RELAXATION AND THERMAL CONDUCTIVITY IN MAGNETIC MATERIALS WITH DISLOCATIONS

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Scattering of spin waves and phonons on dislocations, in ferro- and antiferromagnets, is considered. It is shown that the mean lifetime of a spin wave is proportional, in a ferromagnet, to $\xi^{-1/2}T^{-3/2}$; and in an antiferromagnet, to $\xi^{-1/2}T^{-2}$ (when $T \gg \epsilon_0$): here T is the temperature, ξ is the dislocation concentration, and ϵ_0 is the activation energy of spin waves. The contribution to the heat conductivity by scattering of spin waves and phonons on dislocations is estimated. The spin coefficient of heat conductivity κ_s is given by formula (3.4) in the case of ferromagnets, and by formula (5.10) in the case of antiferromagnets. The phonon coefficient of heat conductivity is given by formula (3.5).

INTRODUCTION

In the study of relaxational and kinetic phenomena in ferro- and antiferromagnets in the low-temperature region, the processes usually taken into account are those connected with the interaction of spin waves with one another, of spin waves with phonons,[1] of spin waves with conduction electrons,[2] and also of spin waves with isolated point defects[3] (chemical impurities and isotopes).

The present paper considers the scattering of spin waves and phonons on dislocations in ferro- and antiferromagnets. It is shown that the dominant role, in the low-temperature region, is played by the scattering of spin waves and phonons on the deformation fields produced by the dislocations in the body, and not on the cores of the dislocations. The mean lifetime of a spin wave under these conditions is proportional to $\xi^{-1/2}T^{-3/2}$ in a ferromagnet, and to $\xi^{-1/2}T^{-2}$ (when $T \gg \epsilon_0$) in an antiferromagnet; the lifetime of a phonon is proportional to $\xi^{-1/2}T^{-2}$ (ξ = dislocation concentration, ϵ_0 = activation energy).

Scattering of spin waves and phonons on dislocations plays a substantial role in ferro- and antiferromagnets at low temperatures. It turns out that in ferromagnets, the basic role in heat conductivity is played by spin waves when $T \ll \Theta_D^2/\Theta_C$ and by phonons when $T \gg \Theta_D^2/\Theta_C$ (Θ_D = Debye temperature, Θ_C = Curie temperature). In antiferromagnets, the basic role in heat conductivity is played by spin waves when $\Theta_N < \Theta_D$ and $\epsilon_0 < T$ (where Θ_N is the Néel temperature), and by phonons when $\Theta_N > \Theta_D$ and $T < \epsilon_0$.

1. HAMILTONIAN FOR INTERACTION OF SPIN WAVES AND PHONONS WITH DISLOCATIONS IN FERROMAGNETS

We consider the interaction of spin waves and phonons with dislocations in ferromagnets. We represent the Hamiltonian of the system in the form

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_p + \mathcal{H}_{sd} + \mathcal{H}_{pd}, \tag{1.1}$$

where the terms on the right are the respective Hamiltonians of the spin-wave system ($\mathcal{H}_s$), of the phonon system ($\mathcal{H}_p$), of the interaction of spin waves with dislocations ($\mathcal{H}_{sd}$), and of the interaction of phonons with dislocations ($\mathcal{H}_{pd}$).

The Hamiltonian $\mathcal{H}_s$ determines the behavior of the spin system in the undeformed lattice:

$$\mathcal{H}_s = \int dV \left\{ \frac{1}{2}\alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_k} - \frac{1}{2}\beta(\mathbf{Mn})^2 - \mathbf{MH} \right\}. \tag{1.2}$$

Here α_{ik} is the exchange-constant tensor, $\mathbf{M}$ is the magnetic-moment density, β is the anisotropy constant, and $\mathbf{n}$ is the unit vector directed along the axis of easiest magnetization.

The Hamiltonian for the interaction of spin waves with dislocations can be represented schematically in the form

$$\mathcal{H}_{sd} = \mathcal{H}_{sd}^{(f)} + \mathcal{H}_{sd}^{(n)}, \tag{1.3}$$

where the first term, $\mathcal{H}_{sd}^{(f)}$, describes the interaction of spin waves with the deformation field produced by the dislocations in the body, and the second term describes their interaction with the dislocation cores. To calculate $\mathcal{H}_{sd}^{(f)}$, we shall start from the Hamiltonian of magnetostrictive interaction

$$\mathcal{H}_{sd^{(f)}} = \int_{V'} \gamma_{ik}(\mathbf{M})\,\varepsilon_{ik}dV + \int_{V} \gamma_{iklm}(\mathbf{M}) \frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_k}\varepsilon_{lm}\,dV, \quad (1.4)$$

where ϵ_{ik} is the dislocation deformation tensor, and where $\gamma_{ik}(\mathbf{M})$ and $\gamma_{iklm}(\mathbf{M})$ are the magnetostriction-constant tensors; the first of these describes magnetoelastic effects under homogeneous magnetization, the second under inhomogeneous. In the isotropic case,

