

Introducing the amplification and loss coefficients per unit length

[3]The case of Q-switching by bleachable filters is not considered here.

$$\alpha(x)=\frac{\omega_0}{\varepsilon_0 c}\varepsilon''(x), \quad \gamma=\frac{\omega_0}{\varepsilon_0 c}\varepsilon_0'',$$

we obtain the following expression for the velocity of the transverse buildup of a giant pulse;

$$v=(\alpha-\gamma)cs. \tag{22}$$

The time for the transverse development of generation τ_{tr} during which generation extends over the whole crystal of diameter 2a can be estimated with the aid of the expression

$$\tau_{tr}\approx\frac{a-x_0}{(\bar{\alpha}-\gamma)cs}, \quad \bar{\alpha}=\frac{1}{a-x_0}\int\limits_{x_0}^{a}\alpha(x)\,dx, \tag{23}$$

where $\bar{\alpha}$ is the average amplification coefficient per unit length in the region of the transverse buildup of the pulse.

We compare the estimate (23) with the exact calculation of the transverse build up given in I. In I we considered a laser with the parameters

$$\bar{\alpha}\approx 0.04\ \mathrm{cm}^{-1}, \quad \gamma\approx 0.02\ \mathrm{cm}^{-1},\ x_0\approx 0.25\ a,\ s=0.7x_0.$$

Then formula (23) yields $\tau_{tr}\approx 7$ nsec which agrees with the time for the transverse buildup of 10 nsec obtained in I by means of a numerical integration of exact equations.

Thus, the time for the transverse buildup of a giant pulse is determined by the shape of the field distribution towards the end of the linear buildup. In order to obtain the shortest possible light pulses it is necessary to reduce the time for the transverse buildup by increasing the degree of homogeneity of the index of refraction and of the population inversion of the crystals and glasses at the time of Q-switching.

5. EFFECT OF INHOMOGENEITY OF THE INDEX OF REFRACTION

In Secs. 2 and 3 an investigation was made of the the effect of a lens-like inhomogeneity in the index of refraction on the field distribution in the region of the linear development of generation. It is of interest (cf., for example,[2]) to investigate the effect of such an inhomogeneity and of a wedge type inhomogeneity on the dynamics of generation of the whole pulse. This can be accomplished by the method of numerical integration of Eqs. (1) and (2) utilized in I. For this it is convenient to represent Eq. (1) in the form of a system of equations for the complex amplitudes $A_k = A_k' + iA_k''$:

$$\dot{A}_k'(t)=\frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty}A_m'(t)\varepsilon_{km}''(t)+\frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty}A_m''(t)\varepsilon_{km}'(t)+(\Omega_k-\Omega_1)A_k''(t),$$

$$\dot{A}_k''(t)=\frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty}A_m''(t)\varepsilon_{km}''(t)-\frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty}A_m'(t)\varepsilon_{km}'(t)-(\Omega_k-\Omega_1)A_k'(t), \tag{24}$$

where

$$\varepsilon_{km}'=\int\limits_{-a}^{a}U_k(x)\,\delta\varepsilon'(x)\,U_m(x)\,dx, \tag{25}$$

while the rest of the notation is the same as in I.

An inhomogeneity of the lens type can be described by specifying $\delta\epsilon'(x)$ in the form

$$\delta\varepsilon'(x)=-\delta\varepsilon_0'(x/a)^2, \tag{26}$$

where $\delta\epsilon_0' > 0$ corresponds to a positive lens, while $\delta\epsilon_0' < 0$ corresponds to a negative lens.

Figures 1 and 2 present a picture of the development of a giant pulse with a small inhomogeneity in the index of refraction of the type of a positive lens ($\delta\epsilon_0' = 2\times 10^{-7}$), where for the sake of definiteness all the remaining parameters of the laser and the initial conditions are retained the same as in I (Sec. 3). First of all we can clearly see a marked decrease in the divergence φ_0 down to a limiting value equal to the divergence of the principal mode. Physically this is associated with the fact that an inhomogeneous distribution of inversion ϵ'' with a maximum at the center is equivalent to a small negative lens which distorts the wave front of the field. The introduction of a small inhomogeneity $\delta\epsilon'$ of opposite properties compensates for this effect and as a result the field approaches a plane wave having minimum divergence. For larger values of $\delta\epsilon_0'$ the effect of the positive lens predominates and the divergence again increases. It is interesting to note that in this case the velocity of propagation of generation in the transverse direction is somewhat lowered –

