ACCOUNT OF HIGHER APPROXIMATIONS WITH RESPECT TO THE NONSPHERICITY PARAMETER IN THE THEORY OF INELASTIC DIFFRACTION SCATTERING

E. V. INOPIN and A. V. SHEBEKO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

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Formulas for the cross sections for inelastic scattering with excitation of collective nuclear levels are derived by the method of approximate separation of variables.[1] No expansion in the nonsphericity parameter is used in the calculation, so that the effect of higher approximations in this parameter can be studied. It is shown that in the case of vibrational levels the inclusion of higher approximations leads only to a simple renormalization of the original S matrix for elastic scattering. In the case of rotational levels the higher approximations lead to a deviation from the Blair phase rule.[6] Observation of these deviations may serve to determine the sign of the nuclear deformation.

1. INTRODUCTION

In a recent paper,[1] a method has been developed for the approximate separation of variables which allows one, under certain circumstances, to reduce the problem of the scattering of a particle in a nonspherical field to the spherical problem. This method has been used in the case where the nucleus is strongly absorbing and $kR \gg 1$, i.e., when the scattering has a diffraction character.

In the present paper, this method is employed to solve the problem of inelastic diffraction scattering with excitation of collective nuclear states. It turns out that the inelastic scattering cross sections can be calculated in closed form without expanding in powers of the nonsphericity parameter. The results obtained allow us to improve considerably the formulas derived under the assumption of a small nonsphericity parameter,[2] and to determine the limits of their applicability.

2. GENERAL EXPRESSIONS FOR THE INELASTIC SCATTERING AMPLITUDES

We shall consider the inelastic scattering problem in the adiabatic approximation.[3] We restrict ourselves to the case of scattering by spinless nuclei. This restriction is not essential, and we shall make the corresponding generalization later.

In the adiabatic approximation the amplitude for inelastic scattering with a transition of a spinless nucleus to a state with spin I and projection M is equal to

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = (IM|f(\mathbf{k}_a, \mathbf{k}_b, \xi)|00), \tag{1}$$

where $\mathbf{k}_a$ and $\mathbf{k}_b$ are the wave vectors of the particle before and after the scattering, with $k_a = k_b = k$; $f(\mathbf{k}_a, \mathbf{k}_b, \xi)$ is the amplitude for scattering on a nuclear potential, and the nuclear coordinates play the role of parameters determining that potential.

For the amplitude $f(\mathbf{k}_a, \mathbf{k}_b, \xi)$ we use an expression found earlier:[1]

$$f(\mathbf{k}_a, \mathbf{k}_b, \xi) = t(\mathbf{k}_a, \mathbf{k}_b, \xi) + t(-\mathbf{k}_b, -\mathbf{k}_a, \xi), \tag{2}$$

where

$$t(\mathbf{k}_a, \mathbf{k}_b, \xi) = \frac{\pi}{ik} \sum_{l'm'lm} Y_{l'm'}(\mathbf{n}_b) Y_{lm}^*(\mathbf{n}_a) i^{l-l'} (Y_{l'm'}, T_{l'} Y_{lm}),$$

$$\mathbf{n}_a = \mathbf{k}_a / k, \qquad \mathbf{n}_b = \mathbf{k}_b / k.$$

The quantities T_l are functions of the nuclear radius

$$R(\mathbf{n}, \xi) = R_0 + \sum_{\lambda\mu} \xi_{\lambda\mu} Y_{\lambda\mu}^*(\mathbf{n}), \quad (\mathbf{n} = \mathbf{r}/r). \tag{3}$$

Substituting (2) and (3) in (1), we obtain after some transformations

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = t_{IM}(\mathbf{k}_a, \mathbf{k}_b) + (-1)^I t_{IM}(\mathbf{k}_b, \mathbf{k}_a), \tag{4}$$

where

$$t_{IM}(\mathbf{k}_a, \mathbf{k}_b) = \frac{\pi}{ik} \sum_{l'm'lm} Y_{l'm'}(\mathbf{n}_b) Y_{lm}^*(\mathbf{n}_a) i^{l-l'} (Y_{l'm'}, T_{l'}^{IM} Y_{lm}), \tag{5}$$

$$T_l^{IM} = (IM|T_l(R(\mathbf{n}, \xi))|00). \tag{6}$$

Let us now make use of the transformation properties of the quantities $t_{IM}(\mathbf{k}_a, \mathbf{k}_b)$ under rotations:

$$t_{IM}(\mathbf{k}_b, \mathbf{k}_a) = \sum_{M'} D^{(I)}_{MM'}(g)\, t_{IM'}(\mathbf{k}_a, \mathbf{k}_b). \qquad (7)$$

