ANGULAR DIVERGENCE OF SOLID STATE LASER RADIATION

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The angular divergence of radiation from neodymium glass and $CaF_2:Sm^{2+}$ lasers is studied for the case of variable resonator and pump parameters. It is shown that angular divergence in homogeneous active media is determined by spatial competition between the transverse resonator modes. Appropriate mode selection can reduce the angular divergence down to the diffraction limit without a significant drop in the output power.

THE angular divergence in a solid state laser with a flat resonator depends upon the following factors:

1. Angular divergence of the individual modes. The angular width of the lowest mode in an undeformed resonator is of the order of $\varphi_d \approx 1.2\lambda/D$, where D is the diameter of the resonator and λ is the radiation wavelength. Inhomogeneities of the active medium, nonuniform distribution of the pumping radiation over the rod cross section, and other similar factors cause deformation of the modes in the resonator and an increase in their angular width (see for example [1–4]).

2. The number m of transverse modes excited in the lasing process. Saturation of population inversion in the active medium is accompanied by a spatial mode competition creating a considerable number of transverse modes.[5] The resulting total angular divergence φ of radiation from a laser with an undeformed resonator and plane mirrors in the form of infinite strips of width D amounts to $\varphi = \varphi_d m$.

The number of excited modes depends on the nature of resonator losses. Nonselective losses (non-active absorption, resonator-mirror transmission, etc.) weakly affect the radiation distribution among the transverse modes. Diffraction losses and scattering are the main factors that substantially determine the number of excited modes. However, the effect of scattering on the angle of laser emission has not yet been studied. In particular, scattering can give rise to coupled modes with a considerable total angle of divergence. Let us also note that the interaction of modes with close frequencies is significantly more complicated than that discussed by Statz and Tang (see for example [5]); mode interaction can be due to interference beats between the modes [6] (this phenomenon seems to be one of the main causes of the spike structure of laser emission). As a result, the applicability of the simple multimode oscillation model of Statz and Tang to the spike-averaged laser processes is not self-evident and requires experimental verification.[1)]

So far, angular divergence in solid state lasers has been studied mainly in ruby lasers.[8–10] Owing to the considerable optical inhomogeneity of ruby crystals, ruby lasers are subject to considerable mode deformation that depends upon specific types of inhomogeneities. In particular, an investigation of angular divergence in ruby laser emission at fairly low power above threshold [9], when individual angular modes were excited, showed that the presence of a gradient in the index of refraction of the crystal gave rise to modes typical of a spherical mirror resonator.

In the case of practical interest, that of high powers above the threshold, when a considerable number of transverse modes are excited, the analysis and interpretation of experimental data on the angular width of laser emission are very difficult when inhomogeneous ruby-type materials are involved.

In the present paper we consider the effect of resonator and pump parameters on the angular divergence emission from neodymium-glass and $CaF_2:Sm^{2+}$ lasers. While the homogeneity of their index of refraction is near perfect, these materials sharply differ in terms of the scattering coefficient: it is 0.02–0.04 cm^{-1} in $CaF_2:Sm^{2+}$ and less than 0.004 cm^{-1} in glass.[2)] The optical inhomogeneity of the active specimens under investigation, including fabrication defects of the

1)See [7] on the applicability of the Statz and Tang model to the determination of radiation distribution in terms of modes with differing axial indices.

2)Integral scattering coefficients are given within the limits 4′ – 3°. The scattering integral coefficients and indicatrices were measured by A. I. Kalyadin with associates.

resonator and thermal deformation at the selected pumping regime, was fairly small; in particular, deviations from the resonator planarity did not exceed 0.05 microns.

The study of the connection between the angular and energy characteristics of a laser with selected angular modes is of particular interest. This selection is accomplished in our work by increasing the resonator length, thereby increasing the contribution of diffraction losses.

