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THERMOMAGNETIC EFFECTS IN AN ELECTRON GAS OF SEMICONDUCTORS HEATED BY A HIGH-FREQUENCY ELECTRIC FIELD

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We consider thermomagnetic effects in semiconductors heated by a high-frequency electromagnetic field. We calculate the thermomagnetic effects for both weak high-frequency field and strong fields. Anomalous and normal skin effects are investigated in the case of strong fields. Degenerate semiconductors and semimetals are considered separately. It is indicated that a number of characteristics of semiconductors can be determined experimentally with the aid of thermomagnetic effects.

BASS and Gurevich [1] (henceforth cited as I) investigated the penetration of a high-frequency electromagnetic field into a halfspace filled with a semiconductor. They have shown that under the adiabatic boundary condition the temperature of the electron gas of a semiconductor depends essentially on the coordinates (and does not depend on the time). As is well known, the presence of a temperature gradient leads, when the circuit is open, to the appearance of a static emf which is collinear with the temperature gradient; when a constant magnetic field is applied, an emf appears also in the plane perpendicular to the temperature gradient. Besides the emf produced upon superposition of an external magnetic field, additional components of the temperature gradient can appear in a plane perpendicular to the initial temperature gradient. Thermomagnetic effects of this kind were investigated in detail in [2]. However, in the case of heating by a high-frequency field, the thermomagnetic effects have a number of specific features, the theoretical study of which we wish to discuss in the present communication.[1)]

As shown in I, the static electric current j and the static heat flux Q are described by the following formulas:

$$\mathbf{j} = e^2I_{10}\mathbf{E}' - eI_{11}\nabla\ln\Theta + [e^2I_{20}\mathbf{E}' - eI_{21}\nabla\ln\Theta, \mathbf{h}] + \mathbf{h}(e^2I_{30}\mathbf{E}' - eI_{31}\nabla\ln\Theta, \mathbf{h}), \tag{1}$$

$$\mathbf{Q} = eI_{11}\mathbf{E}' - I_{12}\nabla\ln\Theta + [eI_{21}\mathbf{E}' - I_{22}\nabla\ln\Theta, \mathbf{h}] + \mathbf{h}(eI_{31}\mathbf{E}' - I_{32}\nabla\ln\Theta, \mathbf{h}).$$

We have introduced here the notation

$$I_{ik} = -\frac{16\sqrt{2}\,\pi m^{1/2}}{3h^3}\left(\frac{eH}{mc}\right)^{i-1}\int_0^\infty \frac{\tau^i(\varepsilon)\,\varepsilon^{k+3/2}}{1+(eH\tau(\varepsilon)/mc)^2}\frac{\partial f_0}{\partial\varepsilon}\,d\varepsilon,$$

e - electron charge, m - electron mass, H - constant magnetic field, $\tau(\epsilon)$ - electron momentum relaxation time with respect to some scattering centers, ϵ - electron energy,

$$f_0(\varepsilon) = \left[1 + \exp\frac{\varepsilon - \mu(\Theta)}{\Theta}\right]^{-1}$$

- equilibrium electron distribution function, Θ - electron temperature determined from the energy-balance and Maxwell's equations (see [1]), $\mu(\Theta)$ - chemical potential of electron gas, $\mathbf{E}' = \mathbf{E} - \nabla[\mu(\Theta)/\Theta]$, $\mathbf{E}$ - static electric field, and $\mathbf{h}$ - unit vector of constant magnetic field.

The dependence of the relaxation time on the energy is determined by the formula

$$\tau(\varepsilon) = \tau_0(T)\,(\varepsilon / T)^q, \tag{2}$$

where T is the lattice temperature and q is determined by the type of scattering (see, for example, [4]).

We shall henceforth consider, in analogy with the procedure proposed in I, an electromagnetic field propagating along the z axis, and consequently the initial temperature gradient will also be directed along this axis. The constant magnetic field is assumed to lie in the zy plane at an angle α to the z axis.

We consider first isothermal thermomagnetic effects in a nondegenerate electron gas. In calculating these effects we assume that there is no static electric current through the semiconductor, j = 0. From this condition we obtain the fields of

1)The occurrence of static fields in a semiconductor irradiated by a high-frequency electromagnetic field was first investigated in [3].

the thermomagnetic effects. We confine ourselves to two limiting cases, strong and weak magnetic fields. Replacing the electron energy ϵ by Θ in the expression for $\tau(\epsilon)$, we obtain in the usual manner the following criteria: the magnetic field is regarded as strong if the following inequality is satisfied

$$(e\tau_0 H / mc)^2 v^{2q} \gg 1 \quad (v = \Theta / T),$$

and weak if the opposite inequality is satisfied.