The rotation g transforms $\mathbf{n}_a$ into $\mathbf{n}_b$ into $\mathbf{n}_a$, and is therefore a rotation by π about an axis directed along the vector $\mathbf{n}_a + \mathbf{n}_b$.

By (4) and (7) we have

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = \sum_{M'} \{\delta_{MM'} + (-1)^I D^{(I)}_{MM'}(g)\}\, t_{IM'}(\mathbf{k}_a, \mathbf{k}_b). \quad (8)$$

Let us consider some general properties of the inelastic scattering amplitude defined by (8).

We introduce a system of coordinates whose z axis is directed along the vector $\mathbf{q} = \mathbf{n}_b - \mathbf{n}_a$ and whose y axis is directed along the vector $\mathbf{K} = \mathbf{n}_a + \mathbf{n}_b$. In this system the Euler angles defining the rotation g (we use the definition of Edmonds[4]) are equal to

$$\alpha = 0, \quad \beta = \pi, \quad \gamma = 0.$$

Then

$$D^{(I)}_{MM'}(g) = (-1)^{I+M}\, \delta_{-MM'}.$$

Using also the fact that in this system we have the relation

$$t_{I-M}(\mathbf{k}_a, \mathbf{k}_b) = t_{IM}(\mathbf{k}_a, \mathbf{k}_b),$$

we obtain

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = [1 + (-1)^M]\, t_{IM}(\mathbf{k}_a, \mathbf{k}_b).$$

From this we have the selection rule: $f_{IM} = 0$ if M is odd. This rule was obtained by Blair and Wilets[5] from the general properties of the scattering amplitude in the adiabatic approximation.

Let us now consider a system of coordinates with a z axis directed along the vector K. In this system the Euler angles are equal to

$$\alpha = \pi, \quad \beta = 0, \quad \gamma = 0.$$

Thus,

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = [1 + (-1)^{I+M}]\, t_{IM}(\mathbf{k}_a, \mathbf{k}_b). \qquad (9)$$

From this expression we obtain the selection rule: $f_{IM} = 0$ if I + M is odd.

It can be shown that this rule, as well as the preceding one, is a consequence of the general symmetry properties of the scattering amplitude in the adiabatic approximation (cf. Appendix A). As far as we know, this rule has not been discussed before.

For the further calculations it is convenient to use a coordinate system where the z axis is directed along $\mathbf{n}_a$ and the y axis is perpendicular to the reaction plane. In this system the Euler angles corresponding to the rotation g are

$$\alpha = 0, \quad \beta = \vartheta, \quad \gamma = \pi,$$

where ϑ is the scattering angle. Thus

$$D^{(I)}_{MM'}(g) = D^{(I)}_{MM'}(0, \vartheta, \pi) = (-1)^{M'} D^{(I)}_{MM'}(\vartheta).$$

We now obtain the following expression for the inelastic scattering amplitude in the chosen coordinate system:

$$f_{IM}(\vartheta) = \sum_{M'} \{\delta_{MM'} + (-1)^{I+M'} D^{(I)}_{MM'}(\vartheta)\}\, t_{IM'}(\vartheta), \quad (10)$$

$$t_{IM'}(\vartheta) = \frac{1}{2ik} \sum_{ll'} \sqrt{\pi(2l+1)}\, i^{l-l'} (Y_{l'-M'}, T_{l'}^{IM'} Y_{l0})\, Y_{l'-M'}(\vartheta, 0). \quad (11)$$