$$\gamma_{ik}(\mathbf{M}) = \gamma_0 M_i M_k + \gamma_1 \mathbf{M}^2\delta_{ik},$$
$$\gamma_{iklm}(\mathbf{M}) = \frac{\Theta_C a^2}{\mu M_0}\left[\frac{1}{2}\beta_1(\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}) + \beta_2\delta_{ik}\delta_{lm}\right] \quad (1.5)$$

where the constants γ_0, γ_1, β_1, and β_2 are of order unity, M_0 is the magnetic-moment density at saturation, a is the lattice constant, and μ is the Bohr magneton. It must be mentioned that in formula (1.4) the integration is extended over V′, the whole volume of the crystal with the exclusion of the dislocation cores. An expression for the Hamiltonian $\mathcal{H}_{sd}^{(n)}$ can be obtained by supposing that near the axis of the dislocation, there is a region with altered values of the exchange constants α'_{ik}, but with $\alpha'_{ik} \sim \alpha_{ik}$:

$$\mathcal{H}_{sd^{(n)}} = \frac{1}{2}\sum_{\nu}\int_{V_\nu} \alpha_{ik}^{\prime(\nu)} \frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_k}dV; \quad (1.6)$$

here the integration is extended over the volume V_ν of the core of the ν-th dislocation, and the summation is over all the dislocations in the ferromagnet.

The Hamiltonian $\mathcal{H}_{pd}$ in the expression (1.1) describes the scattering of phonons on dislocations:

$$\mathcal{H}_{pd} = \frac{1}{6}\int_{V'} dV\,\Lambda_{iklmnp}\,\varepsilon_{ik}\,u_{lm}\,u_{np} + \mathcal{H}_{pd^{(n)}}, \quad (1.7)$$

where Λ_{iklmnp} is a tensor that describes the anharmonicities in the crystal. The Hamiltonian $\mathcal{H}_{pd}^{(n)}$ can be obtained by supposing that along the axis of a dislocation, there is a region with constant but somewhat altered density ρ' and elastic constants λ'_{iklm} $(\rho' \sim \rho,\ \lambda'_{iklm} \sim \lambda_{iklm})$:

$$\mathcal{H}_{pd^{(n)}} \approx \frac{1}{2}\sum_{\nu}\int_{V_\nu} dV\,(\rho'^{(\nu)}\,\dot{u}_i^2 + \lambda_{iklm}^{\prime(\nu)}\,u_{ik}\,u_{lm}). \quad (1.8)$$

The tensor $\epsilon_{ik}(\mathbf{r})$, in the isotropic case, can be represented in the form[4]

$$\varepsilon_{ik}(\mathbf{r}) = \frac{1}{4\pi}\sum_{\nu}\Big\{(\gamma^2 - 1)\oint_{D_\nu}(\delta_{ik} - 3n_i n_k)\,(b^{(\nu)}[\mathbf{n}\tau])R^{-2}\,dl$$
$$+ \left(\gamma^2 - \frac{1}{2}\right)\oint_{D_\nu}(n_i[\tau\mathbf{b}^{(\nu)}]_k + n_k[\tau\mathbf{b}^{(\nu)}]_i)R^{-2}\,dl$$
$$+ \frac{1}{2}\oint_{D_\nu}(b_i^{(\nu)}[\mathbf{n}\tau]_k + b_k^{(\nu)}[\mathbf{n}\tau]_i)R^{-2}\,dl\Big\}. \quad (1.9)^*$$

Here the line integrals are extended along the dislocation lines D; $b^{(\nu)}$ is the Burgers vector of the ν-th dislocation; $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ ($\mathbf{r}'$ is the position vector of a point lying on the dislocation axis); $\mathbf{n} = \mathbf{R}R^{-1}$; $\gamma^2 = \eta/(\lambda + 2\eta)$ (λ and η are the Lamé constants); τ is a unit vector tangent to the dislocation axis at the point $\mathbf{r}'$. The Fourier transform of ϵ_{ik} has the form

$$\varepsilon_{ik}(\mathbf{q}) = \frac{1}{V}\int \varepsilon_{ik}(\mathbf{r})\,e^{-i\mathbf{qr}}\,dV$$
$$= -\frac{i}{qV}\varphi_{iklm}(\mathbf{q}^0)\sum_{\nu} b_l^{(\nu)}\,T_m^{(\nu)}(\mathbf{q})\,e^{-i\mathbf{q}\mathbf{r}^{(\nu)}}, \quad (1.10)$$

$$\varphi_{iklm}(\mathbf{q}^0) = 2(\gamma^2 - 1)\,q_i^0 q_k^0 q_n^0 e_{lmn} + {}^1/_2(e_{iml}q_k^0$$
$$+ e_{knm}\delta_{il}q_n^0 + e_{inm}\delta_{kl}q_n^0 + e_{kml}q_i^0), \quad (1.11)$$

$$T_i^{(\nu)} = \oint_{D_\nu} \tau_i e^{-i\mathbf{ql}}\,dl. \quad (1.12)$$

Here $\mathbf{r}^{(\nu)}$ is the position vector of the ν-th dislocation, $\mathbf{q}^0 = \mathbf{q}q^{-1}$, e_{ikl} is the completely antisymmetric tensor of third rank, and V is the crystal volume.