For weak magnetic fields, the expression for the field of the Nernst-Ettingshausen effect E_X and for the thermal emf E_Z are of the form[2)]:

$$E_x = -\frac{q\Gamma(2q+5/2)}{\Gamma(q+5/2)} \frac{TH\tau_0}{mc} v^q \frac{dv}{dz} \sin\alpha,$$
$$E_z = (1+q)\frac{T}{e}\frac{dv}{dz}. \tag{3}$$

In strong magnetic fields, the formulas for E_X and E_Z are

$$E_x = -\frac{4}{3\sqrt{\pi}} q\Gamma(5/2-q)\frac{T}{e}\frac{mc}{e\tau_0 H} v^{-q}\frac{dv}{dz}\sin\alpha,$$
$$E_z = \frac{T}{e}\frac{dv}{dz}. \tag{4}$$

Besides the isothermal thermomagnetic effects, interest attaches to adiabatic thermomagnetic effects. Adiabatic thermomagnetic effects were calculated under one of two assumptions: $Q_X = Q_Y = j = 0$ or $Q_X = Q_Y = j_Z = E_X = E_Y = 0$. These relations determine in the former case $\partial\Theta/\partial x$, E_X, and E_Z, and in the latter case $\partial\Theta/\partial x$, $\partial\Theta/\partial y$, and E_Z. To calculate these quantities, it is necessary to bear in mind the following circumstance. If uneven heating of the sample is produced by a thermal source, then the presence of a temperature gradient leads to heat fluxes that are connected both with electrons (see formula (1)) and with phonons, and in the calculation of the adiabatic thermomagnetic effects it is necessary to equate to zero the components of the summary heat flux [2]. The situation is different for high-frequency heating. The high-frequency field heats only the electrons, leaving the phonons cold. For this reason it is necessary to equate to zero only the components of the heat flux connected with the electrons.

In the first case, for $\alpha = \pi/2$, we have for both weak and strong magnetic fields, accurate to coefficients of the order of unity (we do not present them here because of their complexity):

$$\frac{\partial\Theta}{\partial x} \sim T\frac{e\tau_0 H}{mc} v^q \frac{dv}{dz}, \quad E_x \sim T\frac{\tau_0 H}{mc} v^q \frac{dv}{dz}, \quad E_z \sim \frac{T}{e}\frac{dv}{dz}. \tag{5}$$

The omitted coefficients are different for weak and strong magnetic fields.

On the other hand, if $Q_X = Q_Y = E_X = E_Y = j_Z = 0$, then different formulas are obtained for the thermomagnetic characteristics. In weak magnetic fields

$$\frac{\partial\Theta}{\partial x} \sim T\frac{e\tau_0 H}{mc} v^q \frac{\partial v}{\partial z}\sin\alpha, \quad \frac{\partial\Theta}{\partial y} \sim T\left(\frac{e\tau_0 H}{mc}\right)^2 v^{2q}\frac{dv}{dz}\sin 2\alpha. \tag{6}$$

In strong magnetic fields

$$\frac{\partial\Theta}{\partial x} \sim T\frac{\partial v}{\partial z}\begin{cases}(e\tau_0 H/mc)^{-1} v^{-q}, & \alpha = \pi/2\\ (e\tau_0 H/mc) v^q \sin\alpha\cos^2\alpha, & \alpha \neq \pi/2\end{cases},$$
$$\frac{\partial\Theta}{\partial y} \sim T\frac{\partial v}{\partial z}\sin 2\alpha. \tag{7}$$

In this case E_Z is described by formula (5) for both weak and strong magnetic fields.

In semimetals and degenerate semiconductors we can obtain an expression for the thermomagnetic characteristics for arbitrary magnetic fields.

