# QUANTUM SIZE EFFECT IN A SEMIMETAL FILM

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We consider the quantum size effect (QSE) in a semimetal film. Formulas are obtained for the carrier density, electric conductivity $\sigma$, the Hall constant R, and the magnetoresistance $\Delta\rho/\rho$. The case of scattering by randomly distributed centers with $\delta$-potential is considered. The effect of the field under the QSE conditions is discussed. It is shown that $\sigma$, $R\sigma$, and $\Delta\rho/\rho$ have an oscillating dependence on the thickness of the semimetal film.

THE quantum size effect (QSE) is defined as the dependence of the thermodynamic properties and kinetic coefficients of solids on their characteristic geometric dimensions, when the latter become comparable with the effective de Broglie wavelength of the elementary excitations.

The first experimental paper reporting observation of QSE for electrons and holes in Bi films was recently published [1]. It is of interest therefore to present the results of a theoretical analysis of a simple model of a semimetal film under the QSE conditions.

Let the band structure of a bulky semimetal have the form shown in Fig. 1a. The electrons and holes in the semimetal film will be regarded as independent particles moving in potential wells with flat bottoms and infinitely high walls; their masses are equal to the corresponding effective masses $m_n$ and $m_p$ in the bulky semimetal.

In the case of a semimetal, the QSE causes the conduction band and the valence band to break up into subbands (similar to the Landau subbands in a quantizing magnetic field), whose numbers correspond to discrete values of the wave vector along the "quantizing" dimension of the film (see Fig. 1b). As a result of the appearance of zero-point oscillations in the conduction and valence bands ($\epsilon_{n,p} = \hbar^2\pi^2/2m_{n,p}a^2$, where a is the film thickness), the band overlap $\Delta$ decreases by an amount $\epsilon_n + \epsilon_p$, and if a is sufficiently small a gap is formed and the semimetal turns into a dielectric.

Owing to the stepwise character of the function of the state density per unit energy interval [2-4], the kinetic and thermodynamic characteristics of the system will oscillate as functions of a, so long as the carrier gas is degenerate. These oscillations (as in the case of a quantizing magnetic field) are connected with abrupt changes of the

density of states on the Fermi surface as the latter goes through different subbands.

We note that the model under consideration corresponds to specular scattering from the surfaces of the film. The conditions for the realization and observation of the QSE consist in the following: $\epsilon_{n,p} \approx \mu_{n,p}$ and $\epsilon_{n,p}\tau_{n,p} \gg \hbar$, where $\mu_{n,p}$ is the carrier Fermi energy, and $\tau_{n,p}$ are the characteristic carrier relaxation times, due to scattering both in the volume and by the surface [2,4].

The QSE in a semimetal film is of interest, in particular, because by varying the thickness it is possible to go over from a degenerate gas carrier into a nondegenerate one. Moreover, it can be assumed that semimetal films are at low temperatures the most favorable objects for experimental
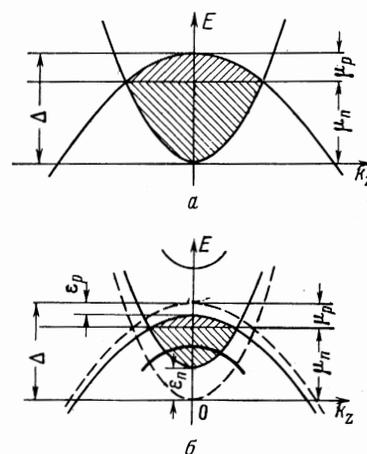


FIG. 1. Band scheme: a — for a bulky semimetal, $\mu_n$, $\mu_p$ — electron and hole Fermi levels, $\triangle$ — band overlap; the shaded areas are those filled with electrons and holes; b — for semimetal film, dashed lines — bands in bulky sample, solid — subbands in film; the shaded regions are those occupied by electrons and holes.

observation of QSE: on the one hand, they have a sufficient number of carriers to screen the scattering centers; on the other hand, the effective de Broglie wavelength of the carriers is sufficiently large to permit them to be weakly scattered by these screened centers.