We shall write the Hamiltonian (1.1) in terms of the creation and annihilation operators $a_{\mathbf{k}}^+$ and $a_{\mathbf{k}}$ of spin waves with wave vector $\mathbf{k}$, and of the corresponding operators $b_{\mathbf{f}s}^+$ and $b_{\mathbf{f}s}$ of phonons with wave vector f and polarization s. It is well known (see, for example, [5]) that

$$M_x \cong \left(\frac{\mu M_0}{2V}\right)^{1/2}\sum_{\mathbf{k}}[a_{\mathbf{k}}e^{i\mathbf{kr}} + a_{\mathbf{k}}^+\,e^{-i\mathbf{kr}}],$$
$$M_y \cong i\left(\frac{\mu M_0}{2V}\right)^{1/2}\sum_{\mathbf{k}}[a_{\mathbf{k}}e^{i\mathbf{kr}} - a_{\mathbf{k}}^+\,e^{-i\mathbf{kr}}], \quad (1.13)$$

and

$$M_z = M_0 - \frac{\mu}{V}\sum_{\mathbf{kk'}} a_{\mathbf{k}}^+ a_{\mathbf{k'}}\,e^{i(\mathbf{k'}-\mathbf{k})\mathbf{r}}$$

$$\mathbf{u}(\mathbf{r}) = \frac{1}{\sqrt{2\rho V}}\sum_{\mathbf{f}s}\frac{\mathbf{e}_{\mathbf{f}s}}{\sqrt{\omega_{\mathbf{f}s}}}[b_{\mathbf{f}s}e^{i\mathbf{fr}} + b_{\mathbf{f}s}^+ e^{-i\mathbf{fr}}], \quad (1.14)$$

*$[\mathbf{n}\tau] \equiv \mathbf{n} \times \tau$.

where[1] ω_{fs} and $\mathbf{e}_{fs}$ are the energy and the polarization vector of a phonon. On using formulas (1.2), (1.3), (1.13), and (1.14) and retaining terms quadratic in the spin-wave operators, we get

$$\mathcal{H}_s = \sum_{\mathbf{k}} \varepsilon(\mathbf{k}) a_{\mathbf{k}}^+ a_{\mathbf{k}}, \tag{1.15}$$

$$\mathcal{H}_{sd} = \sum_{\mathbf{k}\mathbf{k}'} \Phi_{\mathbf{k}\mathbf{k}'} a_{\mathbf{k}}^+ a_{\mathbf{k}'} + \text{h. c.} \tag{1.16}$$

Here $\Phi_{\mathbf{kk}'} = \Phi^{(f)}_{\mathbf{kk}'} + \Phi^{(n)}_{\mathbf{kk}'}$; $\Phi^{(f)}_{\mathbf{kk}'}$ and $\Phi^{(n)}_{\mathbf{kk}'}$ have the form

$$\Phi^{(f)}_{\mathbf{kk}'} = \frac{i}{qV}(\Theta_C a^2 k k' \varphi_{jp} + \gamma_0 \mu M_0 \tilde{\varphi}_{jp}) \sum_{\nu} b_j^{(\nu)} T_p^{(\nu)}(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}^{(\nu)}}, \tag{1.17}$$

$$\Phi^{(n)}_{\mathbf{kk}'} = \Theta_C a^2 k k' \sum_{\nu} \frac{\alpha'}{\alpha} \frac{V_\nu}{V} I^{(\nu)}(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}}, \tag{1.18}$$

in which $\mathbf{q} = \mathbf{k} - \mathbf{k}'$,

$$I^{(\nu)}(\mathbf{q}) = \frac{1}{V_\nu} \int_{V_\nu} dV e^{-i\mathbf{q}\mathbf{r}^{(\nu)}},$$

and the functions φ_{jp} and $\tilde{\varphi}_{jp}$ are

$$\varphi_{jp} = k_l^0 k_m^0 \varphi_{lmjp}(\mathbf{q}^0), \quad \mathbf{k}^0 = \mathbf{k}k^{-1},$$

$$\tilde{\varphi}_{jp} = \{2[\gamma^2 - 3(\gamma^2 - 1) q_z^{02}] \times e_{jlp} q_l^0 + e_{pjz} q_z^0 + e_{lpz} \delta_{jz} q_l^0\}. \tag{1.19}$$

The amplitudes $\Phi^{(f)}_{\mathbf{kk}'}$ and $\Phi^{(n)}_{\mathbf{kk}'}$ in these formulas describe the scattering of spin waves by the dislocation deformation field and by the dislocation cores, respectively. It can be seen from formula (1.17) that when $kR \gg 1$ and $ak \ll 1$ (R = a characteristic dimension of a dislocation loop), $|\Phi^{(f)}_{\mathbf{kk}'}| \gg |\Phi^{(n)}_{\mathbf{kk}'}|$; that is, the fundamental role is played by the scattering of spin waves by the deformation field produced in the body by the dislocations.