In the isothermal case

$$E_y = -\frac{\pi^2 q T^2}{3\varepsilon_0 e}\frac{\beta}{1+\beta^2} v\frac{dv}{dz}\sin\alpha, \tag{8}$$
$$E_z = -\frac{\pi^2}{6}\frac{T^2}{e\varepsilon_0}\left\{1+\frac{2q\beta^2\sin^2\alpha}{1+\beta^2}\right\} v\frac{dv}{dz}.$$

In addition, we present a formula for $\partial\Theta/\partial x$ in the case $j_Z = Q_X = Q_Y = E_X = E_Y = 0$ with $\alpha = \pi/2$:

$$\frac{\partial\Theta}{\partial x} = 2(1+q)T\frac{\beta(1+\beta^4)}{(1+\beta^2)^3}\frac{dv}{dz}; \tag{9}$$

in formulas (8) and (9) we have $\beta = e\tau(\epsilon_0)H/mc$.

Besides the foregoing thermomagnetic characteristics, interest attaches to the total thermal emf V, defined by the relation

$$V = \int_0^\infty E_z\, dz. \tag{10}$$

It follows from the foregoing formulas that $V \sim v_0 - 1$ for a nondegenerate electron gas and $V \sim v_0^2 - 1$ for a degenerate gas. With the aid of the expression for V we can measure directly the electron temperature. For a further investigation it is necessary to know the dependence of the electron temperature v on the frequency and the amplitude of the incident electromagnetic field, the external magnetic field, the coordinate z, and others.

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2)We do not present the expressions for the field E_y—the so-called longitudinal-transverse thermomagnetic effect [5], since its investigation yields nothing new.

The authors of I obtained the corresponding formulas for v. Substitution of these formulas in the expression for the thermomagnetic characteristics solves our problem. We confine ourselves essentially to a study of the thermomagnetic characteristics as functions of the coordinate z. The dependence on the frequency, magnetic field, etc. can be considered similarly.

We assume first that the high-frequency electric field is weak. In this case, the balance equation (1.19, I)[3] can be linearized and can then be easily solved. We seek solutions of (1.19, I) in the form $v = 1 + u$, where $u \ll 1$, under one of the following two boundary conditions on the plane $z = 0$: adiabatic, ($dv/dz = du/dz = 0$) or isothermal ($v = 1\,(u = 0)$). The first condition corresponds to the absence of a heat flux through the plane $x = 0$, and the second to the maintenance of the plane $z = 0$ at the lattice temperature.

Simple calculation leads to the following expressions for u:

$$u = \frac{B}{\delta_0^2 - 4\xi_0^2}\left(e^{-2\xi_0 z} - \frac{2\xi_0}{\delta_0}e^{-\delta_0 z}\right) \quad \text{for} \quad \left.\frac{du}{dz}\right|_{z=0} = 0,$$

$$u = \frac{B}{\delta_0^2 - 4\xi_0^2}\left(e^{-2\xi_0 z} - e^{-\delta_0 z}\right) \quad \text{for} \quad u|_{z=0} = 0. \quad (11)$$

We have introduced here the following notation: $B = \overline{B}_{ik}(1)\widetilde{E}_i^0\widetilde{E}_k^{0*}(\lambda_H T)^{-1}$, where the tensor $\overline{B}_{ik}(1)$ is defined by formula (1.11, I), $\widetilde{E}_i^0$ is the amplitude of the external electromagnetic field in the medium for $z = 0$, λ_H is the electron temperature, ξ_0 is the attenuation of the electromagnetic field in the linear theory, δ_0 is a quantity of the order of $l_e^{-1}(1)$, where $l_e(1)$ stands for the energy mean free path at $v = 1$ (in I they used in lieu of δ_0 a somewhat different quantity δ, connected with δ_0 by the relation $\delta_0 = (2 \pm q)^{-1/2}\delta$).[4]

In the approximation of weak high-frequency electric fields, the z-dependence of the thermomagnetic characteristics is given by the factor du/dz, since v should be set equal to unity in the corresponding formulas at the degree of accuracy under consideration.

