AN EQUATION FOR THE MAGNETIZATION OF A FERRODIELECTRIC IN A TRANSVERSE ALTERNATING MAGNETIC FIELD

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A diagram technique is employed for deducing the equations for the transverse components of the magnetization of a ferrodielectric in an alternating homogeneous magnetic field which is perpendicular to the direction of easy magnetization. The equations derived differ structurally from the equations with a Landau-Lifshitz relaxation term.

IN investigating the behavior of ferrodielectrics in a transverse alternating magnetic field, an equation proposed by Landau and Lifshitz[1] is frequently used for the magnetization:

$$\dot{\mathbf{M}} = -g[\mathbf{M}\mathbf{H}_{\text{eff}}] - \lambda M_0^{-2}[\mathbf{M}[\mathbf{M}\mathbf{H}_{\text{eff}}]], \qquad (1)^*$$

where $\mathbf{M}$ is the magnetization vector, g is the gyromagnetic ratio, λ is the relaxation constant, $M_0 = |\mathbf{M}|$, and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field.

In the spatially homogeneous case, $\mathbf{H}_{\text{eff}} = \mathbf{H}_0 + \beta\mathbf{n}(\mathbf{n}\cdot\mathbf{M}) + \mathbf{h}$, where $\mathbf{H}_0 = \mathbf{n}H_0$ is the dc field applied along the easy axis, β is the anisotropy constant, and $\mathbf{h}$ is the transverse ac field. The relaxation term in Eq. (1) is chosen such that in the absence of the ac field the magnetization vector approaches the direction of $\mathbf{H}_{\text{eff}}$ without changing in magnitude. Sometimes other equations are used (see, for example, [2]), which in the linear approximation to the nonequilibrium addition to the magnetization and ac field are equivalent to Eq. (1) if the corresponding relaxation constants are sufficiently small. All of these equations, as well as (1), are obtained from graphical considerations and hence can pretend to only a qualitative description of the properties of a ferromagnet.

The goal of this paper is the systematic derivation of the linearized equations for the transverse components of the magnetization with account taken of interactions in the spin system. (We note that the case of ferromagnetic resonance was considered earlier in [3] and [4].) In doing this we restrict ourselves to the low-temperature region, where the spin-wave approximation is valid. Since at low temperatures the relaxation processes are determined mainly by interactions between spin waves, we shall not take into account the interaction of spin waves with phonons.[5] In addition, we shall consider the ferromagnet to be ideal, i.e., free of defects.

1. We shall write the Hamiltonian of the system in the form $\mathcal{H} = \mathcal{H}_0$, where $\mathcal{H}_0$ is the sum of the energy of the exchange interaction and the Zeeman energy of the spin system in a steady and homogeneous magnetic field applied along the easy axis; V is the energy associated with the relativistic interactions. The interaction with the high-frequency transverse field $\mathbf{h}$ ($h_1 \cos\omega t$, $h_2 \sin\omega t$, 0) is described by the Hamiltonian $\mathcal{H}_t = -\mathbf{M}\cdot\mathbf{h}$, where $\mathbf{M}$ is the total magnetic moment operator. Using Kubo's formula,[6] we may write, following Konstantinov and Perel',[7] expressions for the average values of the transverse components of the magnetic moment:

$$\langle M^-\rangle = \langle M^-\rangle_- + \langle M^-\rangle_+, \quad M^- = M_x - iM_y,$$

$$\langle M^-\rangle_- = \frac{h_1 + h_2}{4} e^{-i\omega t} Z^{-1} \left\{ \int_0^\beta d\lambda \, \mathrm{Sp} \left[e^{-\beta\mathcal{H}_0} T \right.\right.$$

$$\left. \times \exp\left(\frac{1}{i\hbar} \int_0^{-i\hbar\beta} V_z \, dz\right) M^-_{-i\hbar\lambda} M_0^+ \right] + i\omega \int_{-\infty}^0 d\tau e^{-i\omega\tau} \int_0^\beta d\lambda$$

$$\left.\times \mathrm{Sp}\left[e^{-\beta\mathcal{H}_0} T \exp\left(\frac{1}{i\hbar} \int_C V_z \, dz\right) M^-_{-i\hbar\lambda} M^+_\tau \right]\right\}. \qquad (2)$$

