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VELOCITY OF FIRST SOUND IN He^3-He^4 *SOLUTIONS*

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Measurements are reported of the velocity of first sound in solutions of isotopes of helium with He^3 content up to 20% in the temperature range 1.6—4.0°K. It has been discovered that, for constant temperature, a linear dependence exists for the sound velocity as a function of the concentration.

SEVERAL researches[1-3] have been devoted to the measurement of first sound in solutions of He^3-He^4, in which the solutions with He^3 concentration above 20% were studied in special detail. Detailed data are lacking for much weaker solutions, which makes difficult the interpretation of the results obtained and their comparison with theory. Moreover, a knowledge of the exact values of the velocity of first sound in He^3-He^4 solutions is necessary in the study of a number of properties of solutions, in particular, those of fourth sound. The present research was undertaken with the purpose of obtaining such data.

Measurements were made on solutions with molar concentrations of He^3 of 6.30, 11.05, 15.56 and 19.53%, obtained by mixing the pure isotopes. The error in the determination of the concentration did not exceed ± 0.05% He^3 for all concentrations.

A pulsed ultrasonic method was used for the velocity determination. Piezoelectric quartz crystals were used as the transmitter and receiver of the sound, with a fundamental frequency of 1 MHz. The radiation was excited by a generator of relaxation oscillations with pulses of length 30 μsec, repetition frequency 200 Hz, and initial amplitude 170 V. After passage through the solution under study, the pulses were amplified and fed to an I2-9A detector, which measured small time intervals. This detector allowed us to measure the time intervals between the initial and subsequent pulses with an error not exceeding ± 0.05% of the measured value. A large number of reflected pulses were observed on the screen of the detector. The measurements of the time intervals were carried out from the initial to the first five or six reflected signals; then the result was averaged.

The detector and the transmitter were placed at a distance of 2 cm from one another. This distance was measured at room temperature with an accuracy to within ± 0.01%, and then a correction was made[4] for the thermal compression of the copper resonator at the temperature of liquid helium. The quartz holder guaranteed parallelism of the source with the receiving crystals with sufficient accuracy.

The measured values of the velocity of first sound for different concentrations as a function of the temperature are given in the table. The maximum deviation of the velocity from the mean experimental value did not exceed 0.15%.

Figure 1 gives the temperature dependence of the velocity for the concentrations studied. The singularities corresponding to the transition of the solution to the superfluid state are clearly seen on both curves. Here the values of the temperatures are in excellent agreement with those in the literature.[2,5] The character of the singularities changes with increase in the concentration. The curves corresponding to a weak solution have slopes of different sign at the λ point. Then, as the concentration increases, the difference in the slopes of the curves decreases, and in concentrated solutions, as is seen from the works of Roberts and Sydoriak[2], the kinks are only barely observable.

It should be noted that the measurements of the sound velocity in the region of the relative velocity maximum (which takes place above the λ point for weak solutions and pure He^4) require special care. It was noted that if the measurements were carried out without long isothermal exposures (20—30 min), then the observed maximum and the singularity at the λ point are smeared out.

The dependence on concentration of the first-sound velocity for constant temperature is given

Values of the velocity of first sound in He^3—He^4 solutions with various molar concentrations of He^3.

6.30% He^3		11.05% He^3		15.56% He^3		19.53% He^3	
T, °K	u, m/sec	T, °K	u, m/sec	T, °K	u, m/sec	T, °K	u, m/sec
1,59	228,9	1,62	225.9	1,63	221,4	1.60	219.1
1,74	226.5	1.68	224,7	1,73	219.7	1,66	218,0
1.83	224.7	1,74	222,9	1.80	217.9	1,74	215.8
1.91	222,2	1,82	220,6	1.90	214,2	1,79	214.1
2.00	219,2	1.91	217,8	1.95	211.8	1.85	211,6
2,05	217.4	1,99	214,8	2,01	212.8	1,89	211,1
2,08	215.7	2.02	213.9	2.13	213.2	2,00	211.1
2.09	216,4	2,03	214.5	2.29	212,7	2.08	210,8
2,28	217,6	2.20	215.6	2.48	211,5	2.17	210.4
2,40	218.0	2.28	215.5	2,60	209.8	2.30	209.8
2,54	217,6	2.35	215.2	2,85	206,5	2,41	209.3
2,70	215.9	2,55	213,8	3.07	202,6	2.52	208,0
2,85	213.8	2.70	212.7	3,20	198,9	2.62	207,1
3.00	211.1	2.85	210.5	3,40	193,5	2.86	203.6
3,20	207,4	3,00	207,6	3.60	187,7	2.97	200.8
3.39	202.8	3.17	203,6	3.85	177,8	3,07	199.6
3.55	197.7	3.50	195.1			3.32	194.2
3.99	182.0	3.70	188.6			3,72	179.8
		3.90	180.2				