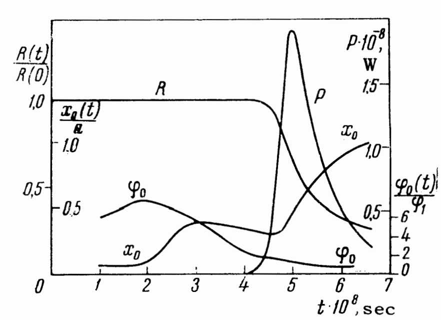

FIG. 1. Variation in the power P(t) of a giant light pulse, the total number of active particles in the resonator R(t), the half-width of the region of generation at half-height $x_0(t)$, the half-divergence of the radiation at half-height $\varphi_0(t)$ in the case that the index of refraction of the medium in the resonator has an inhomogeneity of the positive lens type.

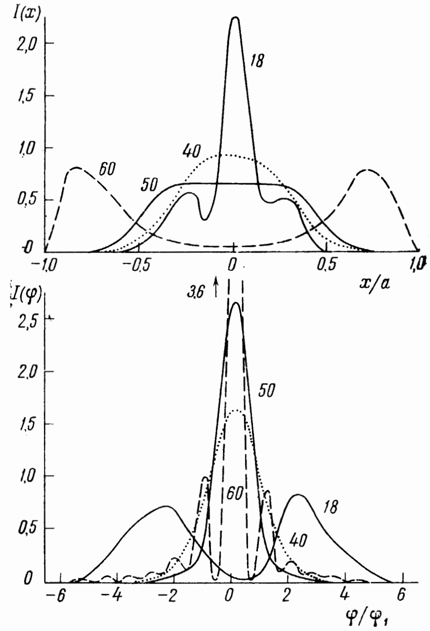

FIG. 2. Instantaneous distributions of intensity over the end I(x) and with respect to the angles I(φ) (a is the half-width of the resonator, $2\varphi_1$ is the divergence with respect to the half-height of the principal type of oscillations) when the index of refraction of the medium in the resonator has an inhomogeneity of the positive lens type (numbers adjacent to the curves represent time in nsec).

the trailing edge of the pulse is delayed as can be seen even in Fig. 1a. This effect is explained by an increase in steepness, i.e., by a decrease in the quantity s at the edges of the initial distribution $I_0(x)$ due to the effect of the positive lens.

Figures 3 and 4 show the development of a giant pulse in the case of a small inhomogeneity in the index of refraction of the negative-lens type ($\delta\epsilon_0' = -2\times 10^{-7}$) and with former values of the remaining parameters of the laser. The negative lens distorts the wave front and, therefore, appreciably increases the divergence of the radiation. In contrast to the case $\delta\epsilon_0' > 0$ a small nega-

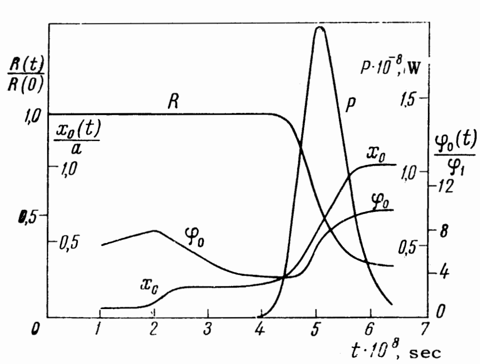

FIG. 3. The same as in Fig. 1, but for the case when the index of refraction of the medium in the resonator has an inhomogeneity of the negative lens type.

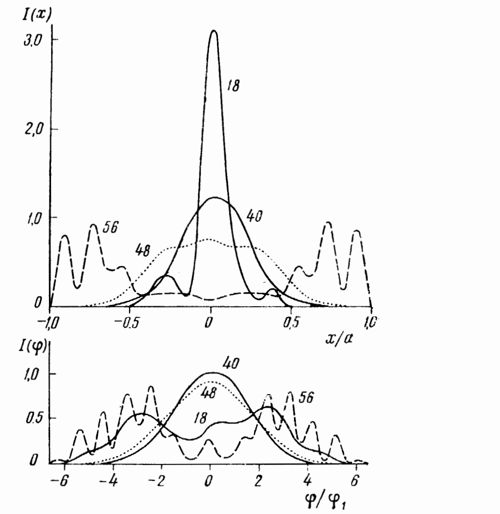

FIG. 4. The same as in Fig. 2, but for the case when the index of refraction of the medium in the resonator has an inhomogeneity of the negative lens type.

tive lens does not increase the time for the transverse development of generation.