For the calculation of $t_{IM}(\vartheta)$ we note first of all that owing to the scalar nature of the operator $R(\mathbf{n}, \xi)$

$$T_l^{IM} = (IM|T_l(R(\mathbf{n}, \xi))|00) = \sqrt{\frac{4\pi}{2I+1}}\, Y_{IM}^*(\mathbf{n})\, T_l^I, \quad (12)$$

$$T_l^I = (I0|T_l(R(\mathbf{n}_a, \xi))|00). \qquad (13)$$

The sum over l obtained after substitution of (12) in (11) is calculated according to[1,2], and we obtain as a result

$$t_{IM}(\vartheta) = \frac{1}{2ik} D^{(I)}_{M0}\left(\frac{\pi}{2}\right) \sum_l \sqrt{\pi(2l+1)}\, T_l^I Y_{lM}\left(\vartheta, \frac{\pi}{2}\right). \quad (14)$$

Only terms with large l make a contribution to this expression. After substitution of (14) in (10) we encounter the sum

$$\Sigma = \sum_{M'} D^{(I)}_{MM'}(\vartheta) D^{(I)}_{M'0}\left(\frac{\pi}{2}\right) Y^*_{lM'}\left(\vartheta, \frac{\pi}{2}\right).$$

Let us distinguish to limiting cases.

In the first case, $kR\vartheta \le 1$, i.e., the region of small angles,

$$\Sigma = D^{(I)}_{M0}\left(\frac{\pi}{2}\right) Y_{lM}^*\left(\vartheta\, \frac{\pi}{2}\right).$$

In the second case, $kR\vartheta \gg 1$, this sum is equal to

$$\Sigma = D^{(I)}_{M0}\left(\frac{\pi}{2} + \vartheta\right) Y_{lI}^*\left(\vartheta, \frac{\pi}{2}\right).$$

This is easy to see recalling $D^{(I)}_{MO}(\pi/2) = 0$ if I + M is odd and using the asymptotic expressions for the spherical functions for $l\vartheta \gg 1$.

Thus we have for scattering angles satisfying the inequality $kR\vartheta \le 1$

$$f_{IM}(\vartheta) = \frac{1}{ik} D^{(I)}_{M0}\left(\frac{\pi}{2}\right) \sum_l \sqrt{\pi(2l+1)}\, T_l^I Y_{lM}\left(\vartheta, \frac{\pi}{2}\right), \quad (15)$$

and in the region of angles $kR\vartheta \gg 1$, where the diffraction maxima and minima are observed and which is of greatest interest, we have

$$f_{IM}(\vartheta)=\frac{1}{2}\left\{D^{(I)}_{M0}\left(\frac{\pi}{2}\right)+D^{(I)}_{M0}\left(\frac{\pi}{2}+\vartheta\right)\right\}f_I(\vartheta), \quad (16)$$

where

$$f_I(\vartheta)=\frac{1}{ik}\sum_{l=0}^{\infty}\sqrt{\pi(2l+1)}\,T_l^I Y_{Il}(\vartheta,0). \quad (17)$$

For the differential cross section $kR\vartheta \gg 1$ we obtain

$$\sigma_I(\vartheta)=\frac{1}{2}[1+P_I(\cos\vartheta)]|f_I(\vartheta)|^2. \quad (18)$$

Formulas (15) to (18) are close in structure to the corresponding expressions obtained by Austern and Blair.[2] However, in our calculations we do not make use of an expansion in the nonsphericity parameter, and therefore the expressions (15) to (18) contain the dependence on the nonsphericity parameter in closed form (in the quantities T_l^I).

The remainder of this paper is devoted to the calculation of the quantities T_l^I. We consider separately the excitation of vibrational levels and the excitation of rotational states.

3. EXCITATION OF VIBRATIONAL LEVELS

In the case of vibrations, the dynamical variables $\xi_{\lambda\mu}$, entering in expression (3) can be expressed in terms of the creation and annihilation operators of phonons of the corresponding multipolarity λ:

$$\xi_{\lambda\mu}=a_\lambda[\eta_{\lambda\mu}^{+}+(-1)^{\mu}\eta_{\lambda-\mu}], \quad (19)$$

where a_λ is a parameter determining the amplitude of the vibrations.