We shall consider the statistics of the carriers, the longitudinal static electric conductivity, the Hall effect, the transverse magnetoresistance, and the field effect. We shall assume that the temperature is sufficiently low, namely $kT/\Delta \ll 1$. In calculating the kinetic coefficients it is assumed that the carriers are scattered by randomly distributed scattering centers with $\delta$-potential.

## 1. STATISTICS OF CARRIERS

Let the film have a thickness a along the x axis, $0 \le x \le a$, and let it have infinite dimensions in the y and z directions. We shall consider electrons, and then generalize the results in obvious fashion to include holes.

In accordance with the assumed model, the single-Particle wave functions and the energy spectrum are:

$$\Psi_{s, k_y, k_z} = \left(\frac{2}{aL_yL_z}\right)^{1/2} \sin\frac{\pi s x}{a} \exp(ik_y y + ik_z z), \quad (1)$$

$$E_{s, k_y, k_z} = \varepsilon_n s^2 + \hbar^2(k_y^2 + k_z^2)/2m_n, \quad (2)$$

where $L_y$ and $L_z$ are the lengths of the sides of the normalization rectangle, $\epsilon_n = \hbar^2\pi^2/2m_na^2$, $s = 1, 2, \ldots$; $k_{y,z} = (2\pi/L_{y,z})s_{y,z}$ and $s_{y,z} = 0$, $\pm 1, \pm 2, \ldots$

For the function of the state density per unit energy interval (without account of spin degeneracy) we get

$$S(E)dE = \frac{V}{4\pi^2}\left(\frac{2m_n}{\hbar^2}\right)^{3/2}\varepsilon_n^{1/2}\left[\left(\frac{E}{\varepsilon_n}\right)^{1/2}\right]dE, \quad (3)$$

where $\lfloor t \rfloor$ is the integer part of t, and $V = aL_yL_z$.

Let the total number of electrons in the conduction band in a volume V be equal to N

$$N = 2\int_{\varepsilon_n}^{\infty} S(E)f_n(E)dE,$$

$$f_n(E) = [1 + \exp(E - \mu_n)/kT]^{-1}. \quad (4)$$

According to (4), the electron "density" is

$$n = \frac{N}{V} = \frac{m_n kT}{\pi\hbar^2 a}\sum_{s=1}^{\infty}\ln\left(1 + \exp\left[\frac{\mu_n - \varepsilon_n s^2}{kT}\right]\right). \quad (5)$$

For the case of complete degeneracy, (4) yields $((\mu_n - \epsilon_n)/kT \gg 1)$

$$n = \frac{\pi}{12a^3}M_n\left[6\frac{\mu_n}{\varepsilon_n} - (M_n + 1)(2M_n + 1)\right], \quad (6)$$

where $M_n = \lfloor\sqrt{\mu_n/\epsilon_n}\rfloor$. The expression for p is obtained by making the substitutions $\mu_n \to \mu_p$ and $m_n \to m_p$.

The Fermi level is determined from the electric neutrality condition n = p. In the region of total degeneracy, we equate (6) to the analogous expression for the holes. Since the left and right sides of the equation have, after dividing by $a^{-3}$, identical forms and depend respectively on the arguments $\mu_n/\epsilon_n$ and $\mu_p/\epsilon_p$, we get

$$\mu_n/\varepsilon_p = \mu_p/\varepsilon_p. \quad (7)$$

Moreover,

$$\mu_n + \mu_p = \Delta. \quad (7a)$$

Hence

$$\mu_n = \Delta\frac{m_p}{m_n + m_p}, \quad \mu_p = \Delta\frac{m_n}{m_n + m_p}. \quad (7b)$$

Consequently, $\mu_{n,p}$ does not depend on the thickness. This is true only when the electron and hole gases are degenerate. Thus, the motion of the bands as the thickness varies consists in the following: The Fermi level, reckoned from the bottom of the conduction band in the bulky sample, stays in place when a decreases, the electron subbands slide past it upward, and the hole subbands slide downward. According to (7), for the same thickness, $\mu_n$ and $\mu_p$ pass through the extrema of the electron and hole subbands with the same numbers. The band overlap vanishes at a thickness $\bar{a}$ at which $\epsilon_n(\bar{a}) + \epsilon_p(\bar{a}) = \Delta$:

$$\bar{a} = \pi\hbar/\sqrt{2M\Delta}, \quad M^{-1} = m_n^{-1} + m_p^{-1}. \quad (8)$$

