KINETIC INSTABILITY OF A PLASMA LOCATED IN A STRONG HIGH FREQUENCY FIELD

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It is demonstrated that, owing to the Cerenkov effect, instability with respect to buildup of potential oscillations is possible in a plasma situated in a high frequency electric field. Such a kinetic instability, in contrast to the hydrodynamic instability previously discussed[1], should also be possible in a plasma in which the electron Langmuir frequency is smaller than the external field frequency.

1. It was shown in a study of the stability of the plasma in a strong high-frequency field $\mathbf{E}(t) = \mathbf{E}\sin\omega_0 t$[1] that instability against buildup of oscillations of the potential field sets in at external-field frequencies ω_0 close to the electron Langmuir frequency $\omega_{Le} = \sqrt{4\pi e^2 N_e/m}$ and at lower frequencies. This instability is not connected with the Cerenkov effect, and is hydrodynamic in this sense. On the other hand, it was shown[2] that the plasma is stable against buildup of potential oscillations under conditions of very high external-field frequencies, when the plasma is transparent. In this communication we show that a transparent nonisothermal plasma situated in a strong high frequency field is unstable if the frequency of the external field is not too high. The resultant instability is due to the Cerenkov effect on the plasma particles and is kinetic in this sense. To determine the stability of a nonisothermal plasma we can use the dispersion equation for potential oscillations, obtained in[1]

$$1+\frac{1}{\delta\varepsilon_i(\omega+i\gamma,\mathbf{k})} = \sum_{n=-\infty}^{+\infty} J_n^2(a)\left[1-\frac{1}{1+\delta\varepsilon_e(n\omega_0+\omega+i\gamma,\mathbf{k})}\right] \quad (1.1)$$

With the aid of this equation it is possible to investigate the oscillations whose frequency is small compared with the external-field frequency (see[1,3]). Here J_n is a Bessel function, $a = e\mathbf{E}\cdot\mathbf{k}/m\omega_0^2$, the indices i and e correspond to ions and electrons, and, finally, the function

$$\delta\varepsilon_a(\omega+i\gamma,\mathbf{k}) = \frac{4\pi e_a^2}{k^2}\int d\mathbf{p}\mathbf{k}\frac{\partial f_a}{\partial\mathbf{p}}\frac{1}{\omega+i\gamma-\mathbf{kv}} \quad (1.2)$$

represents the well known expression describing the contribution made by particles of species a to the longitudinal dielectric constant. The particle distribution functions f_a will be assumed to be isotropic functions of the momentum.

In the instability considered below the increment γ is small compared with the frequency ω. Therefore we shall assume such a situation to hold in the analysis that follows. For the damping of the waves to be small it is necessary to assume that the phase velocity (ω/k) of the oscillations is large compared with the thermal velocity of the ions. Then the left side of Eq. (1.1) can be approximately represented in the form:

$$1-\frac{\omega^2+2i\omega\gamma}{\omega_{Li}^2}+i\frac{4\pi^2 e_i^2}{k^2}\frac{\omega^4}{\omega_{Li}^4}\int d\mathbf{p}\,\mathbf{k}\frac{\partial f_i}{\partial\mathbf{p}}\delta(\omega-\mathbf{kv}). \quad (1.3)$$

The right side of (1.1) assumes a specially simple form for short waves, much shorter than the electronic radius of Debye screening. In this case $\delta\epsilon_e$ is small compared with unity. Therefore the right side of Eq. (1.1) can be written in the form

$$\sum_{n=-\infty}^{+\infty} J_n^2(a)\,\delta\varepsilon_e(n\omega_0+\omega,\mathbf{k}) = \frac{4\pi e^2}{k^2}\int d\mathbf{p}\mathbf{k}\frac{\partial f_e}{\partial\mathbf{p}}\sum_{n=-\infty}^{+\infty}\frac{J_n^2(a)}{\omega+i\gamma-n\omega_0-\mathbf{kv}}. \quad (1.4)$$