EXPERIMENTAL METHOD

The photographic method was used to measure radiation divergence. Photographic film was placed in the focal plane of a lens whose focal length ranged from 0.3 to 2 m. A stepped attenuator was printed on the same film using the light of the investigated laser. The laser emission intensity was measured with a photomultiplier and a calorimeter. A straight xenon flash lamp and an elliptical reflector were used to pump the cylindrical rods. The resonator length was varied in the experiments from 0.5 to 300 cm. Mirrors deposited on the rod ends were used in short resonators (up to 4 cm), and external mirrors in long resonators. The $CaF_2:Sm^{2+}$ crystal rods were 5 mm in diameter and 5—40 mm long; the glass rods were 5 and 8 mm in diameter and 40 mm long. The lateral surface of the rods was ground dull. The resonator diameter could be reduced with diaphragms ($D = 1.3$ and 3 mm). In the case of $CaF_2:Sm^{2+}$, the crystal was kept in a vacuum cell with high-precision flat windows to avoid deformation of the resonator. The crystal was cooled down to 30—35°K and its temperature monitored with a thermocouple.

The pumping regime was chosen to produce instantaneous high powers above threshold with minimal pump energies. The pump energy did not exceed 50—70 J for $CaF_2:Sm^{2+}$ and 130 J for glass. The thermal deformation of the rods at these energies amounted to less than 0.05 micros.[11,12] According to special measurements [13], such a pumping system maintains the pump radiation density at the rod center within 1.5—2 times the radiation density at the rod surface ($D = 0.5$ cm).

Particular attention was paid to the adjustment of the resonator. Adjustment with the autocollimator was supplemented by a fine adjustment made by setting at the minimum the threshold pump intensity and by ensuring circular symmetry of the radiation angular distribution.

RESULTS OF MEASUREMENTS

Figures 1 and 2 show examples of photographs and microphotographs of the angular distribution of laser radiation. Lasing occurs in $CaF_2:Sm^{2+}$ at a pump pulse length t_p considerably longer than the luminescence decay time τ. In this case it is possible to achieve generation at fairly low excess pump power when the individual low order modes are excited (Fig. 1 b and c). In neodymium glass, $t_p \ll \tau$ and generation requires a high excess pump power. No sharply defined structure is observed in the angular distribution of emission in this case (Fig. 1 a). The picture is similar for $CaF_2:Sm^{2+}$ at high excess pump power (Fig. 1 d). The photograph clearly shows the rings whose angular distances are equal to those in a Fabry-Perot etalon with a base $L - l(1 - \kappa^{-1})$; here L and l are the lengths of the resonator and rod respectively, and κ is the index of refraction of the rod material.

It should be noted that the rings are also observed in neodymium glass; however, their intensity is much weaker than in the case of $CaF_2:Sm^{2+}$. According to measurements with a calorimeter and diaphragms placed in the focal plane of the lens, the portion of emission energy falling on the central spot in the angular distribution is 50—70% for $CaF_2:Sm^{2+}$ and over 95% for neodymium glass.

We have found that variations in excess pump

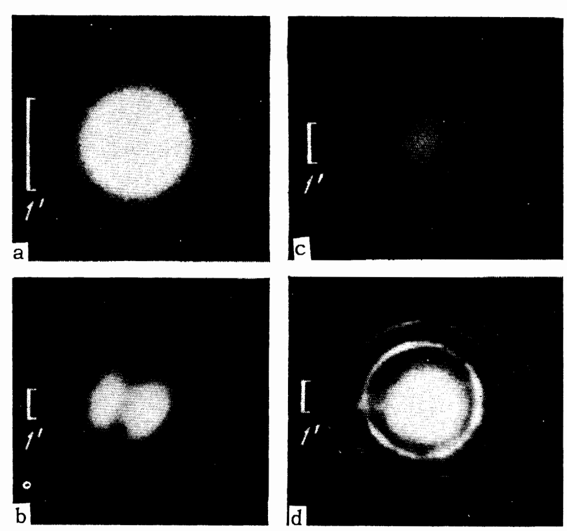

FIG. 1. Photographs of the angular distribution of laser emission. a — neodymium glass; red length $l = 4.0$ cm, diameter $D = 0.8$ cm, resonator length $L = 100$ cm, excess pump power $n = 20$. b, c, d — $CaF_2:Sm^{2+}$; $l = 4$ cm, $D = 0.5$ cm. b — $L =$ cm, $n \approx 1$, TEM_{00} mode. c — $L = 60$ cm, $n \approx 1$, TEM_{01} mode. d — $L = 45$ cm, $n = 16$.