When $a < \bar{a}$, a gap is formed:

$$\varepsilon_g = \Delta(\bar{a}^2/a^2 - 1). \quad (9)$$

We present expressions for n, p, and $\mu_n$ as functions of a. Owing to the smallness of $kT/\Delta$, we can assume the carrier gases to be degenerate almost up to the thickness $\bar{a}$ (for example, in Bi at T = 4.2 K we have $kT/\Delta \approx 10^{-2}$). We find that

$$\frac{n}{n_\infty} = \frac{1}{4}\left(\frac{\bar{a}}{a}\right)A\left[6\frac{a^2}{\bar{a}^2} - (A + 1)(2A + 1)\right], \quad a > \bar{a}, (10a)$$

$$\frac{n}{n_\infty} = \frac{3kT}{\Delta}\left(\frac{m_n + m_p}{M}\right)^{1/2}\frac{\bar{a}}{a}e^{-\varepsilon_g/2kT}, \quad a < \bar{a}. \quad (10b)$$

Here $A = \lfloor a/\bar{a}\rfloor$, $n_\infty = (2\mu_n m_n)^{3/2}/3\pi^2\hbar^3$ is the concentration of electrons in a bulky semimetal.

For the Fermi level at $a > \bar{a}$, we obtain expression (7b), and at $a < \bar{a}$ we get

$$\mu_n - \varepsilon_n = -\varepsilon_g/2 - (kT/2)\ln(m_n/m_p). \quad (11)$$

The dependence of $n/n_\infty$ on $a/\bar{a}$, according to (10a) is shown in Fig. 2. The function $n(a)/n_\infty$ is continuous, but its derivative experiences a
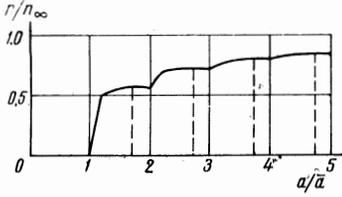
FIG. 2. Dependence of the carrier density on the thickness. The dashed lines show the points at which the density has a maximum.

jump at the point $a/\bar{a} = 1, 2, \ldots$ In each section $s < a/\bar{a} < s + 1$, the function $n(a)/n_\infty$ has a maximum at the point $2^{-1/2}(s+1)^{1/2}(2s+1)$, $s = 1, 2, \ldots$ The appearance of the maximum is connected with the fact that in each section $(s, s+1)$ there exists a region in which the number of allowed states under the Fermi level increases more slowly with increasing thickness, than the volume of the system $V$. The relative magnitudes of these maxima are small, $\sim 1\%$.

## 2. LONGITUDINAL STATIC ELECTRIC CONDUCTIVITY

Let us calculate the electric conductivity in a constant uniform electric field of intensity $F$, directed along the $z$ axis. Let the carriers (for concreteness, electrons) be elastically scattered by randomly distributed centers with $\delta$-like scattering potential:

$$V(\mathbf{r}) = \sum_{j=1}^{N} U\delta(\mathbf{r} - \mathbf{R}_j), \qquad (12)$$

where $\mathbf{r}$ is the carrier coordinate, $\mathbf{R}_j$ the coordinate of the scattering center, $U$ the strength of the potential of the scattering center, and $N$ the number of scattering centers in the volume $V = aL_yL_z$. The total current along the $z$ axis is

$$J = \int dy\, dx\, \mathrm{Sp}\,(\hat{j}_z\hat{\rho}). \qquad (13)$$

Here $\hat{j}_z$ is the operator of the $z$-component of the current density and is defined by the matrix elements

$$\langle sq|\hat{j}_z|s'q'\rangle = \frac{ie\hbar}{2m_n}\Big(\psi_{s'q'}\frac{\partial}{\partial x}\psi_{sq}^* - \psi_{sq}^*\frac{\partial}{\partial x}\psi_{s'q'}\Big);$$