This expression is always small compared with unity. Therefore for the frequency of such shortwave oscillations we obtain immediately

$$\omega^2 = \omega_{Li}^2\left\{1+O\left(\frac{1}{k^2 r_{De}^2}\right)\right\}, \quad (1.5)$$

where $\omega_{Li} = \sqrt{4\pi e_i^2 N_i/m_i}$ is the Langmuir frequency of the ions. The Cerenkov effect on the ions leads to dissipation of the oscillation energy. The corresponding contribution to the increment for a Maxwellian distribution of the ions is negative and of the form

$$\gamma_i = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\exp\left[-\frac{1}{2}s_i^2\right], \tag{1.6}$$

where $s_i = \omega/kv_{Ti}$, and $r_{Di} = \sqrt{\kappa T_i/4\pi e_i^2 N_i}$ is the Debye radius of the ions.

A positive contribution to the increment leading to the instability can be due to the electrons. In order to assess such a possibility, we must consider the imaginary part of the formula (1.4). This can be readily done in general form if the phase velocity of the oscillations is small compared with the thermal velocity of the electrons. In this case we can expand (1.4) in powers of ω. The zeroth term of such an expansion makes no contribution to the imaginary part if, as has been assumed, the distribution of the electrons is an even function of the momentum. As a result we get for the imaginary part of expression (1.4)

$$\operatorname{Im}\sum_{n=-\infty}^{+\infty} J_n^2(a)\,\delta\varepsilon_e(n\omega_0+\omega, k) \cong \frac{4\pi^2 e^2}{mk^3}\omega\sum_{n=-\infty}^{\infty} J_n^2(a)\frac{d^2\bar{f}_e}{dv^2}\Big|_{v=l\omega_0/k}. \tag{1.7}$$

For instability to set in it is necessary that this expression be negative for positive ω. This calls for a negative second derivative, with respect to the velocity, of the electron distribution function $\bar{f}_e$ integrated over the momenta perpendicular to the wave vector **k**. This condition is satisfied for a large number of distributions. We shall consider separately, using two plasma-electron momentum distributions as examples, the regions in which the Cerenkov effect leads to buildup of oscillations, and will obtain expressions for the corresponding oscillation increments.

2. In order to understand better the properties of the discussed instability, let us consider the case of an electron distribution

$$f(p) = \frac{1}{\pi^2}\frac{mv_0}{(p^2+m^2v_0^2)^2}. \tag{2.1}$$

Such a distribution function yields for the dielectric constant an expression

$$\delta\varepsilon_e(\omega, k) = -\frac{\omega_{Le}^2}{(\omega+ikv_0)^2}. \tag{2.2}$$

Substituting (2.2) in the right side of (1.1), we can sum the series (see[1]). We then obtain

$$1+\frac{1}{\delta\varepsilon_i(\omega+i\gamma, k)} = \frac{\pi}{2}\frac{\omega_{Le}}{\omega_0}\left\{\frac{J_\nu(a)J_{-\nu}(a)}{\sin\pi\nu} - \frac{J_\mu(a)J_{-\mu}(a)}{\sin\pi\mu}\right\}, \tag{2.3}$$

where

$$\nu = (ikv_0 + i\gamma + \omega + \omega_{Le})/\omega_0,$$

$$\mu = (ikv_0 + i\gamma + \omega - \omega_{Le})/\omega_0.$$

In the case of interest to us, that of plasma oscillation frequencies that are small compared with the frequency of the external field, we can expand the right side of (2.3) in powers of $\omega + i\gamma$. As a result we obtain[1)]

$$1+\frac{1}{\delta\varepsilon_i(\omega+i\gamma, k)} = 2\frac{\omega_{Le}}{\omega_0}\operatorname{Re}\left\{\left[1+i\frac{\omega}{\omega_0}\frac{d}{d\rho}\right]\left(\frac{d}{d\ln a}\ln\frac{J_\rho(a)}{J_{-\rho}(a)}\right)^{-1}\right\}, \tag{2.4}$$

where $\rho = (ikv_0 + \omega_{Le})/\omega_0$. We have left out γ from the term that is linear in the oscillation frequency, since this entire term is relatively small.