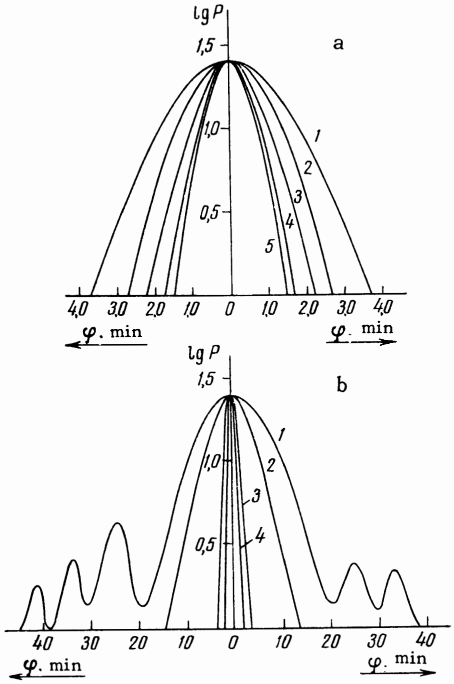

FIG. 2. Angular distribution of laser emission. a – neodymium glass; D = 0.5 cm, $\ln R^{-1} = 0.19$. 1 – L = 7.5 cm, n = 41; 2 – L = 16 cm, n = 41; 3 – L = 30 cm, n = 39; 4 – L = 60 cm, n = 28; 5 – L = 120 cm, n = 23. b – $CaF_2:Sm^{2+}$; D = 0.5 cm. 1 – L = 2 cm, n = 20; 2 – L = 4 cm, n = 16; 3 – L = 45 cm, n = 3.6; 4 – L = 150 cm, n ≈ 3.

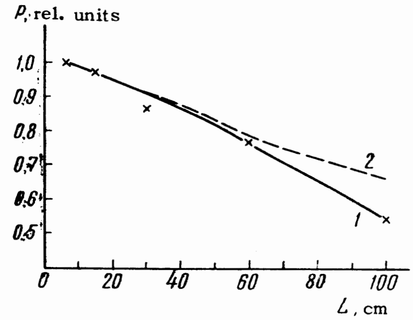

FIG. 3. Output power as a function of resonator length for a neodymium glass laser. D = 0.5 cm, $\ln R^{-1} = 0.19$, n = 23 – 41. Plot 1 corresponds to the experiment, plot 2 to formula (3).

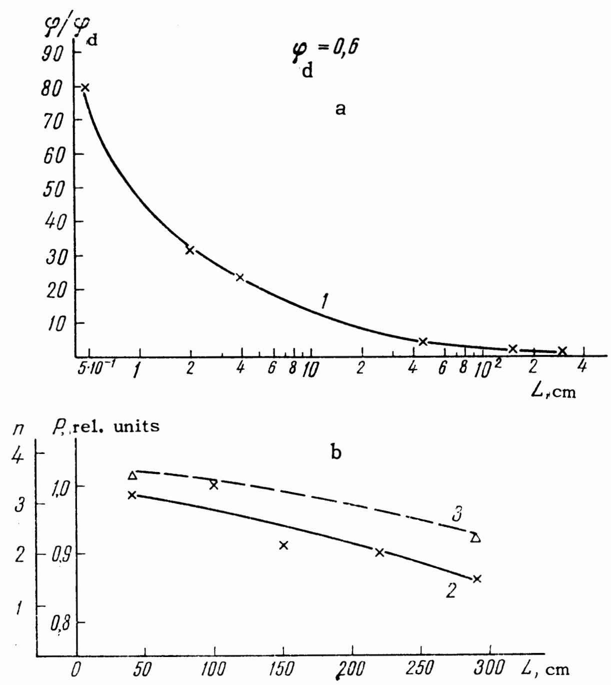

FIG. 4. Angular divergence φ(a, plot 1), output power P (b, plot 2), and excess pump power n (b, plot 3) as functions of resonator length for a $CaF_2:Sm^{2+}$ laser. $\ln R^{-1} = 0.5$.