Similarly, by use of the expressions (1.6), (1.7), and (1.14), we write the Hamiltonian $\mathcal{H}_{pd}$ in the second-quantization representation:

$$\mathcal{H}_{pd} = \sum_{\mathbf{f}s\mathbf{f}'s'} \chi_{\mathbf{f}s,\,\mathbf{f}'s'} b_{\mathbf{f}s}^+ b_{\mathbf{f}'s'} + \text{h. c.} \tag{1.20}$$

here $\chi = \chi^{(f)} + \chi^{(n)}$;

$$\chi^{(f)}_{\mathbf{f}s,\,\mathbf{f}'s'} = -\frac{ic\sqrt{ff'}}{qV} \varkappa_{jp}(\mathbf{f},\mathbf{f}') \sum_{\nu} b_j^{(\nu)} T_p^{(\nu)}(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}^{(\nu)}}, \tag{1.21}$$

$$\chi^{(n)}_{\mathbf{f}s,\,\mathbf{f}'s'} = \frac{c\sqrt{ff'}}{V} \sum_{\nu} \varkappa^{(\nu)}(\mathbf{f},\mathbf{f}') V_\nu I^{(\nu)}(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}^{(\nu)}}, \tag{1.22}$$

$$\varkappa_{jp} = \frac{1}{4\rho c^2} \Lambda_{lmnqrt} e_{\mathbf{f}s,\,n} e_{\mathbf{f}'s',\,n} f_m^0 f_q'^0 \varphi_{rtjp}(\mathbf{q}^0), \tag{1.23}$$

$$\varkappa^{(\nu)} = a_1 \frac{\rho'}{\rho} + a_2 \frac{\lambda'_{iklm}}{\rho c^2} e_{\mathbf{f}s,\,i} e_{\mathbf{f}'s',\,l} f_k^0 f_m^0 \tag{1.24}$$

where $\mathbf{q} = \mathbf{f} - \mathbf{f}'$, a_1 and a_2 are constants of order unity, and c is the speed of sound. We remark that in formula (1.20) the amplitude $\chi^{(f)}$ describes the scattering of phonons on the deformation field of the dislocations, whereas $\chi^{(n)}$ describes the scattering on the dislocation cores. Terms corresponding to phonon-phonon interaction have been omitted.

The fundamental contribution to scattering processes is made by phonons with energy $\Theta_D af \sim T$, where $\Theta_D = c/a$ is a temperature of the order of the Debye temperature. If $af \ll 1$, the scattering of phonons on the cores can be neglected. In this case the Hamiltonian $\mathcal{H}_{pd}$ has the form

$$\mathcal{H}_{pd} \approx \sum_{\mathbf{f}s,\,\mathbf{f}'s'} \chi^{(f)}_{\mathbf{f}s,\,\mathbf{f}'s'} b^+_{\mathbf{f}s} b_{\mathbf{f}'s'} + \text{h. c.} \tag{1.25}$$

2. KINETIC EQUATIONS FOR PHONONS AND SPIN WAVES

If we know the interaction Hamiltonian in the second-quantization representation,

$$\mathcal{H}_{int} = \mathcal{H}_{sd} + \mathcal{H}_{pd}, \tag{2.1}$$

we can determine the heat-conductivity coefficient of the ferromagnet so far as it is due to scattering of spin waves and phonons on dislocations. For this purpose, we write the kinetic equations that determine the distribution functions of the spin waves $n_{\mathbf{k}}$ and of the phonons $N_{\mathbf{f}s}$ in the presence of a weak temperature gradient:

$$n_{\mathbf{k}}^0(n_{\mathbf{k}}^0 + 1)\varepsilon_{\mathbf{k}} T^{-2}(\mathbf{v}_{\mathbf{k}} \nabla T) = \dot{n}_{\mathbf{k}}^{\,st} \equiv L_{\mathbf{k}}^{sd}\{n\},$$
$$N^0_{\mathbf{f}s}(N^0_{\mathbf{f}s} + 1)\,\omega_{\mathbf{f}s} T^{-2}(\mathbf{c}_{\mathbf{f}s} \nabla T) \equiv \dot{N}^{st}_{\mathbf{f}s} \equiv L^{pd}_{\mathbf{f}s}\{N\}, \tag{2.2}$$

where $\mathbf{v}_{\mathbf{k}} = \partial\epsilon_{\mathbf{k}}/\partial\mathbf{k}$ is the group velocity of a spin wave, $\mathbf{c}_{\mathbf{f}s}$ is the velocity of a phonon with polarization s and wave vector $\mathbf{f}$, and $n^0_{\mathbf{k}}$ and $N^0_{\mathbf{f}s}$ are the equilibrium distribution functions of the spin waves and of the phonons.