Further, if the frequency of the external field greatly exceeds the Langmuir frequency of the electrons, then, expanding (2.4) in powers of (ω_{Le}/ω_0), we obtain

$$1+\frac{1}{\delta\varepsilon_i} = 2\frac{\omega_{Le}^2}{\omega_0^2}\left[1-i\frac{\omega}{\omega_0}\frac{d}{dy}\right]\frac{d}{dy}\int_0^\pi dt\, J_0(2a\sin t)\frac{e^{-2yt}}{e^{-2\pi y}-1}$$

$$= 2\frac{\omega_{Le}^2}{\omega_0^2}\left[1-i\frac{\omega}{\omega_0}\frac{d}{dy}\right]\frac{d}{dy}\operatorname{Im}\left\{\frac{d}{d\ln a}\ln\frac{J_{iy}(a)}{J_{-iy}(a)}\right\}^{-1},$$

$$y = kv_0/\omega_0. \tag{2.5}$$

For very short waves, when $kv_0 \gg \omega_0$, formula (2.5) assumes the following simple form:

$$1+\frac{1}{\delta\varepsilon_i} = \frac{\omega_{Le}^2 kv_0}{[k^2v_0^2+a^2\omega_0^2]^{3/2}}\left\{1+i\frac{\omega}{kv_0}\frac{2k^2v_0^2-a^2\omega_0^2}{k^2v_0^2+a^2\omega_0^2}\right\}. \tag{2.6}$$

The imaginary part of the right side becomes negative when

$$a > \sqrt{2}kv_0/\omega_0. \tag{2.7}$$

In other words, the instability becomes possible under conditions when the velocity of the oscillations of the electron in the external electric field exceeds $\sqrt{2}\,v_0$. Taking formula (1.6) into account, we obtain from (2.6) the following expression for the oscillation increment:

[1)]The following integral representation of the right-hand side of (2.4) is also convenient:

$$2\frac{\omega_{Le}}{\omega_0}\left[1-i\frac{\omega}{\omega_0}\frac{d}{dy}\right]\operatorname{Im}\int_0^\pi dt\exp\left[-2\left(y+i\frac{\omega_{Le}}{\omega_0}\right)t\right] \times J_0(2a\sin t)\left\{\exp\left[-2\pi\left(y+i\frac{\omega_{Le}}{\omega_0}\right)\right]-1\right\}^{-1}, \quad y=\frac{kv_0}{\omega_0}.$$

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\exp\left(-\frac{1}{2}s_i^2\right) + \frac{1}{2}\frac{\omega_{Li}^2\omega_{Le}^2[a^2\omega_0^2 - 2k^2v_0^2]}{[a^2\omega_0^2 + k^2v_0^2]^{5/2}} \quad (2.8)$$

By virtue of the smallness of the right side of (2.6), the oscillation frequency is given by expression (1.5). According to (2.8), the instability sets in when the wavelength is not too small.

$$\lambda = \frac{1}{k} > r_{Di}\left\{2\ln\frac{v_0^3\omega_{Li}^2}{v_{Ti}^3\,\omega_{Le}^2}\right\}^{1/2}. \quad (2.9)$$

The last inequality is compatible with the condition for the applicability of formula (2.6) if

$$\omega_{Le} < \omega_0 < \frac{v_0}{r_{Di}}\left\{2\ln\frac{v_0^3\omega_{Li}^2}{v_{Ti}^3\,\omega_{Le}^2}\right\}^{-1/2}. \quad (2.10)$$