power (from 2 to 10 for $CaF_2:Sm^{2+}$ and from 15 to 40 for glass) and in the reflection coefficient R of the resonator mirror (from 20 to 95% for $CaF_2:Sm^{2+}$ and from 50 to 97% for glass) are practically without effect on the angular divergence of radiation.

We investigated the angular divergence, emission power, and threshold pump power as functions of resonator length. The results are given in Table I and Figs. 3 and 4. Instantaneous excess pump power n is determined according to the method given in[14], and laser emission power P is determined from the oscilloscope traces of laser emission and corresponds to the spike-averaged maximum peak power.[14] The angular widths φ given in Fig. 3 and Table I correspond to the angular divergence of laser emission at the 0.5 level of maximum emission power.

The above data indicate that an increase in resonator length L leads to a considerable reduction in the angular width of the laser emission. At the same time, the emission power and threshold pump power vary negligibly as L increases up to lengths causing the angular emission width to approach $(1.5-2)\varphi_d$. The drop in output power becomes significant with further increase of L. The angular width of the neodymium laser emission is approximately proportional to $L^{-1/2}$, and when

Table I. Neodymium Glass Laser

D, cm	0.8(φ_d=0.56')			0,5(φ_d=0.89')							0.3(φ_d=1,48')		0,13 (φ_d=3,4')
L, cm	21	100	200	7,5	16	17.5	30	60	100	120	17.5	60	17.5
φ/φ_d	4.5	1.0	0,6	3.5	2.7	2.2	2.06	1,67	1,39	1.33	1,26	1.23	1.0
n				41	41	40	39	28	25	23	34	16	18

$L = 7.5$ cm, φ/φ_d is only 3.5 ($D = 0.5$ cm). In the case of $CaF_2:Sm^{2+}$, the dependence of the angle φ on the resonator length is well approximated by $L^{-1/2}$ and the absolute values of φ are considerably higher than in the case of neodymium laser (at the same L). We note that the angular dimension of the central spot of $CaF_2:Sm^{2+}$ emission is fairly close to $[2\lambda/(L - l(1-\kappa^{-1}))]^{1/2}$.

DISCUSSION OF RESULTS

Even a preliminary examination of the obtained results leads to the conclusion that a very careful adjustment can ensure an insignificant mode deformation due to resonator deviations from the ideal shape. In particular, according to Figs. 1 b and c, the angular width of the TEM_{00} and TEM_{01} modes excited at very low excess pump power is not over 1.5 times the width of the corresponding modes of a flat resonator. Consequently, we can assume that the angular width of neodymium laser emission observed in the experiment is determined by a large number of transverse modes due to spatial competition, and that diffraction losses play a decisive role in the competition. In fact, the results of studies [14-16] of losses in $CaF_2:Sm^{2+}$ and neodymium-glass laser resonators indicate that while losses in CaF_2Sm^{2+} are mainly due to the scattering of light in the crystal, diffraction losses assume the major importance in neodymium glass.