Here $\langle M^-\rangle_+$ and $\langle M^-\rangle_-$ are the positive- and negative-frequency parts of $\langle M^-\rangle$, which are obtained from each other by the replacement $\omega \to -\omega$ and $h_2 \to -h_2$, $Z = \mathrm{Sp}\exp(-\beta\mathcal{H}_0)$, $\beta = 1/T$. The dummy subscripts on the operators indicate the Heisenberg representation with the Hamiltonian $\mathcal{H}_0$; for example,

$$V_z = \exp\left(-\frac{1}{i\hbar}\mathcal{H}_0 z\right) V \exp\left(\frac{1}{i\hbar}\mathcal{H}_0 z\right).$$

The integration over z in the first term is carried out along the imaginary axis, and in the second along a contour joining the points τ and $\tau - i\hbar\beta$ and passing through the point $-i\hbar\lambda$ (see [7]);

*$[\mathbf{M}\mathbf{H}_{\text{eff}}] \equiv \mathbf{M} \times \mathbf{H}_{\text{eff}}$.

T-ordering is carried out along these contours.

We note that since the total moment commutes with the Hamiltonian of the exchange interaction, it falls out from the time dependence of the operators M^- and M^+. From this, in particular, it follows that the exchange interaction by itself does not lead to a broadening or a shift of the ferromagnetic resonance line.

2. In the spin-wave approximation the transverse components of the moment may be represented in the form of an expansion in powers of the Bose operators b^+ and b by means of the Holstein-Primakoff formulas.[8]

Limiting ourselves to terms of the fourth order, we have

$$\mathscr{H}_0 = E_0 + \sum_{\mathbf{k}} (\Theta_C a^2 k^2 + \mu H_0) b_{\mathbf{k}}^+ b_{\mathbf{k}} + \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3+\mathbf{k}_4} \Phi_{1,2;3,4} b_{\mathbf{k}_1}^+ b_{\mathbf{k}_2}^+ b_{\mathbf{k}_3} b_{\mathbf{k}_4}, \tag{3}$$

where Θ_C is the Curie temperature, a is the lattice constant, and $\mathbf{k}$ is the wave vector of the spin wave; $\mu = g\hbar$; the explicit form of the amplitudes $\Phi_{1,2;3,4}$ associated with the exchange interaction are given, for example, in the review.[5]

We shall for simplicity take into account only anisotropy energy, considering that we have a uniaxial crystal with a positive anisotropy constant. Then the anisotropy energy is equal to $\frac{1}{2}\beta\int(M_x^2 + M_y^2)dv$, and, as is easy to show,

$$V = -\frac{\beta\mu^2}{2v} \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3+\mathbf{k}_4} b_{\mathbf{k}_1}^+ b_{\mathbf{k}_2}^+ b_{\mathbf{k}_3} b_{\mathbf{k}_4}, \tag{4}$$

where v is the volume of the body. The part of the considered interaction that is quadratic in the operators can be included in the Hamiltonian $\mathscr{H}_0$ if to the applied field one adds an anisotropy field βM_0.

In deriving the equations for the magnetization we shall make use of the graphical technique proposed by Konstantinov and Perel'.[7] If we write down the expansions for M^+ and M^- in Bose operators, then, as can be seen from (2), the quantity $\langle M^-\rangle_-$ is represented in the form of a sum of terms of different structure. We consider only the first term of this sum, corresponding to the approximation $M^+ = (2\mu M_0 v)^{1/2} b_0^+$, $M^- = (2\mu M_0 v)^{1/2} b_0$. It can be shown that the remaining terms, which contain a greater number of the operators b^+ and b, lead to a small contribution with respect to temperature.

We introduce the symbols K_- and L_- for the first and second terms in Eq. (2): $\langle M^-\rangle_- = K_- + L_-$. The first term corresponds to the static magnetization. It can be calculated by means of the thermodynamic theory of perturbations with respect to the interaction V. The second term L_- is associated with time dispersion. The perturbation series for this term contains the dangerous denominators $\omega_0 - \omega$ ($\omega_0 = gH_0$), which correspond to ferromagnetic resonance. The summation of the series reduces to a solution of an equation which in this case is algebraic and has the form

$$i(\omega_0 - \omega)L_- = R_- + W_- L_-. \tag{5}$$

In this equation R_- means the sum of diagrams that do not contain horizontal irreducible parts,[1)] without the right edge vertical section; W_- is the sum of horizontal irreducible parts without the free sections and the extreme left line.