In other words, this is possible when the average energy of the disordered motion of the electron greatly exceeds the ionic energy, corresponding to the case of a non-isothermal plasma. In the "long-wave" limit, when $|kv_0 + i\omega_{Le}| \ll \omega_0$, we can write formula (2.4) in the following simple form

$$1 + \frac{1}{\delta\varepsilon_i(\omega + i\gamma, k)} = \frac{\omega_{Le}^2 J_0^2(a)}{k^2v_0^2 + \omega_{Le}^2} + 2i\frac{\omega k v_0 \omega_{Le}^2}{\omega_0^4} \times\left\{\frac{\omega_0^4 J_0^2(a)}{[k^2v_0^2 + \omega_{Le}^2]^2} - \frac{2}{\pi}\int_0^\pi dt\, t^2(2\pi^2 - t^2)J_0(2a\sin t)\right\}. \quad (2.11)$$

It follows therefore, that in such a limit the instability is possible only in the region of small values of $J_0(a)$. Thus, for example, in the vicinity of the point $J_0(a_r) = 0$ we can write for the frequency and the increment of the oscillations the following formulas:

$$\omega^2 = \omega_{Li}^2\left\{1 + 3k^2r_{Di}^2 - \frac{\omega_{Le}^2 J_1^2(a_r)}{k^2v_0^2 + \omega_{Le}^2}(a - a_r)^2\right\}, \quad (2.12)$$

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\exp\left[-\frac{1}{2}s_i^2\right] + \omega_{Li}^2\omega_{Le}^2\frac{kv_0}{\omega_0^4} \times\left\{C_r - \frac{\omega_0^4 J_1^2(a_r)}{[k^2v_0^2 + \omega_{Le}^2]^2}(a - a_r)^2\right\}, \quad (2.13)$$

where

$$C_r = \frac{2}{\pi}\int_0^\pi dt\, t^2(2\pi^2 - t^2)J_0(2a_r\sin t). \quad (2.14)$$

When $a_1 = 2.4$ we have $C_1 = 1.86$. For large values of a_r, we get the asymptotic relation $C_r = \pi^3/8a_r$. Also at large values of a_r, we can write the following formula, which determines the limits of the region near a_r in which the electronic part of the increment (2.13) is negative:

$$a = a_r \pm \frac{\pi^2}{4}\left[\frac{\omega_{Le}^2}{\omega_0^2} + a_r^2\frac{v_0^2}{v_E^2}\right]. \quad (2.15)$$

It is obvious that formula (2.15) holds true only when the velocity of the electron oscillations $v_E = eE/m\omega_0$ greatly exceeds v_0.

According to formula (2.13), instability occurs when the following inequalities are satisfied

$$\lambda \equiv \frac{1}{k} > r_{Di}\left\{2\ln\left[\sqrt{\frac{\pi}{8}}\frac{\omega_0^4 v_{Ti}}{\omega_{Le}^2\omega_{Li}^2 C_r v_0 (kr_{Di})^4}\right]\right\}^{1/2} \gg r_{Di}. \quad (2.16)$$

Simultaneously, according to the condition for the applicability of (2.11), it is also necessary to satisfy the inequality $\lambda \gg v_0/\omega_0$. At the same time, by virtue of the fact that a_r is not small, the wavelength of the growing oscillations is small compared with the amplitude of the oscillations of the electron in the external electric field. This is the condition determining the electric field intensity at which instability with increment (2.13) sets in. We note, finally, that we can confine ourselves to the term linear in ω in the right side of (2.4) only when $\omega \ll kv_0$ or $kv_0 \ll \omega_{Le}$. Therefore, just as in the case (2.10), we must have $mv_0^2 \gg \kappa T_i$. In the case of short waves, we obtain the instability condition in the region of large values of a. On the other hand, the limit of the instability region for large values of a also lies in the region of small values of kv_0/ω_0. The corresponding asymptotic expressions for the right sides of the dispersion relations (2.4) and (2.5) can be obtained relatively simply. For simplicity, we shall write here only the dispersion relation (2.5) in the limit of large a and small $y = kv_0/\omega_0$. Namely:

$$1 + \frac{1}{\delta\varepsilon_i(\omega, k)} = -\frac{\pi}{2a}\frac{\omega_{Le}^2}{\omega_0^2}\left\{\frac{\sin^2(a - \pi/4)}{\mathrm{ch}^2(\pi y/2)} - \frac{\cos^2(a - \pi/4)}{\mathrm{sh}^2(\pi y/2)}\right\} + i\frac{\omega\omega_{Le}^2}{\omega_0^3}\frac{8\pi^2}{a}\frac{e^{-2\pi y} + e^{-4\pi y}}{(1 - e^{-2\pi y})^3}[1 + F_1(y)\sin 2a], \quad (2.17)$$

$$F_1(y) = \frac{1}{4}\frac{1 + 6e^{2\pi y} + e^{4\pi y}}{e^{\pi y}(1 + e^{2\pi y})}. \quad (2.18)$$

Figure 1 shows the limits of the region in which the right side of (2.5) has a negative imaginary part. The ordinates represent $kv_0/a\omega_0$, and the abscissas the values of a. It is precisely within the shaded region that the Cerenkov effect on the electrons leads to buildup of oscillations. The asymptotic formula (2.18) shows the correct variation at the lower limit of this region, beyond the extreme right side of the figure. The upper limit (also on the right) is obtained from (2.7).

3. Let us consider now the condition for the occurrence of instability in the case when the electron

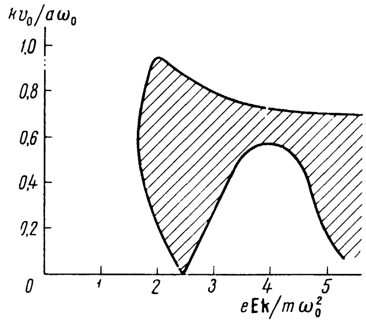

FIG. 1

distribution can be regarded as Maxwellian with a temperature T_e. Then

$$\delta\varepsilon_e(\omega, k) = \frac{1}{k^2 r_{De}^2}\left[1 - J_+\left(\frac{\omega}{k v_{Te}}\right)\right], \qquad (3.1)$$

where $r_{De} = \sqrt{\kappa T_e/4\pi e^2 N_e}$ is the electronic Debye radius, $v_{Te} = \sqrt{\kappa T_e/m}$ is the thermal velocity of the electrons, and

$$J_+(x) = x e^{-x^2/2} \int_{i\infty}^{x} dt\, e^{t^2/2}.$$

Substituting (3.1) in the right side of (1.1) we obtain the following dispersion equation:

$$\frac{1}{\delta\varepsilon_i(\omega + i\gamma, k)} = -\sum_{n=-\infty}^{+\infty} J_n^2(a) \times \left\{1 + \frac{1}{k^2 r_{De}^2}\left[1 - J_+\left(\frac{n\omega_0 + \omega + i\gamma}{k v_{Te}}\right)\right]\right\}^{-1}. \qquad (3.2)$$

We turn first to the case of long waves $kv_{Te} \ll \omega_0$, but at the same time we do not assume the frequency of the external field to be close to ω_{Le}. We can then write near the zeroes of the function J_0 the following expression for the instability increment

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\left(\exp\left[-\frac{1}{2}s_i^2\right] + \frac{r_{Di}^3\omega_{Li}}{r_{De}^3\omega_{Le}} J_1^2(a_r)\left\{\frac{(a - a_r)^2}{[1 + (kr_{De})^{-2}]^2} - \frac{2\omega_0^6}{k^2 v_{Te}^2[\omega_0^2 - \omega_{Le}^2]^2}\exp\left[-\frac{\omega_0^2}{2k^2 v_{Te}^2}\right]\right\}\right). \qquad (3.3)$$