An inhomogeneity in the index of refraction of the wedge type was taken in the form

$$\delta\varepsilon'(x) = -\delta\varepsilon_0'(x/a). \qquad (27)$$

Figures 5 and 6 show the development of a giant pulse in this case for $\delta\epsilon_0' = 2\times 10^{-7}$ and with the values of all the remaining parameters the same. At the top of Fig. 5 there is also shown the displacement of the field distribution along the end during the time of generation (the solid lines correspond to the half-heights of the distribution I(x) at a given instant). It can be seen that towards the end of the linear buildup of generation ($t \approx 44$ nsec) the distribution has been displaced towards the "open" end of the resonator. Therefore, generation at the opposite end of the resonator begins much later, and this leads to an increase in the

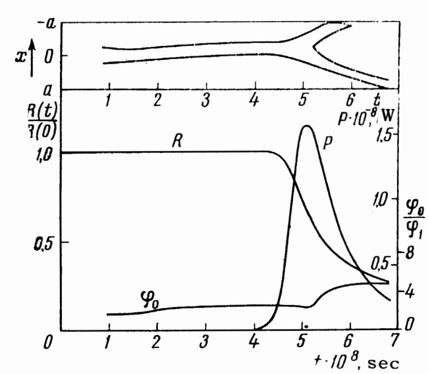

FIG. 5. The same as in Fig. 1, but for the case when the index of refraction of the medium in the resonator has an inhomogeneity of the wedge type. At the top is shown the motion of the field distribution along the end of the generator (solid lines correspond to the half-maxima of the distribution.)

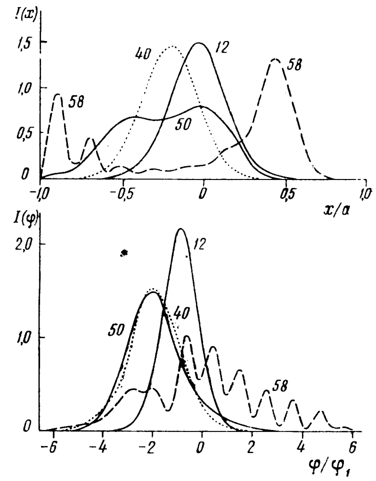

FIG. 6. The same as in Fig. 2, but for the case when the index of refraction of the medium in the resonator has an inhomogeneity of the wedge type.

time of the transverse development and, consequently, of the duration of the giant pulse. This result agrees with the experiments carried out in [2].

6. CONCLUSION

In I and in the present paper (II) we investigated the dynamics of processes in a Q-switched laser. This work was stimulated by absence of data on the space-time development of generation in spite of the evident importance of having such data for utilizing giant light pulses in investigations of nonlinear interaction between radiation and matter. As the result of a consistent investigation of a comparatively simple model of a Q-switched laser an analytic investigation has been given of the two principal phases of the development of a giant pulse - the phase of linear development of generation beginning from amplification of spontaneous radiation in various types of oscillations, and the phase of nonlinear transverse development of generation during which the giant light pulse proper is radiated. Moreover, for a detailed investigation of the picture of the development of a pulse as a whole, numerical integration of equations was carried out.

Experiments which followed later [2,3] have confirmed the existence of a transverse development of a giant pulse, and recent experiments on nonlinear amplification [4] have shown the importance of this effect for the propagation of a giant pulse in a nonlinear medium. The knowledge of the transverse development of a giant pulse is apparently essential for an exact determination of the true intensity of the light field in experiments on multiphoton processes (report by N. G. Basov at the colloquium of the laboratory of oscillations and quantum radiophysics, Physics Institute, Academy of Sciences, U.S.S.R., 1965). The theory developed above also yields recommendations regarding the construction of quantum generators of giant light pulses of shortest possible duration and of smallest possible divergence of the radiation.

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