It is easy to determine the collision integrals L, once we know the probabilities of scattering of spin waves and of phonons on dislocations. We get

$$L_{\mathbf{k}}^{sd}\{n\} = 2\pi \sum_{\mathbf{k}'} \overline{|\Phi^{(f)}_{\mathbf{kk}'}|^2} (n_{\mathbf{k}'} - n_{\mathbf{k}})\,\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}), \tag{2.3}$$

$$L_{\mathbf{f}s}^{pd}\{N\} = 2\pi \sum_{\mathbf{f}'s'} \overline{|\chi_{\mathbf{f}s,\,\mathbf{f}'s'}|^2} (N_{\mathbf{f}'s'} - N_{\mathbf{f}s})\,\delta(\omega_{\mathbf{f}'s'} - \omega_{\mathbf{f}s}), \tag{2.4}$$

[1]We use a system of units in which Planck's constant $\hbar = 1$.

where the line denotes an average over the random distribution of dislocations in the body. The lifetimes of a spin wave and of a phonon are, respectively,

$$1/\tau_{\mathbf{k}}{}^{sd} = 2\pi \sum_{\mathbf{k}'} \overline{|\Phi^{(f)}_{\mathbf{kk}'}|^2} \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}),$$

$$1/\tau_{\mathbf{f}s}{}^{pd} = 2\pi \sum_{\mathbf{f}'s'} \overline{|\chi^{(f)}_{\mathbf{f}s;\,\mathbf{f}'s'}|^2} \delta(\omega_{\mathbf{f}'s'} - \omega_{\mathbf{f}s}). \quad (2.5)$$

(We assume that the length of the free path l of a quasiparticle is larger than the mean distance d between dislocations.)

The averaging reduces to a determination of the average of the expression

$$\sum_{\nu\nu'} \overline{b_i^{(\nu)} b_k^{(\nu')} T_l^{(\nu)}(\mathbf{q}) T_m^{(\nu')*}(\mathbf{q}) \exp\{-i\mathbf{q}(\mathbf{r}^{(\nu)} - \mathbf{r}^{(\nu')})\}}$$
$$= \frac{1}{3} \sum_{\nu} b^{(\nu)2} \overline{T_l^{(\nu)}(\mathbf{q}) T_m^{(\nu')*}(\mathbf{q})} \delta_{ik}. \quad (2.6)$$

The asterisk serves to denote the complex conjugate. For simplicity, we consider dislocations of circular form. Then we find

$$\overline{T_l^{(\nu)}(\mathbf{q}) T_m^{(\nu)*}(\mathbf{q})} = 2\pi^2 R^{(\nu)2} (\delta_{lm} - q_l^0 q_m^0) \int_0^1 J_2(2qRx)\, dx, \quad (2.7)$$

where $R^{(\nu)}$ is the radius of a dislocation loop and J_2 is the Bessel function of second order. On using formulas (2.6) and (2.7) and supposing that $kR \gg 1$, we get for the values of $\tau^{sd}_{\mathbf{k}}$ and $\tau^{pd}_{\mathbf{f}s}$

$$\frac{1}{\tau_{\mathbf{k}}{}^{sd}} \approx \Theta_C (ak)^3 \sum_{\nu} \frac{(b^{(\nu)} R^{(\nu)})^2}{aV},$$

$$\frac{1}{\tau_{\mathbf{f}}{}^{pd}} = \sum_s \frac{1}{\tau_{\mathbf{f}s}{}^{pd}} \approx \Theta_D (af)^2 \sum_{\nu} \frac{(b^{(\nu)} R^{(\nu)})^2}{aV}. \quad (2.8)$$

If the distance between dislocations is of the order of the dislocation dimensions, then the expressions (2.8) have the simpler form

$$1/\tau_{\mathbf{k}}{}^{sd} \approx \Theta_C (ak)^3 \xi^{1/2} a, \quad 1/\tau_{\mathbf{f}}{}^{pd} \approx \Theta_D (af)^2 \xi^{1/2} a, \quad (2.8')$$

where $\xi = (n/V)^{2/3}$ is the dislocation concentration; n is the total number of dislocations in the volume of the crystal.

On averaging $1/\tau$ over the equilibrium distribution n^0, N^0, we find the mean lifetime of spin waves and of phonons with respect to scattering of them on dislocations:

$$1/\tau^{sd} = \Theta_C (T/\Theta_C)^{3/2} \xi^{1/2} a, \quad 1/\tau^{pd} \approx \Theta_D (T/\Theta_D)^2 \xi^{1/2} a. \quad (2.9)$$

3. COEFFICIENT OF HEAT CONDUCTIVITY

We shall now proceed to the determination of the spin (κ_s) and phonon (κ_p) heat conductivities in a ferromagnet with dislocations. For this purpose it is necessary to solve the kinetic equations (2.2). We shall seek a solution of these equations in the form

$$n_{\mathbf{k}} = n_{\mathbf{k}}{}^0 + n_{\mathbf{k}}{}^0 (n_{\mathbf{k}}{}^0 + 1) \varepsilon_{\mathbf{k}} G(\varepsilon_{\mathbf{k}}) T^{-2} (\mathbf{k} \nabla T),$$
$$N_{\mathbf{f}s} = N^0_{\mathbf{f}s} + N^0_{\mathbf{f}s} (N^0_{\mathbf{f}s} + 1) \omega_{\mathbf{f}s} F(\omega_{\mathbf{f}s}) T^{-2} (\mathbf{f} \nabla T). \quad (3.1)$$