Let us denote the matrix elements corresponding to the excitation of n phonon states with spin I by T_l^{nI}:

$$T_l^{nI}=(nI|T_l(R(\mathbf{n}_\alpha,\xi))|0)=(nI|e^{\alpha\partial/\partial R_0}|0)T_l(R_0),$$

$$\alpha=R(\mathbf{n}_\alpha,\xi)-R_0=\sum_\lambda b_\lambda(\eta_{\lambda0}^{+}+\eta_{\lambda0}),$$

$$b_\lambda=\sqrt{\frac{2\lambda+1}{4\pi}}\,a_\lambda. \quad (20)$$

Let us show that the elements T_l^{nI} are simply connected with the average values of the quantities T_l in the ground state of the nucleus,

$$\tilde{T}_l(R_0)=(0|T_l(R(\mathbf{n}_\alpha,\xi))|0). \quad (21)$$

To establish this connection we use the operator relation

$$[a, b^p]=p[a,b]b^{p-1}, \quad (22)$$

where p is an integer. This equality holds if the commutator of the operators a and b is a c-number. We also note the relation

$$[a, e^b]=[a,b]e^b, \quad (23)$$

which is a consequence of (22).

Applying now (23) to (20), we obtain for a one-phonon transition

$$T_l^{1I}=b_I\frac{\partial}{\partial R_0}\tilde{T}_l(R_0). \quad (24)$$

Analogously, if two phonons with angular momenta I_1M_1 and I_2M_2 are excited, we have

$$T_l^{2I}=[1+\delta_{I_1I_2}]^{-1/2}(I_1I_200|I0)b_{I_1}b_{I_2}\frac{\partial^2}{\partial R_0^2}\tilde{T}_l(R_0). \quad (25)$$

Expressions (20), (24), and (25) show that the elastic scattering amplitude is expressed through the quantities $\tilde{T}_l(R_0)$, and the inelastic scattering amplitudes through the derivatives of the same quantity $\tilde{T}_l(R_0)$ with respect to R_0. Our formulas differ from the corresponding formulas of Austern and Blair[2] obtained in the first non-vanishing approximation in the nonsphericity parameter in that the quantity $T_l(R_0)$ is replaced by $\tilde{T}_l(R_0)$ defined by (21).

Thus the inclusion of the higher-order terms in the nonsphericity parameter leads to a simple renormalization of the scattering matrix. If we take into account that the elastic scattering matrix is given phenomenologically in the analysis of the experimental data on diffraction scattering, then it becomes understandable why the first approximation in the nonsphericity parameter yields good results in the comparison of theory and experiment (in particular, the Blair phase rule is fulfilled[6]), notwithstanding the fact that the condition for the applicability of perturbation theory,

$$k\Delta\vartheta \ll 1 \quad (26)$$

is usually not satisfied. The quantity Δ in (26) is the nonsphericity parameter defined by

$$\Delta^2=(0|\alpha^2|0). \quad (27)$$

It is easy to express the connection between $\tilde{T}_l(R_0)$ and $T_l(R_0)$ given by (20) in a simpler form. Indeed, one can show using (22) (cf. Appendix B) that, if a certain process is described by the amplitude $f(\vartheta, R_0)$ without account of the higher approximations in the nonsphericity parameter, then one obtains for the amplitude $\tilde{f}(\vartheta, R_0)$ including the higher approximations

$$\tilde{f}(\vartheta,R_0)=\exp\left\{\frac{1}{2}\Delta^2\frac{\partial^2}{\partial R_0^2}\right\}f(\vartheta,R_0).$$

Since the amplitudes for diffraction scattering are oscillating functions of the scattering angle with a frequency $\omega = kR_0$, we may write to a good approximation

$$\tilde{f}(\vartheta, R_0) = \exp\{-{}^1/_2 k^2\Delta^2\vartheta^2\} f(\vartheta, R_0).$$

The exponential factor in the last formula is the same for the elastic and inelastic scattering amplitudes. It takes account of the effect of higher approximations. If $k\Delta\vartheta \ll 1$, this factor can be disregarded, in complete accord with what has been said before.

These results generalize the results obtained earlier by us[7] for the special case of the excitation of nuclear monopole vibrations.