We obtain accordingly for the frequency

$$\omega^2 = \omega_{Li}^2\left\{1 + \frac{J_1^2(a_r)\,(a - a_r)^2}{1 + (kr_{De})^{-2}} - \frac{\pi\omega_{Le}/\omega_0}{\sin \pi\omega_{Le}/\omega_0} J_{\omega_{Le}/\omega_0}(a_r) J_{-\omega_{Le}/\omega_0}(a_r)\right\}. \qquad (3.4)$$

From this it follows that for instability to occur it is necessary to satisfy the inequality

$$2k^2 \ln\frac{2\omega_{Li}\omega_0^6 (r_{Di}/r_{De})^3 J_1^2(a_r)}{\omega_{Le}k^2 v_{Te}^2(\omega_0^2 - \omega_{Le}^2)^2} > \frac{\omega_0^2}{v_{Te}^2} - \frac{1}{r_{Di}^2} \equiv \frac{\omega_0^2}{v_{Te}^2}\left\{1 - \frac{\omega_{Le}^2}{\omega_0^2}\left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\right\}. \qquad (3.5)$$

It is much easier to satisfy this inequality for a plasma with an electron temperature much higher than the ion temperature. In particular, this is seen especially clearly if $\omega_0^2 T_i < \omega_{Le}^2 T_e$. One must not forget, however, that in the considered limit of long waves the instability occurs only under conditions when the wavelength is smaller than the amplitude of the oscillations of the electron in the external field. This means that the velocity of such electron oscillations should greatly exceed its thermal velocity.

An analysis of the dispersion equation (3.2) becomes especially simple in the limit of short waves, shorter than the Debye radius of the electrons. In this case the dispersion equation (3.2) can be written in the form

$$1 + \frac{1}{\delta\varepsilon_i(\omega + i\gamma, k)} = \frac{1}{(kr_{De})^2}\int_0^\infty dt\, te^{-\frac{1}{2}t^2} J_0\left(2a \sin\frac{\omega_0 t}{2kv_{Te}}\right)\exp\left(it\frac{\omega + i\gamma}{kv_{Te}}\right)$$
$$\cong \frac{1}{(kr_{De})^2}\int_0^\infty dt\, te^{-\frac{1}{2}t^2} J_0\left(2a \sin\frac{\omega_0 t}{2kv_{Te}}\right)\left(1 + i\frac{\omega t}{kv_{Te}}\right). \qquad (3.6)$$

The integral in the right side of this equation can be readily integrated if $\omega_0 \ll kv_{Te}$. We then obtain

$$\omega^2 = \omega_{Li}^2\left\{1 + 3k^2 r_{Di}^2 - O\left(\frac{1}{k^2 r_{De}^2}\right)\right\}, \qquad (3.7)$$

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{De})^3}\left(\frac{r_{De}^3}{r_{Di}^3}\exp\left[-\frac{1}{2}s_i^2\right] + \frac{\omega_{Li}}{\omega_{Le}}\left\{\left[1 - \frac{a^2\omega_0^2}{2k^2v_{Te}^2}\right] I_0\left(\frac{a^2\omega_0^2}{4k^2v_{Te}^2}\right) + \frac{1}{2}\frac{a^2\omega_0^2}{k^2v_{Te}^2} I_1\left(\frac{a^2\omega_0^2}{4k^2v_{Te}^2}\right)\right\}\exp\left[-\frac{\omega_0^2 a^2}{4k^2v_{Te}^2}\right]\right). \qquad (3.8)$$

In the limit when the intensity of the external electric field vanishes, formula (3.8) gives the usual expression for the decrement of the ion-sound oscillations. In the opposite limit of strong fields, when $a\omega_0 \gg kv_{Te}$, we obtain from (3.8)