$q = (k_y, k_z)$ and $\hat{\rho}$ is the single-particle density matrix in the representation of the wave function (1). Since only the diagonal matrix elements of $\hat{\rho}$ and $f(sq) = \langle sq|\hat{\rho}|sq\rangle$ remain after integration over the cross section of the film in (13), we get

$$J = -\frac{e\hbar}{m_nL_z}\sum_{sq} k_z f(sq). \qquad (14)$$

where $e$ is the absolute magnitude of the electron charge. The current density and the electric conductivity are defined as follows:

$$j = J/L_ya; \qquad \sigma = j/F. \qquad (15)$$

The diagonal elements of the density matrix can be obtained from the kinetic equation:

$$\frac{eF}{\hbar}\frac{\partial f}{\partial k_z} = \frac{f - f_0}{\tau_{sq}}, \qquad (16)$$

where the relaxation time in the state $(sq)$ is

$$\frac{1}{\tau_{sq}} = \sum_{s'q'}\frac{k_z - k_z'}{k_z}\langle w(sq; s'q')\rangle. \qquad (17)$$

Calculation shows that the transition probability, averaged over the configurations of the scattering centers, is

$$\langle w(sq; s'q')\rangle = \frac{2\pi}{\hbar}U^2\Big[\frac{N}{V^2}\Big(1 + \frac{\delta_{ss'}}{2}\Big) + \frac{N(N-1)}{V^2}\delta_{ss'}\delta_{qq'}\Big]. \qquad (18)$$

Substituting (18) in (17) we get

$$\frac{1}{\tau_{sq}} = B_n\frac{m_n}{\hbar^3 a}\Big\{[\sqrt{E/\varepsilon_n}] + \frac{1}{2}\Big\}, \qquad (19)$$

where $B_n = NU^2/V$. Thus, $\tau_{sq}$ is a discontinuous stepwise function of the energy. Substituting the solution of (16)

$$f(sq) = f_0 + \frac{eF}{\hbar}\tau_{sq}\frac{\partial f_0}{\partial k_z}$$

in (14), we get, in accord with (15),

$$\sigma = -\frac{2e^2\hbar^5}{m_n^3 L_yL_zB_n}\sum_{sq} k_z^2\frac{\partial f_0/\partial E}{[\sqrt{E/\varepsilon_n}] + 1/2}. \qquad (20)$$

For a completely degenerate gas of electrons $\partial f_0/\partial E = -\delta(E - \mu_n)$, and (20) yields

$$\sigma = enu_n, \qquad (21)$$

where the electron mobility is

$$u_n = u_{n\infty}\frac{a}{\bar{a}}\frac{2}{2M_n + 1}, \qquad (22)$$

$u_{n\infty}$ is the electron mobility in the bulky sample, and the concentration $n$ is given by the formula (6). The hole mobility $u_p$ is given by (22), in which the index $n$ should be replaced by $p$.

In a semimetal, the electric conductivity is

$$\sigma = eu_n n + eu_p p.$$

In the thickness region $a > \bar{a}$, we obtain, inserting expressions (6) and (22),

$$\frac{\sigma}{\sigma_\infty} = \frac{a}{\bar{a}}\frac{2n/n_\infty}{2[a/\bar{a}] + 1} \qquad (23)$$

and

$$\frac{u_{n,p}}{u_{n,p\infty}} = \frac{a}{\bar{a}}\frac{2}{2[a/\bar{a}] + 1} \qquad (24)$$

Here $\sigma_\infty$ and $u_{n,p\infty}$ are the conductivity and mobility in the bulky sample. The dependence of $\sigma/\sigma_\infty$ on $a/\bar{a}$ for the region $a > \bar{a}$ is shown in Fig. 3. As seen from (23), (24) and Fig. 3, the electric conductivity oscillates with a period $\bar{a}$.
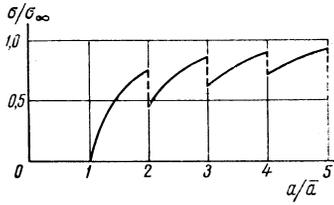
FIG. 3. Thickness dependence of the electric conductivity.

In the thickness region $a < \bar{a}$, assuming that the carriers are only in the first subbands (low temperatures), we get from (20)

$$\frac{\sigma}{\sigma_\infty} = 3\frac{kT}{\Delta}\left(\frac{m_n + m_p}{M}\right)^{1/2} e^{-\varepsilon_g/2kT}, \qquad (25)$$

$$u_{n,p} / u_{n,p\infty} = 2a / \bar{a}. \qquad (26)$$

We note that when $a < \bar{a}$ (24) goes over into (26).