In the most interesting region of angles, $kr\vartheta \gg 1$, one can establish a direct connection between the inelastic and elastic scattering amplitudes. Using (24) and the asymptotic form of the spherical functions for $l \gg 1$, we obtain for the excitation of a one-phonon state with spin I

$$f_I^{(1)}(\vartheta) = (-1)^{I/2} b_I \frac{\partial}{\partial R_0} f_0(\vartheta, R_0), \tag{28}$$

if I is even, and

$$f_I^{(1)}(\vartheta) = (-1)^{(I-1)/2} \frac{b_I}{kR_0} \frac{\partial^2}{\partial\vartheta\,\partial R_0} f_0(\vartheta, R_0), \tag{29}$$

if I is odd. In these formulas

$$f_0(\vartheta, R_0) = \frac{1}{ik}\sum_{l=0}^{\infty} \sqrt{\pi(2l+1)}\, \tilde{T}_l(R_0)\, Y_{l0}(\vartheta, 0) \tag{30}$$

is, according to (17), the elastic scattering amplitude.

Similarly, we find for a two-phonon excitation, using (25),

$$f_I^{(2)}(\vartheta) = (-1)^{I/2}[1 + \delta_{I_1 I_2}]^{-1/2} \times (I_1 I_2 00 | I0)\, b_{I_1} b_{I_2} \frac{\partial^2}{\partial R_0^2} f_0(\vartheta, R_0), \tag{31}$$

if I is even, and

$$f_I^{(2)}(\vartheta) = (-1)^{(I-1)/2}[1 + \delta_{I_1 I_2}]^{-1/2} \times (I_1 I_2 00 | I0) \frac{b_{I_1} b_{I_2}}{kR_0} \frac{\partial^3}{\partial\vartheta\,\partial R_0^2} f_0(\vartheta, R_0), \tag{32}$$

if I is odd.

Since $f_0(\vartheta, R_0)$ is an oscillating function of the scattering angle ϑ with frequency $\omega = kR_0$, then n-fold differentiation of $f_0(\vartheta, R_0)$ with respect to R_0 leads to a change of its phase by $n\pi/2$. The differentiation with respect to ϑ gives an additional phase shift of $\pi/2$ corresponding to an odd spin of the final state of the nucleus.

Thus (28) to (32) lead to the Blair rule mentioned earlier, which states that the oscillations of the cross section of inelastic scattering with excitation of an n phonon state with spin I are in phase with the oscillations of the elastic scattering cross section if I + n is even and in opposite phase if I + n is odd.

Our results can be easily extended to the case where the target nucleus has spin J_0. Denoting the spin of the nucleus in the excited state by J and the angular momentum of the even-even core in the same state by I, we find that the corresponding scattering amplitude is obtained by multiplying f_{IM}, the amplitude for scattering on a spinless nucleus, by the factor $(J_0 I M_0 M | JM + M_0)$. The corresponding cross section differs from the cross section for scattering on a spinless nucleus by the factor $(2J+1)\,[(2J_0+1)(2I+1)]^{-1}$.

4. EXCITATION OF ROTATIONAL LEVELS

In the consideration of transitions of the rotational type we shall assume that $R(n, \xi)$ contains only quadrupole terms. Then the elements T_l^I corresponding to the excitation of a rotational state with spin I can be written in the form

$$T_l^I = (2I+1)^{1/2} \int_0^1 P_I(x) \exp\left\{\sqrt{5}\,\Delta P_2(x) \frac{\partial}{\partial R_0}\right\} dx \cdot T_l(R_0), \tag{33}$$

where the nonsphericity parameter Δ is given by (27), as before.

Thus the problem reduces to the consideration of integrals of the type

$$A_I(s) = \int_0^1 e^{sP_2(x)} P_I(x)\, dx. \tag{34}$$

Here it is convenient to use the relations

$$A_2(s) = \frac{d}{ds} A_0(s),$$

$$(2I00 | I+20)^2 A_{I+2}(s) = \frac{d}{ds} A_I(s) - (2I00 | I0)^2 A_I(s) - (2I00 | I-20)^2 A_{I-2}(s) \quad (I = 2, 4, \ldots). \tag{35}$$

The integral $A_0(s)$ is equal to[8]

$$A_0(s) = \int_0^1 e^{sP_2(x)}\, dx = e^{-s/2}\Phi\left(\frac{1}{2}, \frac{3}{2}; \frac{3}{2}s\right) \tag{36}$$

where $\Phi(a, c; x)$ is a confluent hypergeometric function.