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\exp\left[-\frac{1}{2}s_i^2\right] + \frac{\omega_{Li}^2\omega_{Le}^2}{|\mathbf{kv}_E|^3}, \qquad (3.9)$$

where $\mathbf{v}_E = eE/m\omega_0$ is the velocity of the electron oscillations. The second positive term of the right

z	x = 2	3	4	6	9	z	x = 2	3	4	6	9
0.9	−1,65	−0,932	−0,562	−0.22	−0,02	2.0	34.1	20,14	8,27	3.11	1.52
1.0	−2.30	−0,79	−0.109	0,42	0.61	2,2	125.6	16.04	5,81	2.06	0,97
1.2	−3,61	−0 40	0,736	1,25	1 28	2.4	92 8	8.90	3,25	1,16	0,54
1.4	−6,012	1,05	2.50	2.38	1,88	2,6	40.0	3,95	1,64	0,58	0,28
1.6	−9.30	5,45	5,44	3,44	2.19	2,8	14,93	1.95	0.76	0,28	0,13
1.8	−7.76	14.37	8,32	3,78	2.04	3.0	3.50	0,66	0.28	0,11	0,05

side of this formula, due to the Cerenkov effect on the electrons, leads to growth of the oscillations if

$$\frac{1}{k} > r_{Di}\left\{2\ln\left[\left|\frac{\mathbf{k}\mathbf{v}_E}{kv_{Te}}\right|^3\frac{\omega_{Li}^2}{\omega_{Le}^2}\right]\right\}^{1/2}.$$

According to (3.8), the condition for the instability turns out to be weaker. Indeed, the electronic part of the increment becomes positive when

$$a\omega_0 \equiv |\mathbf{k}\mathbf{v}_E| \cong 1.8\,kv_{Te}. \tag{3.10}$$

This means that at frequencies of the external field lower than

$$\cong 0.55\left|\frac{eE}{mv_{Te}}\right| \equiv 0.55\omega_{Le}\left(\frac{E^2/4\pi}{N_e\varkappa T_e}\right)^{1/2}, \tag{3.11}$$

the Cerenkov effect on the electrons leads to a buildup of the oscillation. The corresponding condition for the wavelengths of the growing oscillations is of the form

$$\lambda\!\!\!^{-} = \frac{1}{k} > r_{Di}\left\{\ln\left[\frac{T_e^3 e_i^2 m_i}{T_i^3 e^2 m}\right]\right\}^{1/2}. \tag{3.12}$$

Inasmuch as we have assumed above that $kr_{De} \gg 1$, to satisfy the inequality (3.12) it is necessary that the temperature of the electrons greatly exceed the temperature of the ions.

Figure 2 shows the limits of the region in which the imaginary part of the right side of (3.6) is negative (the ordinates are the values of $kv_{Te}/\mathbf{k}\cdot\mathbf{v}_E$, and the abscissas the values $a = e\mathbf{E}\cdot\mathbf{k}/m\omega_0^2$). Just as in Fig. 1, the Cerenkov effect on the electrons lead to buildup oscillations inside the region outlined by the solid curve.

Inasmuch, on one hand, that the ion damping is relatively small for wavelengths which are much larger than the Debye radius of the ions, and on the other hand, according to Fig. 2, the buildup of oscillations under conditions when formula (3.6) is valid is possible only for values of a that are not small, in order for the instability to set in it is necessary that the amplitude of the electron oscillations be much larger than the ionic Debye radius.

The region of buildup on electrons, outlined by the solid curve in Fig. 2, corresponds to the case of wavelengths which are much shorter than the electronic Debye radius. The left edge of the boundary of such a region corresponds to small values of kv_{Te}/ω_0. The latter is possible only when the frequency of the external field greatly exceeds the electronic Langmuir frequency. To determine where the limits of the buildup region shifts when the frequency of the external field is reduced, it is necessary to carry out an additional analysis of formula (3.2). We will note that greatest interest is attached to the left part of such a boundary, for which, according to Fig. 2, kv_{Te}/ω_0 is of the order of unity.