A quantitative analysis of multimode generation in the presence of considerable light scattering in the active medium is not possible at this time. To analyze the operation of the neodymium laser we use the results of [4,5] on multimode generation with allowance for spatial competition of angular modes in a flat resonator in the form of infinite strip mirrors having a width D and spaced at a distance L from one another.[3)] These papers present the following relation to determine the number of excited modes m:

$$\frac{n}{D}\int_0^D \frac{(1-\cos 2\pi jx/D)\,dx}{1+\sum\limits_{k=1}^{j} A_k(1-\cos 2\pi kx/D)} \begin{cases} = 1+\Delta_j, & A_j > 0, \\ \leqslant 1+\Delta_j, & A_j = 0, \end{cases} \tag{1}$$

where n is excess pump power above threshold, A_j is a quantity proportional to the emission density of the mode with index j (the proportionality factor depends only upon the parameters of the active medium), and $\Delta_j = \sigma_j/\rho$ is the ratio of diffraction losses σ_j for the mode with index j to the nonselective losses $\rho \approx \ln R^{-1}$ determined mainly by the reflection coefficient R of the resonator mirror.

Using the expansion

$$\left[1+\sum_{k=1}^{m} A_k\left(1-\cos\frac{2\pi kx}{D}\right)\right]^{-1} \approx (1+\Sigma)^{-1}\left[1+(1+\Sigma)^{-1}\sum_k A_k\cos\frac{2\pi kx}{D}\right],$$

where $\Sigma \equiv \sum\limits_{k=1}^{m} A_k$, we can integrate (1) and find expressions for A_j and Σ[4]:

$$A_j = 2(1+\Sigma)\left[1-\frac{1+\Sigma}{n}(1+\Delta_j)\right],$$

$$1+\Sigma = \frac{n}{2}\frac{m-{}^1\!/_2}{m+\Sigma_\Delta}\left(1+\left[1+\frac{2(m+\Sigma_\Delta)}{n(m-{}^1\!/_2)^2}\right]^{1/2}\right),$$

$$\Sigma_\Delta \equiv \sum_{k=1}^{m}\Delta_k.$$

Using these expressions we can readily find the following approximate relations for the number m ($m \gtrsim 3$) of modes and output power P:

$$\frac{4}{3}m^3 + 2m^2 \approx \frac{n-1}{n}\frac{1}{\Delta_1}, \tag{2}$$

$$P \sim \frac{m-{}^1\!/_2}{m}\frac{\rho}{\rho+\sigma_{av}}\left(1-\frac{m}{m-{}^1\!/_2}\frac{\rho+\sigma_{av}}{\rho}\frac{1}{n}\right) \approx \frac{\rho}{\rho+\sigma_{av}}\left(1-\frac{\rho+\sigma_{av}}{\rho}\frac{1}{n}\right), \tag{3}$$

where

$$\sigma_{av} = \frac{1}{m}\sum_{k=1}^{m}\sigma_k$$

are the average diffraction losses for m excited modes (for a double pass through the resonator). The derivation of (2) was based on Vaĭnshteĭn's data [17] dealing with losses in a strip resonator:

$$\sigma_j \approx 2j^2(\sqrt{L\lambda}/D)^3.$$

According to (2), the number m of excited modes is weakly dependent on the transmission of the resonator mirror and on the excess pump power n when $n > 2$, which is in a good agreement with experimental data.

Figure 5 gives the calculated dependence of m on the quantity $1/\sqrt{L}$ for the case of D =

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3) From now on we assume that the angular divergence of radiation in a resonator with round mirrors having a diameter D is approximately equal to the angular divergence (in the corresponding direction) of a strip resonator having a strip width D; see also [4] on the angular divergence in a square-mirror resonator.

Table II

Angle, min	Scattering coefficient, $cm^{-1} rad^{-1}$	Laser emission intensity (Fig. 2b, 1) rel. un.	Angle, min	Scattering coefficient, $cm^{-1} rad^{-1}$	Laser emission intensity (Fig. 2b, 1) rel. un.
0	—	25	15	28	—
5	400	20	20	12.5	—
7	160	—	24	8	2.5
10	80	—			

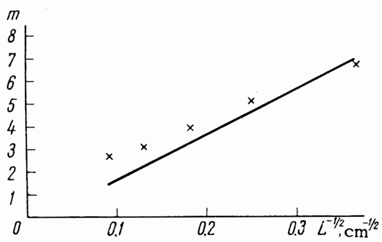

FIG. 5. Solid line corresponds to formula (2), D = 0.5 cm, $\lambda = 1.06 \times 10^{-4}$ cm, $\rho = 0.19$; $n \gg 1$. Crosses correspond to experimental data (see Fig. 2a).