Making the substitution $s \to \sqrt{5}\Delta\, \partial/\partial R_0$, we obtain for the amplitude for inelastic scattering with excitation of a rotational level with spin I the following expression:

$$f_I(\vartheta) = (-1)^{I/2}(2I+1)^{1/2} A_I\left(\sqrt{5}\,\Delta \frac{\partial}{\partial R_0}\right) f(\vartheta, R_0), \tag{37}$$

where

$$f(\vartheta, R_0) = \frac{1}{ik}\sum_{l=0}^{\infty} \sqrt{\pi(2l+1)}\, T_l(R_0) Y_{l0}(\vartheta, 0) \qquad (38)$$

is the amplitude for scattering in the corresponding spherically symmetric field.

Expressions (34) to (38) permit us to obtain a rigorous condition for the applicability of the first approximation in the nonsphericity parameter Δ. Indeed, $f(\vartheta, R_0)$ is an oscillating function of the scattering angle ϑ with frequency $\omega = kR_0$; therefore the operator $\partial/\partial R_0$ acting on $f(\vartheta, R_0)$ yields a factor $k\vartheta$, so that the desired condition is of the form

$$^1/_{11}\sqrt{5/\pi}\,\beta_2 kR_0\vartheta \ll 1, \qquad (39)$$

where β_2 is the usual parameter characterizing the quadrupole deformation. Condition (39) differs from the earlier condition[6] $\beta_2 kR_0\vartheta \ll 1$ by a factor of order 1/10. This explains why the first approximation in the nonsphericity parameter can yield satisfactory results for the excitation of rotational levels, when $kR_0\vartheta \gg 1$.

It is interesting to consider the effect of the higher approximations in the nonsphericity parameter. One of the most important questions is whether the Blair phase rule is violated. Assuming that the amplitude $f(\vartheta, R_0)$ has the form[9]

$$f(\vartheta, R_0) = F(\vartheta)\cos(kR_0\vartheta + \gamma),$$

and taking account of the next order in Δ, we obtain from (37), using (34) to (36),

$$f_0(\vartheta) = F(\vartheta)\cos(kR_0\vartheta + \gamma),$$

$$f_2(\vartheta) = \frac{\beta_2}{2\sqrt{\pi}} kR_0\vartheta F(\vartheta)\sin[(kR_0 + \delta)\vartheta + \gamma],$$

$$f_4(\vartheta) = -\frac{3}{28\pi}(\beta_2 kR_0\vartheta)^2 F(\vartheta)\cos\left[\left(kR_0 + \frac{14}{11}\delta\right)\vartheta + \gamma\right],$$

$$\delta = {}^1/_{14}\sqrt{5/\pi}\,\beta_2 kR_0.$$

These formulas show that the inclusion of the next approximation in the nonsphericity parameter has no effect on the elastic scattering, but leads to a modification of the frequency of the oscillations of the inelastic scattering cross section and hence, to a deviation from the Blair phase rule. This effect depends on the sign of the deformation parameter β_2: for positive deformations the frequency increases, while it decreases for negative deformations. Hence we have the possibility to determine the sign of the deformation of the nucleus experimentally.

Let us consider, for example, the scattering into the angle $\vartheta = \vartheta_n$, where

$$\vartheta_n = (kR_0)^{-1}[(n-1)\pi/2 - \gamma]$$

(n is an integer). The elastic cross section must have a minimum at this angle, and the inelastic cross section a maximum if I = 2 and $\delta = 0$. We obtain the following shift in the position of this maximum relative to the minimum of the elastic scattering cross section for $\delta \neq 0$:

$$\Delta\vartheta_n = -{}^1/_{14}\sqrt{5/\pi}\,\beta_2\vartheta_n.$$

If we set $\beta_2 = 0.3$, $\vartheta_n = 90°$, then $\Delta\vartheta_n = 2.4°$. Such an angular shift is quite accessible to experimental determination.