Let us write down in place of Eq. (3.8) the corresponding approximation obtained by assuming that ω/kv_{Te} is small. Indeed, assuming that ω_0/kv_{Te} is not small compared with unity, we get

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{(kr_{Di})^3}\exp\left(-\frac{\omega^2}{2k^2v_{Ti}^2}\right) - \sqrt{\frac{\pi}{8}}\frac{\omega_{Li}^2}{\omega_{Le}^2}\frac{kv_{Te}J_1^2(a)}{(k^2r_{De}^2+1)^2} \times\left\{\frac{J_0^2(a)}{J_1^2(a)} - F\left(\frac{\omega_0^2}{\omega_{Le}^2};\ \frac{\omega_0}{kv_{Te}}\right)\right\}, \tag{3.13}$$

where

$$F(x, \sqrt{2}z) = \frac{2(z^2+{}^1/_2x)^2e^{-z^2}}{\left[{}^1/_2x + z^2 - 2z^3e^{-z^2}\int\limits_0^z dt\,e^{t^2}\right]^2 + \pi z^6e^{-2z^2}} \times\left\{2z^2 - 1 - 2z^3\left(2\left[\frac{x}{2} + z^2 - \frac{2z^3}{e^{z^2}}\int\limits_0^z dte^{t^2}\right]\right.\right. \times\left.\left[z + \frac{1-2z^2}{e^{z^2}}\int\limits_0^z dte^{t^2}\right] - \frac{\pi z^3(1-2z^2)}{e^{2z^2}}\right) \times\left.\left(\left[\frac{x}{2} + z^2 - \frac{2z^3}{e^{z^2}}\int\limits_0^z dte^{t^2}\right]^2 + \frac{\pi z^6}{e^{2z^2}}\right)^{-1}\right\}. \tag{3.14}$$

The table lists the values of this function. In Fig. 2 the dashed curve is the boundary of the region of positive electron increment, determined with the aid of the equation

$$\frac{J_0^2(a)}{J_1^2(a)} = F\left(\frac{\omega_0^2}{\omega_{Le}^2},\ \frac{\omega_0}{kv_{Te}}\right), \tag{3.15}$$

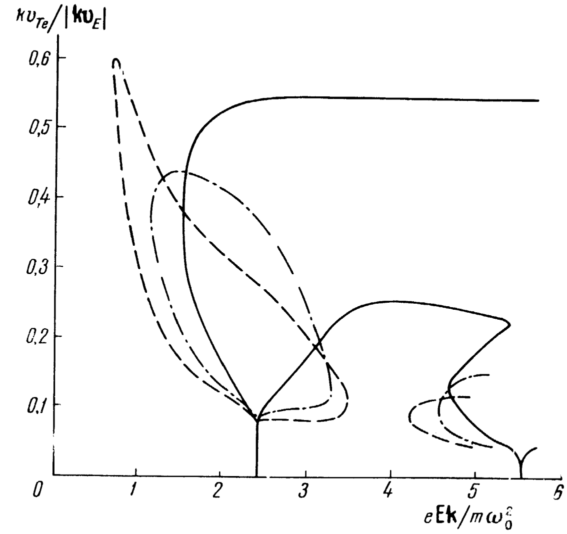

FIG. 2

for the case of an external-field frequency which is double the Langmuir frequency of the plasma electrons. The dash-dot curve corresponds to the case $\omega_0 = 3\omega_{Le}$.

Thus, according to Fig. 2, we see that the region of the instability breaks up into individual subregions with decreasing frequency of the external field. In addition, the left part of the boundary of the region of possible instability shifts towards longer wavelengths and lower velocities of the electron oscillations in the external field. The latter corresponds to an increase, according to the table, in the values of the wavelengths for which the function F is positive. At negative values of this function, expression (3.13) is always negative and the oscillations do not build up).

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