## 3. HALL CONSTANT AND MAGNETORESISTANCE

Let us consider the dependence of the Hall constant $R$ and of the magnetoresistance $\Delta\rho/\rho$ on the thickness for the case when the magnetic field $H$ is directed along the x axis (perpendicular to the plane of the film), and the current flows along the z axis. At such a geometry, we can take the expressions for $R$ and $\Delta\rho/\rho$ obtained from the kinetic equation, and substitute in them the expressions obtained in the preceding section for the relaxation time $\tau_{sq}$ and for the mobility $u_{n,p}$. (It is assumed that the cyclotron radius $r_H \gg a$.) Thus, we get for the Hall constant:

when $a > \bar{a}$

$$R = -(u_p - u_n) / nec(u_n + u_p); \qquad (27)$$

when $a < \bar{a}$

$$R = -\frac{3\pi}{8nec}\frac{u_p - u_n}{u_n + u_p}, \qquad \left(\frac{u_{p,n}H}{c}\right)^2 \ll 1;$$

$$R = -\frac{45\pi}{64nec}\frac{u_p - u_n}{u_n + u_p}, \qquad \left(\frac{u_{p,n}H}{c}\right)^2 \gg 1. \qquad (28)$$

It follows from these formulas that the $R(a)$ dependence is contained only in the factor $1/n$, so that its form can be readily visualized with the aid of the plot in Fig. 2. A characteristic feature is the sharp increase of $R$ when $a < \bar{a}$, , and the absence of noticeable oscillations in the region $a > \bar{a}$. The Hall mobility $u_H = R\sigma c$ is shown in Fig. 4.

For the magnetoresistance we get:
when $a > \bar{a}$

$$\Delta\rho / \rho = (u_n u_p / c^2)H^2, \qquad (29)$$

when $a < \bar{a}$

$$\frac{\Delta\rho}{\rho} = \frac{9\pi H^2}{16c^2}\left[\left(1 - \frac{\pi}{4}\right)(u_n - u_p)^2 + u_n u_p\right], \left(\frac{u_{n,p}H}{c}\right)^2 \ll 1;$$

$$\frac{\Delta\rho}{\rho} = \frac{9\pi}{32}\frac{u_n u_p}{c^2}H^2, \qquad \left(\frac{u_{p,n}H}{c}\right)^2 \gg 1. \qquad (30)$$

These dependences are shown in Fig. 5. The magnetoresistance oscillates with a period $\bar{a}$.

We note that the obtained jumps in $\sigma$, $u_H$, and $\Delta\rho/\rho$ at the point $a = s\bar{a}$, $s = 1, 2, 3, \ldots$ are connected with the fact that no account is taken of the influence of scattering on the change in the state density.

## 4. FIELD EFFECT UNDER QSE CONDITIONS

The QSE can be investigated in a film of fixed thickness, by varying in it the position of the Fermi level. This can be done, for example, by means of the field effect. As is well known, the field effect consists in charging the investigated film, which serves as one electrode of a capacitor, varying thereby the carrier density in it.

Let us consider approximately the variation of the electric conductivity under the influence of the field effect. We assume that the charge conservation condition can be written in the form

$$CV / ea = n - p, \qquad (31)$$

where $C$ is the capacitance per unit area, $V$ the potential of the second capacitor electrode, and the densities $n$ and $p$ are given by (6) for the
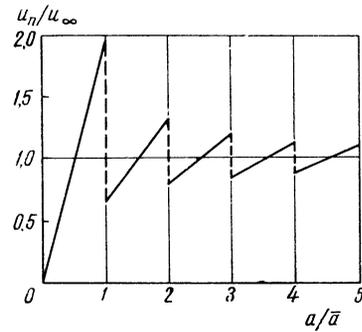


FIG. 4. Dependence of Hall mobility on the thickness.
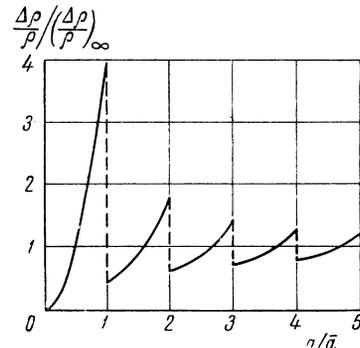


FIG. 5. Dependence of the magnetoresistance on the thickness (in units of magnetoresistance for the bulky sample).

electrons and by an analogous expressions for the holes. Expression (31) presupposes that the form of the potential trough does not change when the electric field is applied. In a rigorous analysis of the field effect it would be necessary to solve simultaneously the Poisson equation and the quantum kinetic equation.