In conclusion we quote the formulas for the cross section for scattering on a nucleus with spin J_0. This problem is easily reduced to the previous one, and we obtain for the cross section for scattering accompanied by the nuclear transition $J_0 \to J$

$$\sigma_{J_0J}(\vartheta) = \sum_{IM} \frac{2J+1}{2I+1}(JJ_0 - KK|I0)^2\sigma_{IM}(\vartheta),$$

where K is the projection of the nuclear spin on the symmetry axis. From this we obtain for $kR\vartheta \gg 1$, using (18),

$$\sigma_{J_0J}(\vartheta) = \sum_I \frac{2J+1}{2I+1}(JJ_0 - KK|I0)^2\sigma_I(\vartheta).$$

APPENDIX A

THE SELECTION RULE (9)

Let us show that the selection rule (9) is a direct consequence of the adiabatic approximation.

We write the inelastic scattering amplitude in the adiabatic approximation in the following form:

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = (\Phi_{IM}{}^b(\xi)\,|f(\mathbf{k}_a, \mathbf{k}_b, \xi)|\,\Phi_{00}{}^a(\xi)). \qquad (A.1)$$

We make a rotation through the angle π about the z axis [rotation $C_z(\pi)$] in a coordinate system whose z axis is directed along the vector $\mathbf{K} = \mathbf{n}_a + \mathbf{n}_b$. As a result we obtain

$$f(\mathbf{k}_a, \mathbf{k}_b, \xi) = f(\mathbf{k}_b, \mathbf{k}_a, \xi') = f(-\mathbf{k}_a, -\mathbf{k}_b, \xi'), \qquad (A.2)$$

where ξ' are the values of the nuclear coordinates after this rotation. In deriving (A.2) we have also used the invariance of the elastic scattering process under time reversal.

Making further an inversion, we obtain from (A.2)

$$f(\mathbf{k}_a, \mathbf{k}_b, \xi) = f(\mathbf{k}_a, \mathbf{k}_b, \xi''), \qquad (A.3)$$

where ξ'' are the values of the nuclear coordinates after $C_z(\pi)$ and the inversion P. Evidently $\xi = PC_z(\pi)\xi''$, therefore

$$\Phi_{IM}{}^b(\xi) = P_b(-1)^M\Phi_{IM}{}^b(\xi''), \qquad \Phi_{00}{}^a(\xi) = P_a\Phi_{00}{}^a(\xi''), \qquad (A.4)$$

After the change of integration variables $\xi'' \to \xi$ we obtain

$$f_{IM}(\mathbf{k}_a, \mathbf{k}_b) = P_a P_b (-1)^M f_{IM}(\mathbf{k}_a, \mathbf{k}_b). \quad (A.5)$$

In the case of interest to us $P_a = +1$, $P_b = (-1)^I$, and (A.5) leads directly to the desired result.

APPENDIX B

CALCULATION OF $(0|e^{\alpha}|0)$

First of all we have

$$(0|e^{\alpha}|0) = \sum_{p=0}^{\infty} \frac{1}{p!}(0|\alpha^p|0). \quad (B.1)$$

The average value $(0|\alpha^p|0)$ can be written in the form

$$(0|\alpha^p|0) = \sum_{\lambda} b_{\lambda}(0|\eta_{\lambda}\alpha^{p-1}|0). \quad (B.2)$$

Applying (22), we obtain

$$(0|\eta_{\lambda}\alpha^{p-1}|0) = (p-1) b_{\lambda}(0|\alpha^{p-2}|0). \quad (B.3)$$

If we further use

$$\sum_{\lambda} b_{\lambda}^2 = (0|\alpha^2|0),$$

then we find the following recurrence relation, using (B.2) and (B.3):

$$(0|\alpha^p|0) = (p-1)(0|\alpha^2|0)(0|\alpha^{p-2}|0). \quad (B.4)$$

This implies

$$(0|\alpha^p|0) = \begin{cases} \frac{(2k)!}{2^k k!}(0|\alpha^2|0)^k, & \text{if } p = 2k \\ 0, & \text{if } p = 2k+1, \end{cases}$$

i.e.,

$$(0|e^{\alpha}|0) = \exp\{{}^1/_2(0|\alpha^2|0)\}. \quad (B.5)$$

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