0.5 cm, $\rho = \ln R^{-1} = 0.19$, and $n \gg 1$. The same figure also shows the experimental data. The experimental value of m is defined here as φ_0/φ_d, where φ_0 is the angular width of laser emission at the 0.1 level of the maximum intensity (according to Fig. 2 a, φ_0 is close to the total angular width of the emission). The graph in Fig. 5 shows fully satisfactory agreement between the calculated and experimental data, considering the approximate nature of the calculation. The deviation between experimental and calculated data in the region of small m may be due to a mode deformation that causes both the broadening of the angular mode divergence and the deviation from the relation $\Delta_j \sim j^2$ in the region of small j.

In calculating the generated power we assumed that the average diffraction losses are approximately equal in round and square mirror resonators. Furthermore, we can obtain an expression similar to (3) for the square mirror resonator, provided diffraction losses are averaged over all the excited modes; we can also show that the resulting quantity is approximately twice as large as the average diffraction losses in the strip-mirror resonator. The results of such a calculation are given in Fig. 3. A fully satisfactory agreement between experiment and theory is again evident here.

It is quite significant that the angular mode selection as the resonator length is increased does not result in any marked drop in output power up to lengths corresponding to a fairly small number of excited modes. This can be in part explained by the fact that as the resonator is lengthened the average losses determining the output power (see (3)) vary slowly because of the reduction in the number of excited modes, regardless of the increase in the losses for individual angular modes.

The above analysis of the experimental data allows us to assume that the angular divergence of neodymium laser emission is determined mainly by spatial competition leading to the excitation of a large number of angular modes. We can also conclude that under our experimental conditions the interaction between modes due to beats does not significantly affect the time-averaged characteristics of the laser.[4)] We further note that the width of the angular modes obtained in the presence of the actual deviations of the resonator from planarity (provided they retain their axial symmetry) does not appear to be larger than 1.5 times the width of the angular modes in an ideal flat resonator.[5)]

The above experimental data on the energy relationships and angular divergence of $CaF_2:Sm^{2+}$ laser emission lead to some preliminary conclusions concerning the mechanism of formation of angular divergence of emission in the presence of considerable light scattering.

The relatively large angular divergence of the emission seems in this case to be due to the formation of coupled modes, each of which represents a set of a large number of synchronized modes of an ideal resonator. The angular divergence of such a complex mode is much larger than φ_d; this is also indicated by the results obtained in [7]. Mode coupling in each set is effected by scattered radiation and is weak, owing to the relatively low intensity of this radiation (see Table II). Therefore a reduction in the angular divergence of the emission (as the resonator length increases) down to the values $\sim 2\varphi_d$ does not result in a significant drop of power output.

4)A study of losses in a neodymium laser resonator leads to a similar conclusion [16]

5)If we assume that the deviations of the resonator from planarity are due to spherical aberration in the specimen, then we find, using [18,19], that the width of the lowest mode (TEM_{00}) differs from φ_d (D = 0.5 cm) by a factor 1.3 – 1.4.

Given the above mechanism of angular divergence, the size of the central spot in the distribution should be determined from the condition that the phase shift of the synchronized modes per pass through the resonator be small and should, therefore, be proportional to $L^{-1/2}$, as was indeed observed.

The data given in Table II and the results of measuring the portion of emission energy falling on the central spot indicate that the emission intensity in the angular-distribution rings is fairly large, in spite of the fact that the intensity of the scattered radiation of the central spot should be small at the corresponding angles. This circumstance, together with the periodic distribution of the rings, further indicates that we are dealing with the excitation of complex modes whose emission fills both the central spot and the angular distribution rings.

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