We shall assume that $a > \bar{a}$. From (31) we determine $\mu_n(V)$ and $\mu_p(V)$ which we then substitute in the expression for the electric conductivity $\sigma = en\mu_n + ep\mu_p$, and obtain $\sigma(V)$.

The character of the $\sigma(V)$ dependence for the case of scattering by an impurity with a $\delta$-potential is clear from the following reasoning. When $V$ increases, the film becomes negatively charged, and consequently $\mu_n$ increases and $\mu_p$ decreases. In the sections $s < \lfloor\sqrt{\mu_p/\epsilon_p}\rfloor < s + 1$, the hole conductivity $\sigma_p = ep\mu_p$ is continuous; in the sections $s < \lfloor\sqrt{\mu_n/\epsilon_n}\rfloor < s + 1$ it is the electron conductivity which is continuous; here $s$ are integers. Since, according to (31), $\mu_n$ and $\mu_p$ depend on $V$ linearly in these sections and since $\sigma$ depends linearly on $\mu_n$ and $\mu_p$, $\sigma$ depends linearly on $V$ in the continuity sections. Further, when $\mu_p$ decreases and passes through the extremum of the hole subband, $\sigma_p$ increases jumpwise; when $\mu_n$ increases and passes through the bottom of the electron subband, $\sigma_n$ decreases jumpwise.

Assume that all the holes are "filled" when $V = V_n$, so that $p = 0$. Then when $V > V_n$ there remain only sections of linear growth $\sigma = \sigma_n$ and points of jumpwise drops.

When $V < 0$ and $V$ increases in absolute magnitude, there will also be such sections of linear variation of $\sigma$ and points of jumpwise decreases on crossing the extrema of the hole subbands and jumpwise increases on crossing the minima of the electron subbands. When $V < V_p$, when $n = 0$, there remain only sections of linear increase of $\sigma = \sigma_p$ when $V$ decreases, and points of jumpwise drops in $\sigma$.

It is easy to find that

$$V_n = \frac{\pi e}{12Ca^2}\bar{M}_n\left\{6\frac{\Delta - \varepsilon_p}{\varepsilon_n} - (\bar{M}_n + 1)(2\bar{M}_n + 1)\right\}, \quad (32a)$$

$$V_p = -\frac{\pi e}{12Ca^2}\bar{M}_p\left\{6\frac{\Delta - \varepsilon_n}{\varepsilon_p} - (\bar{M}_p + 1)(2\bar{M}_p + 1)\right\};$$

$$\bar{M}_n = [\sqrt{(\Delta - \varepsilon_p)/\varepsilon_n}], \quad \bar{M}_p = [\sqrt{(\Delta - \varepsilon_n)/\varepsilon_p}]. \quad (32b)$$

Let $\epsilon_p \ll \Delta$ and $\epsilon_n \ll \Delta$; then

$$V_p \approx \frac{\pi}{3}\frac{e}{C\bar{a}^2}\frac{a}{\bar{a}}\left(\frac{m_p}{M}\right)^{3/2}, \quad V_n \approx \frac{\pi}{3}\frac{e}{C\bar{a}^2}\frac{a}{\bar{a}}\left(\frac{m_n}{M}\right)^{3/2}. \quad (33)$$

When $a/\bar{a} \approx 4$, $m_n/M \approx 1$, and $C \approx 100$ cm we have $V_n \approx 300$ V.