It follows from (11) that

$$\frac{du}{dz} = \frac{2B\xi_0}{\delta_0^2 - 4\xi_0^2}\left(e^{-\delta_0 z} - e^{-2\xi_0 z}\right) \quad \text{for} \quad \left.\frac{du}{dz}\right|_{z=0} = 0,$$

$$\frac{du}{dz} = \frac{B}{\delta_0^2 - 4\xi_0^2}\left(\delta_0 e^{-\delta_0 z} - 2\xi_0 e^{-2\xi_0 z}\right) \quad \text{for} \quad u|_{z=0} = 0. \quad (12)$$

If the boundary conditions at $z = 0$ are adiabatic, then the function du/dz has a minimum at the point

$$z_m = \frac{\ln(2\xi_0/\delta_0)}{2\xi_0 - \delta_0},$$

the value of du/dz at the minimum being determined by the following relation

$$\left(\frac{du}{dz}\right)_{min} = \frac{2B\xi_0}{\delta_0^2 - 4\xi_0^2}\left[\left(\frac{\delta_0}{2\xi_0}\right)^{\delta_0/(2\xi_0-\delta_0)} - \left(\frac{\delta_0}{2\xi_0}\right)^{2\xi_0/(2\xi_0-\delta_0)}\right]. \quad (13)$$

It is easy to see that $du/dz < 0$ for all z.

When the isothermal boundary condition is satisfied, du/dz vanishes at the point

$$z_0 = \frac{\ln(2\xi_0/\delta_0)}{2\xi_0 - \delta_0}$$

and has a maximum at

$$z_m = \frac{2\ln 2\xi_0/\delta_0}{2\xi_0 - \delta_0}.$$

At the minimum point we have

$$\left(\frac{du}{dz}\right)_{min} = \frac{B}{\delta_0^2 - 4\xi_0^2}\left[\delta_0\left(\frac{\delta_0}{2\xi_0}\right)^{2\delta_0/(2\xi_0-\delta_0)} - 2\xi_0\left(\frac{\delta_0}{2\xi_0}\right)^{4\xi_0/(2\xi_0-\delta_0)}\right]. \quad (14)$$

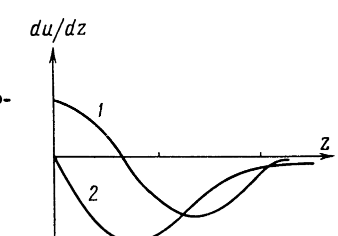
It is obvious that $du/dz > 0$ when $z < z_0$ and $du/dz < 0$ when $z > z_0$. The diagram shows the schematic variation of du/dz for adiabatic and isothermal boundary conditions.

We now proceed to the case of strong high-frequency electric fields. We note that the dependence of the thermomagnetic effects on the coordinate z is determined by the function $F_k(z)$:

$$F_k(z) = v^k(z)\,dv(z)\,/\,dz. \quad (15)$$

$k = \pm q$ for effects that are odd in the magnetic field, and $k = 0.2q$ for even effects. In a degenerate gas $k = 0.1$.

Let us substitute in (15) the values of v calculated in I. As indicated in that paper, we must distinguish here between the anomalous and normal skin effects. We consider first a nondegenerate electron gas. In the case of a sharply anomalous skin effect, in the region of strong heating of



Variation of du/dz under isothermal and adiabatic boundary conditions: curve 1—$v|_{z=0} = 1$, curve 2—$(dv/dz)|_{z=0} = 0$.

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[3]We shall henceforth refer to the formulas of I by placing I alongside the number of the formula.

[4]Formulas for ξ_0 and δ_0 are given in I.

the electron gas ($v(z) \gg 1$), the temperature is determined by formula (2.10, I), which takes the form

$$v = v_0\left\{[1-(2\pm q-r)\,\delta(v_0)\,z]^{2/(2\pm q-r)} - \frac{\delta(v_0)}{\xi(v_0)}\,e^{-2\xi(v_0)z}\right\}, \tag{16}$$

here v_0 is the dimensionless temperature of the electron gas at $z = 0$

$$\delta(v_0) = [2(2\pm q-r)]^{-1/2}\delta_0 v_0^{(r-2\mp q)/2},$$

$\xi(v_0)$ is the damping of the electromagnetic waves near the plane $z = 0$. The formulas for v_0 and $\xi(v_0)$ in different particular cases are given in I (see (2.21, I), (2.22, I), and others).

The upper sign in front of q in (16) corresponds to the absence of a magnetic field or to an arbitrary magnetic field perpendicular to the z axis, while the lower sign corresponds to a strong magnetic field perpendicular to the z axis; r characterizes the scattering with energy transfer (see I).