Replacing L_- by $\langle M^-\rangle_- - K_-$ in Eq. (5), we find

$$i(\omega_0 - \omega)\langle M^-\rangle_- - W_-\langle M^-\rangle_- = R_- + i(\omega_0 - \omega)K_- - W_- K_-.$$

The coefficients of the obtained equation do not contain dangerous denominators, and in calculating them one may consider only terms quadratic in the interactions. Then it turns out that in the right part of the equality the second order terms cancel out, and as a result only terms of zero order in the interaction remain. After the calculations we finally obtain a system of equations for the magnetization:

$$-i\omega M_-^- = -i\omega_0 M_-^- + igM_0\frac{h_1 + h_2}{2}e^{-i\omega t} + W_- M_-^-,$$
$$i\omega M_+^- = -i\omega_0 M_+^- + igM_0\frac{h_1 - h_2}{2}e^{i\omega t} + W_+ M_+^-, \tag{6}$$

where

$$W_- = -\frac{2\pi}{\hbar}\left(\frac{\beta\mu^2}{v}\right)^2 \sum_{\mathbf{k}_1=\mathbf{k}_2+\mathbf{k}_3} [n_1(n_2+1)(n_3+1) - (n_1+1)n_2 n_3]\,\delta_+(\varepsilon_1 + \hbar\omega - \varepsilon_2 - \varepsilon_3),$$

$$n_k = (e^{\beta\varepsilon_k} - 1)^{-1}, \qquad \delta_+(x) = \delta(x) + \frac{i}{\pi}\mathrm{P}\frac{1}{x},$$

$$W_+(\omega) = W_-(-\omega),$$

(the brackets $\langle\ldots\rangle$ are left out).

It is to be noted that in deriving Eqs. (6) we neglected the contribution from terms of the fourth order in the Hamiltonian $\mathscr{H}_0$ (see Eq. (3)). As follows from the calculations, taking these terms into account leads to imaginary and frequency-independent additions to the quantities W_+ and W_-. This gives an insignificant shift of the frequency

1)We follow the terminology proposed in[7].

of ferromagnetic resonance ω_0 which, of course, is proportional to the relativistic interaction constant β. We note also that, although in the derivation of Eqs. (6) we used the Hamiltonian for the relativistic interaction in its simplest form (4), the structure of the equations is in fact preserved even in the general case. (The expressions for W_+ and W_- will of course be different in this case.)

3. The equations (6) we have obtained differ from the linearized Landau-Lifshitz equations, which can be written in the form ($\nu = \lambda\omega_0/gM_0$):

$$-i\omega M_-^- = -i\omega_0 M_-^- + igM_0\left(1-\frac{i\nu}{\omega_0}\right)\frac{h_1+h_2}{2}e^{-i\omega t} - \nu M_-^-,$$

$$i\omega M_+^- = -i\omega_0 M_+^- + igM_0\left(1-\frac{i\nu}{\omega_0}\right)\frac{h_1-h_2}{2}e^{i\omega t} - \nu M_+^-. \tag{7}$$

A comparison of Eqs. (6) and (7) shows that the Landau-Lifshitz equation (1) should be altered in the following way: 1) in the term $\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\text{eff}})$, the field $\mathbf{H}_{\text{eff}} = \mathbf{H}_0 + \mathbf{n}(\mathbf{M}\cdot\mathbf{n})$ does not contain the alternating field $\mathbf{h}$; 2) the coefficient λ needs to be considered as independent of the frequency ω and different for left and right field polarizations, whereby $\lambda_{\text{right}}(\omega) = \lambda_{\text{left}}(-\omega)$.

These differences can significantly affect physical results. As an example we give the expression for the coefficient of absorption Γ of the alternating magnetic field. According to the Landau-Lifshitz equation,

$$\Gamma = \frac{4\pi gM_0\omega^2\nu}{(h_1^2+h_2^2)\omega_0}\left[\frac{(h_1+h_2)^2}{(\omega_0-\omega)^2+\nu^2}+\frac{(h_1-h_2)^2}{(\omega_0+\omega)^2+\nu^2}\right],$$

whereas, if we start from Eq. (6),

$$\Gamma = \frac{4\pi gM_0\omega}{h_1^2+h_2^2}\left[\frac{(h_1+h_2)^2\nu_-}{(\omega_0-\omega-\delta_-)^2+\nu_-^2} - \frac{(h_1-h_2)^2\nu_+}{(\omega_0+\omega-\delta_+)^2+\nu_+^2}\right],$$

where

$$W_\pm = -\nu_\pm + i\delta_\pm.$$

In particular, for the case of circular polarization ($h_1 = h_2$) far from resonance we have, respectively,

$$\Gamma = 8\pi gM_0\frac{\omega^2\nu}{\omega_0(\omega_0-\omega)^2}, \qquad \Gamma = 8\pi gM_0\frac{\omega\nu_-}{(\omega_0-\omega)^2}.$$

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