The coefficients of heat conductivity are expressed in terms of the functions G and F in the following manner:

$$\varkappa_s = -\frac{2}{3} \frac{T}{(2\pi)^3} \int \left(\frac{\varepsilon_{\mathbf{k}}}{T}\right)^3 n_{\mathbf{k}}{}^0 (n_{\mathbf{k}}{}^0 + 1) G(\varepsilon_{\mathbf{k}})\, d\mathbf{k},$$

$$\varkappa_p = -\frac{1}{3} \frac{T}{(2\pi)^3} \sum_s \int \left(\frac{\omega_{\mathbf{f}s}}{T}\right)^3 N_{\mathbf{f}s}{}^0 (N_{\mathbf{f}s}{}^0 + 1) F(\omega_{\mathbf{f}s})\, d\mathbf{f}. \quad (3.2)$$

On solving Eqs. (2.2) for the functions $G(\epsilon)$ and $F(\omega)$, we get

$$G^{-1}(\varepsilon_{\mathbf{k}}) \cong \xi (ak)^2, \quad F^{-1}(\omega_{\mathbf{f}s}) \cong \xi (af)^2. \quad (3.3)$$

Finally, on substituting (3.3) in (3.2), we find

$$\varkappa_s \cong \frac{\Theta_C}{a} \left(\frac{T}{\Theta_C}\right)^{3/2} \frac{1}{a^2 \xi} \text{ (when } H = 0), \quad (3.4)$$

$$\varkappa_p \cong \frac{\Theta_D}{a} \left(\frac{T}{\Theta_D}\right)^2 \frac{1}{a^2 \xi}. \quad (3.5)$$

The whole coefficient of heat conductivity of the ferromagnet is

$$\varkappa = \varkappa_s + \varkappa_p.$$

From formulas (3.4) and (3.5) it follows that at temperatures $T \ll \Theta_D^2/\Theta_C$ the heat is transported mainly by spin waves, at $T \gg \Theta_D^2/\Theta_C$ mainly by phonons. In a magnetic field, κ_s has the form

$$\varkappa_s \cong \frac{1}{\Theta_C{}^{1/2} a^3 \xi} \frac{\partial}{\partial T} \left\{ T^{5/2} \int_{\mu H/T}^{\infty} \frac{x^2 dx}{(e^x - 1)(x - \mu H/T)^{1/2}} \right\}. \quad (3.6)$$

If the inequality $\mu H \gg T$ is satisfied, then the spin heat conductivity will decrease exponentially with increase of the field H:

$$\varkappa_s \approx \frac{\Theta_C}{a} \left(\frac{\mu H}{T}\right)^3 \left(\frac{T}{\Theta_C}\right)^{3/2} \frac{e^{-\mu H/T}}{\xi a^2} \quad (3.7)$$

This means that the fundamental role in heat-transport processes will now be played by phonons.

We shall compare the heat-conductivity coefficient κ thus obtained with the heat-conductivity coefficient produced in a ferromagnet by umklapp processes, and also by scattering of spin waves and phonons on impurities. On the basis of the results of references [3] and [6], the heat conductivity will be determined by the following formulas.

A. Impurities paramagnetic: then

1) when $\frac{\Theta_C}{\Theta_D}\zeta_p \gg \xi a^2 \gg \left(\frac{T}{\Theta_D}\right)^3\zeta$, $\Theta_D \ll \Theta_C$

$$\varkappa \approx \frac{\Theta_D}{a}\left(\frac{T}{\Theta_D}\right)^2; \tag{3.8}$$

2) when $\xi a^2 \gg \left(\frac{T}{\Theta_D}\right)^3\zeta$, $T \gg \frac{\Theta_D^2}{\Theta_C}$, $\Theta_D \gg \Theta_C$

$$\varkappa \approx \frac{\Theta_D}{a}\left(\frac{T}{\Theta_D}\right)^2\frac{1}{\xi a^2}; \tag{3.9}$$

3) when $\left(\frac{\Theta_C}{\Theta_D}\right)\zeta_p \gg \xi a^2 \gg \left(\frac{J_{12}}{\Theta_C}\right)^2\frac{\Theta_C}{\Theta_D}\left(\frac{T}{\Theta_C}\right)^2\zeta_p$ and $\Theta_D \gg \Theta_C$

$$\text{we get } \varkappa \approx \frac{\Theta_D}{a}\left(\frac{T}{\Theta_D}\right)^2\frac{1}{\xi a^2}; \tag{3.10}$$

but if $\xi a^2 \gg (\Theta_C/\Theta_D)\zeta_p$, $\Theta_D \gg \Theta_C$, then

$$\varkappa \approx \frac{\Theta_C}{a\zeta_p}\left(\frac{T}{\Theta_C}\right)^2, \tag{3.11}$$

and if $\xi a^2 \ll \left(\frac{J_{12}}{\Theta_C}\right)^2\frac{\Theta_C}{\Theta_D}\left(\frac{T}{\Theta_C}\right)^2\zeta_p$, $\Theta_D \gg \Theta_C$

$$\varkappa \approx \left(\frac{\Theta_C}{J_{12}}\right)^2\frac{\Theta_C}{a\zeta_p}. \tag{3.12}$$