As indicated in I, under conditions of a sharply anomalous skin effect ($(v_0) \ll \xi(v_0)$) the second term in the curly brackets in (16) is small compared with the first, but it must be taken into account in the calculation of the derivatives. Simple calculation leads to the following expression for $F_k(z)$:

$$F_k(z) = 2\delta(v_0)\,v_0^{k+1}[1-(2\pm q-r)\,\delta(v_0)\,z]^{2k/(2\pm q-r)} \times \{e^{-2\xi(v_0)z} - (1-(2\pm q-r)\,\delta(v_0)\,z]^{(r\mp q)/(2\pm q-r)}\}. \tag{17}$$

This expression has a minimum at the point z_m:

$$z_m = \frac{1}{2\xi(v_0)}\ln\frac{\xi(v_0)}{\delta(v_0)}, \tag{18}$$

and the minimum value of $F_k(z_m)$ is of the form

$$F_k(z_m) = -2\delta(v_0)\,v_0^{k+1}. \tag{19}$$

Deep inside the sample, where the heating is small, $v = 1 + S_v\exp(-\delta_0 z)$, and the temperature-interaction multiplier (see formulas (2.12, I) and (2.13, I)) is by definition much smaller then unity. In this case

$$F_k(z) = -\delta_0 S_v e^{-\delta_0 z}. \tag{20}$$

Under the normal skin effect ($\delta(v_0) \gg \xi(v_0)$), it is necessary to consider separately two cases. In the first case, when the damping is small compared with wavelength, the following formulas hold when $v(z) \gg 1$:

$$v(z) = v_0\left(1-\frac{2q}{r+q}\,\xi(v_0)\,z\right)^{1/q}, \tag{21}$$

$$F_k(z) = -\frac{2}{r+q}\,\xi(v_0)\,v_0^{k+1}\left[1-\frac{2q}{r+q}\,\xi(v_0)\,z\right]^{(k-q+1)/q} \tag{22}$$

On the other hand, if $v(z) = 1 + u(z)$, with $u(z) \ll 1$, then

$$u(z) = S_v e^{-2\xi_0 z}, \qquad \frac{du}{dz} = -2\xi_0 S_v e^{-2\xi_0 z}. \tag{23}$$

Formula (21) and the first formula of (23) correspond to formulas (3.8, I) and (3.9, I).

If the damping is strong (cyclotron and magnetoplasma resonances, low-frequency field (see I)), then the temperature v is determined by formula (3.14, I), which is of the form

$$v = v_0\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega z}{c\,|\zeta(v_0)|}\right)^{-2/q}, \tag{24}$$

where ω is the frequency of the incident field, c the velocity of light in vacuum, and $\zeta(v_0)$ the surface impedance of the half-space, calculated in I. With this,

$$F_k(z) = -\frac{2}{[(2r-q)(r-q)]^{1/2}}\frac{\omega v_0^{k+1}}{c\,|\zeta(v_0)|} \times\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega z}{c\,|\zeta(v_0)|}\right)^{-(2k+q+2)/q}. \tag{25}$$

We note that under the normal skin effect $F_k(z)$ and du/dz are negative for arbitrary z.

In order to obtain the results for a fully degenerate electron gas, it is necessary to put $q = 0$ and $r = 1$ in formulas (16)–(25), and $\xi(v_0)$ and $\delta(v_0)$ must be replaced by the analogous quantities for the degenerate gas; these, as can be readily shown, do not depend on the electron temperature.

We note that an investigation of thermomagnetic phenomena, besides measurements of the impedance of the reflected high-frequency field, makes it possible to obtain additional information on the heating of the electron gas in semiconductors. For degenerate semiconductors and semimetals, this is the only method, since the electron-gas heating does not influence the impedance in this case. Besides the electron temperature, which was already referred to above, it is also possible to determine with the aid of thermomagnetic effects the attenuation depth of the electromagnetic field, of the temperature, etc.

Thermomagnetic effects are especially convenient for differentiation between the anomalous and normal skin effects.

One of the criteria of the anomalous skin effect is non-monotonicity of $F_k(z)$. In strong magnetic fields it is possible to propose one more criterion for this purpose.

As shown in I, v_0 is a function of the amplitude and geometry of the heating high-frequency field, and also of the constant magnetic field. This causes, in particular, the thermal emf to depend on the magnetic field (even if the magnetic field is directed along the primary temperature gradient (z axis)). If the anomalous skin effect takes place, then the temperature, and consequently also the thermal emf, varies strongly in a strong magnetic field perpendicular to the z axis, compared with an oblique magnetic field (cf. 2.34, I). This phenomenon is not observed in the normal skin effect.

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