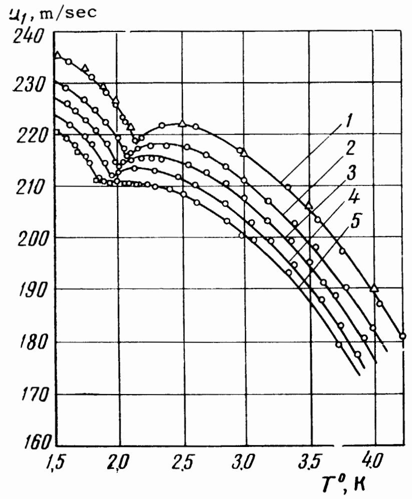

FIG. 1. Temperature dependence of the velocity of first sound: curve 1 – He^4, 2 – 6.3% He^3, 3 – 11.05% He^3, 4 – 15.56% He^3, 5 – 19.53% He^3 (the molar concentrations are shown); △ – according to [3], □ – according to [2], ○ – present research.

in Fig. 2. For clarity, the λ line and the line of stratification, which bounds the region of the superfluid solutions, are shown in the drawing. In this region, the isotherms are practically straight lines, along which lie the experimental data obtained by different authors. Such a dependence of the sound velocity on the concentration can be explained in the following way.

The velocity of first sound u_1 in the He^3—He^4 solutions is, according to Khalatnikov,[6] described by the expression

$$u_1^2 = \left(\frac{\partial P}{\partial \rho}\right)_{c,T}\left[1+\frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)^2\right], \qquad (1)$$

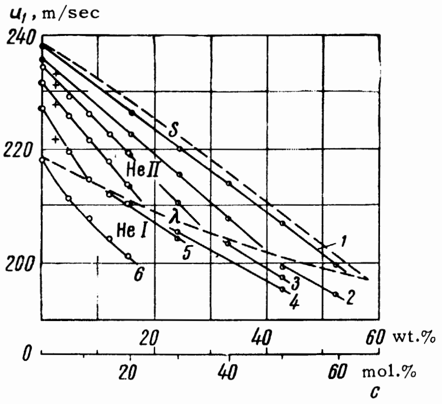

FIG. 2. Dependence of the velocity of first sound on the concentration: curve 1 – T = 1°K, 2 – 1.4°K, 3 – 1.6°K, 4 – 1.8°K, 5 – 2.0°K, 6 – 3.0°K; + – according to [1], ◐ – according to [2], ● – according to [3], ○ – present research. The dashed curves indicate the λ lines and the S line of the stratification of the solutions.

where ρ_s, ρ_n, and ρ are the densities of the superfluid component and the normal component and the total density of a solution with weight concentration c; P is the pressure. This formula was obtained from the set of hydrodynamic equations under the assumption that He^3 is completely entrained by the normal motion, and also neglecting the quantity u_2^2/u_1^2 in comparison with unity. The latter circumstance shows that the formula (1) is an approximation with accuracy to within ~ 1%.

With the same accuracy, the density of the solutions[7] can be written in the form

$$\rho(T) = c\rho_{30}(T) + (1-c)\rho_{40}(T), \qquad (2)$$

where ρ_{30} and ρ_{40} are the densities corresponding to He^3 and He^4 for the same temperature.

Inasmuch as the relation (2) is satisfied over a very broad range of temperatures (and, apparently also pressures), then it is natural to suppose that it is also satisfied for pressures which somewhat exceed the equilibrium vapor pressure of He^3, i.e.,

$$\rho(P, T) = c\rho_{30}(P, T) + (1 - c)\rho_{40}(P, T). \qquad (3)$$

By differentiating Eq. (3) with respect to the pressure at the point $P = P_{30}$, where P_{30} is the vapor pressure of He^3 at the given temperature T, we have

$$\left(\frac{\partial\rho(P,T)}{\partial P}\right)_{T,c}\Bigg|_{P=P_{30}} = cu_{30}^{-2}(P_{30}, T) + (1-c)u_{40}^{-2}(P_{30}, T), \qquad (4)$$

where u_{30} and u_{40} are the velocities of sound in pure He^3 and He^4.

By neglecting the dependence of u_{40} and $\partial\rho/\partial P$ on the pressure in the pressure range previously mentioned, we get, in accord with (1),

$$u_1^2 = u_{40}^2\left[1+\frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial\rho}{\partial c}\right)^2\right]\Big/\left[1+c\left(\frac{u_{40}^2}{u_{30}^2}-1\right)\right], \qquad (5)$$

in which all the quantities which enter are taken along the liquid-vapor equilibrium curve.

The obtained expression (5) for the velocity of first sound in the solution agrees with the experimental data within ~ 1%. Evidently, this indirectly supports the use of the assumption made in the derivation of Eq. (1) regarding the complete entrainment of the He^3 by the normal component of the solution.

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203