When $V > V_n$ and $V < V_p$, the distance between the successive values of the potentials, at which $\sigma$ changes in a jump, is $\pi es(2s + 1)/2a^2C$, where $s$ is the number of the corresponding electron $(V > V_n)$ or hole $(V < V_p)$ subband. When $V > V_n$, the electric conductivity in the section $s < \sqrt{\mu_n/\epsilon_n} < s + 1$ changes in accordance with law

$$\sigma = \frac{CV}{\bar{a}}u_{n\infty}\frac{2}{2s + 1}. \quad (34)$$

Making the substitution $u_{n\infty} \to u_{p\infty}$, we obtain the corresponding formula for $V < V_p$.

Let us consider the region $V_p < V < V_n$. Let $a/\bar{a} = k + \delta$, where $k$ is an integer and $0 \le \delta < 1$. If $\delta = 0$, then $\sigma(V)$ experiences a jump at $V = 0$. This jump is equal to

$$\lim \sigma(+0) - \lim \sigma(-0) = \frac{e\pi}{3\bar{a}^3}(u_{p\infty} - u_{n\infty})$$
$$\times \frac{(k - 1)(4k + 1)}{k^2(4k^2 - 1)}. \quad (35)$$

Thus, this quantity has the sign of the difference $u_{p\infty} - u_{n\infty}$. When $V$ increases, the first jump in $\sigma$ takes place on crossing the $(k - 1)$-st hole subband if $m_n < m_p$, or on crossing the $(k + 1)$-st electron subband if $m_n > m_p$. A similar picture takes place also for $V_p < V < 0$.

The magnitude of the effect can be characterized by the derivatives

$$\left(\frac{d\sigma}{dV}\right)_{+0} = 2C\left[\frac{k}{2k + 1}m_n u_{n\infty} - \frac{k - 1}{2k - 1}m_p u_{p\infty}\right]$$
$$\times \{\bar{a}[k(m_n + m_p) - m_p]\}^{-1},$$

$$\left(\frac{d\sigma}{dV}\right)_{-0} = 2C\left[\frac{k - 1}{2k - 1}m_n u_{n\infty} - \frac{k}{2k + 1}m_p u_{p\infty}\right] \quad (36)$$
$$\times \{\bar{a}[k(m_n + m_p) - m_n]\}^{-1}.$$

We see therefore that in the vicinity of $V = 0$, in the continuity sections, $\sigma$ can either increase or decrease with variation of $V$, depending on the relation between the parameters of the system.

If $\delta \ne 0$, then in the vicinity of $V = 0$ the value of $\sigma(V)$ changes continuously. The effect can be continuously. The effect can be characterized by the quantity

$$\left(\frac{d\sigma}{dV}\right)_0 = \frac{2C}{(2k + 1)\bar{a}}\left(\frac{m_n}{m_n + m_p}u_{n\infty} - \frac{m_p}{m_n + m_p}u_{p\infty}\right). \quad (37)$$

The sign of the effect can differ, depending on the relations between the parameters in the system. The form of the $\sigma(V)$ dependence is shown in Fig. 6.

In exactly the same way we can determine the change of other kinetic coefficients in the field effect. We note, in particular, that the Hall con-
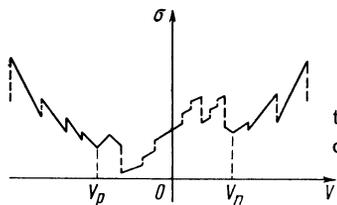
FIG. 6. Qualitative plot of the variation of the electric conductivity in the field effect.

cussions, and also the participants of the seminar of the Institute of Radio Engineering and Electronics, headed by M. I. Elinson and S. G. Kalashnikov for a discussion of the results. I am grateful to M. Ya. Azbel' for a discussion concerning the work and for useful remarks.

stant R in the vicinity $V_p < V < V_n$ can reverse sign several times. This follows from the fact that when a change in V takes place corresponding, for example, to a growth of $\sigma_n$, jumps take place in which $\sigma_p$ increases the jumps in which $\sigma_n$ decreases.

Let us make several remarks concerning comparison with experiment. The very fact that the kinetic coefficients depend in oscillatory fashion on the thickness agrees with observations reported in [1]. The period of the oscillations for bismuth films, calculated from formula (8), is also in satisfactory quantitative agreement. The variation of R(a) in the vicinity a $\approx \bar{a}$ does not agree.

In conclusion, I am grateful to Sh. M. Kogan, Yu. F. Ogrin, and V. N. Lutskiĭ for numerous dis-

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