B. Impurities diamagnetic: then

4) when $\xi a^2 \gg \left(\frac{T}{\Theta_D}\right)^3\zeta$, $\Theta_D \ll \Theta_C$, $T \ll \frac{\Theta_D^2}{\Theta_C}$

$$\varkappa \approx \frac{\Theta_C}{a\zeta_d}\ln\frac{T}{\mu M_0}; \tag{3.13}$$

5) when $\xi a^2 \gg \left(\frac{T}{\Theta_C}\right)^{3/2}\ln^{-1}\left(\frac{T}{\mu M_0}\right)\zeta_d$, $\Theta_D \gg \Theta_C$

$$\varkappa \approx \frac{\Theta_C}{a}\left(\frac{T}{\Theta_C}\right)^{3/2}\frac{1}{\xi a^2}, \tag{3.14}$$

and if $\zeta_d\frac{\Theta_C}{\Theta_D}\left(\frac{T}{\Theta_C}\right)^2\ln^{-1}\left(\frac{T}{\mu M_0}\right) \ll \xi a^2 \ll \left(\frac{T}{\Theta_C}\right)^{3/2}\ln^{-1}\left(\frac{T}{\mu M_0}\right)\zeta_d$,

$$\Theta_D \gg \Theta_C, \text{ then } \varkappa \approx \frac{\Theta_C}{a\zeta_d}\ln\left(\frac{T}{\mu M_0}\right), \tag{3.15}$$

whereas if $\left(\frac{T}{\Theta_D}\right)^3\zeta \ll \xi a^2 \ll \zeta_d\frac{\Theta_C}{\Theta_D}\left(\frac{T}{\Theta_C}\right)^2$, $\Theta_D \gg \Theta_C$, then

$$\varkappa \approx \frac{\Theta_D}{a}\left(\frac{T}{\Theta_D}\right)^2\frac{1}{\xi a^2}. \tag{3.16}$$

In formulas (3.8) to (3.16) the following notation is used: ζ_p and ζ_d are the concentrations of paramagnetic and diamagnetic impurities, whereas ζ is the total concentration of impurities with allowance for atoms of the rare isotope; J_{12} is the exchange integral between paramagnetic impurity atoms and nearest neighbors of the basic material.

According to [6], the value of the heat-conductivity coefficient produced by umklapp processes is

$$\varkappa \sim {}^1/_9 mc^2(C_p + 2C_s)^2 a^5 \exp(\pi\Theta_D/T), \quad \Theta_D \ll \Theta_C,$$
$$\varkappa \sim (T/a)\exp(\pi^2\Theta_C/T), \quad \Theta_D \gg \Theta_C, \tag{3.17}$$

where $C_S \sim a^{-3}(T/\Theta_C)^{3/2}$ and $C_p \sim a^{-3}(T/\Theta_D)^3$ are the heat capacities of spin waves and phonons, respectively. On comparing the expressions (3.17) and (3.4), one can observe that the scattering of spin waves and phonons on dislocations plays a fundamental role when the following inequalities are satisfied:

a) if $\Theta_D \ll \Theta_C$, $T \gg \Theta^2_D/\Theta_C$, then

$$\xi a^2 \gg \left(\frac{9\Theta_D}{mc^2}\right)\left(\frac{\Theta_D}{T}\right)^4\exp\left(-\frac{\pi\Theta_D}{T}\right); \tag{3.18}$$

b) if $\Theta_D \ll \Theta_C$. $T \ll \Theta^2_D/\Theta_C$, then

$$\xi a^2 \gg \frac{\Theta_C}{mc^2}\left(\frac{\Theta_C}{T}\right)^{3/2}\exp\left(-\frac{\pi\Theta_D}{T}\right); \tag{3.19}$$

c) if $\Theta_D \gg \Theta_C$, then

$$\xi a^2 \gg \left(\frac{T}{\Theta_C}\right)^{1/2}\exp\left(-\frac{\pi^2\Theta_C}{T}\right). \tag{3.20}$$

4. HAMILTONIAN FOR INTERACTION OF SPIN WAVES AND PHONONS IN ANTIFERROMAGNETS

We shall consider the scattering of spin waves on dislocations and shall estimate the spin heat conductivity and relaxation time in antiferromagnets.

In a uniaxial antiferromagnet, whose ground state in the absence of an external magnetic field H is determined by two compensated sublattices,[7] the magnetic moments $\mathbf{M}_1(\mathbf{r})$ and $\mathbf{M}_2(\mathbf{r})$ are antiparallel and are along the axis of easiest magnetization. The Hamiltonian of the system of spin waves, phonons, and dislocations has the form

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_{sd} + \mathcal{H}_{pd}, \tag{4.1}$$

where

$$\mathcal{H}_s = \int dV\left\{\frac{\alpha}{2}\left[\left(\frac{\partial\mathbf{M}_1}{\partial x_i}\right)^2 + \left(\frac{\partial\mathbf{M}_2}{\partial x_i}\right)^2\right] + \alpha'\frac{\partial\mathbf{M}_1}{\partial x_i}\frac{\partial\mathbf{M}_2}{\partial x_i} + \delta\right.$$
$$\times(\mathbf{M}_1\mathbf{M}_2) - \frac{\beta}{2}[(\mathbf{n}\mathbf{M}_1)^2 + (\mathbf{n}\mathbf{M}_2)^2] - \beta'(\mathbf{n}\mathbf{M}_1)(\mathbf{n}\mathbf{M}_2)$$
$$\left. - (\mathbf{M}_1 + \mathbf{M}_2, \mathbf{H})\right\}. \tag{4.1'}$$

Here the constants α, α', and δ are connected with the exchange interactions within and between the sublattices of the antiferromagnet; β and β' are magnetic anisotropy constants and are due to relativistic interactions (spin-spin and spin-orbit);

$\mathbf{n}$ is the unit vector directed along the z axis, the axis of easiest magnetization. Furthermore

$$\mathscr{H}_{sd} = \mathscr{H}_{sd^{(f)}} + \mathscr{H}_{sd^{(n)}},$$

where

$$\mathscr{H}_{sd^{(f)}} = \int\limits_{V'} dV \left[\gamma_{ik}(\mathbf{M}_1, \mathbf{M}_2)\varepsilon_{ik} + \gamma_{iksp}^{jj'} \frac{\partial \mathbf{M}_j}{\partial x_i}\frac{\partial \mathbf{M}_{j'}}{\partial x_k}\varepsilon_{sp} \right], \quad (4.2)$$

$$\mathscr{H}_{sd^{(n)}} = \sum_{v}\int\limits_{V_v} dV \left\{ \frac{1}{2}\Delta\alpha \left[\left(\frac{\partial \mathbf{M}_1}{\partial x_i}\right)^2 + \left(\frac{\partial \mathbf{M}_2}{\partial x_i}\right)^2 \right] + \Delta\alpha' \frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i} + \Delta\delta(\mathbf{M}_1\mathbf{M}_2) \right\}. \quad (4.3)$$

As in ferromagnets, we shall consider the medium isotropic in its magnetoelastic properties. Then we get for the magnetostriction tensors

$$\gamma_{ik}(\mathbf{M}_1, \mathbf{M}_2) = \gamma_1(M_{1i}M_{1k} + M_{2i}M_{2k}) + \gamma_2(M_{1i}M_{2k} + M_{1k}M_{2i}) + \delta_{ik}[\delta\gamma_3(\mathbf{M}_1\mathbf{M}_2) + \gamma_4(M_1^2 + M_2^2)],$$

$$\gamma_{iksp}^{jj'} = \alpha^{jj'}[{}^1/{}_2\beta_1(\delta_{is}\delta_{kp} + \delta_{ip}\delta_{ks}) + \beta_2\delta_{ik}\delta_{sp}], \quad (4.4)$$

where $\alpha^{11} = \alpha^{22} = \alpha_1$, $\alpha^{12} = \alpha^{21} = \alpha_1'$; the quantities γ_i, β_1 and β_2 are dimensionless and have order of magnitude unity; j, j′ = 1, 2; σ_{ik} is the Kronecker symbol.

We write the Hamiltonian (4.1) in terms of the generation and absorption operators for the spin waves, c_{jk}^+ and c_{jk}, and for the phonons, b_{fs}^+ and b_{fs}. On going over from the sublattice magnetic-moment operators to the Holstein-Primakoff operators[8]

$$M_{xj} \cong \left(\frac{\mu M_0}{2V}\right)^{1/2} \sum_{\mathbf{k}} [a_{j\mathbf{k}}e^{i\mathbf{kr}} + a_{j\mathbf{k}}^+e^{-i\mathbf{kr}}],$$

$$M_{yj} \cong (-1)^{j+1} i\left(\frac{\mu M_0}{2V}\right)^{1/2} \sum_{\mathbf{k}} [a_{j\mathbf{k}}e^{i\mathbf{kr}} - a_{j\mathbf{k}}^+e^{-i\mathbf{kr}}],$$

$$M_{zj} = (-1)^{j+1}\left[M_0 - \frac{\mu}{V}\sum_{\mathbf{kk'}} a_{j\mathbf{k}}^+ a_{j\mathbf{k}'} e^{i(\mathbf{k'}-\mathbf{k})\mathbf{r}} \right] \quad (4.5)$$

(j = 1, 2) and on using formula (1.14) and the canonical transformation

$$a_{1\mathbf{k}} = u_{11}c_{1\mathbf{k}} + v_{12}^* c_{2,-\mathbf{k}}^+, \quad a_{2\mathbf{k}} = u_{22}c_{2\mathbf{k}} + v_{21}^* c_{1,-\mathbf{k}}^+, \quad (4.6)$$

where u and v have the form

$$u_{11} = u_{22} = \frac{B}{[B^2 - (A_1 - \varepsilon_1)^2]^{1/2}},$$

$$v_{21} = v_{12} = -\frac{A_1 - \varepsilon_1}{[B^2 - (A_1 - \varepsilon_1)^2]^{1/2}}, \quad (4.7)$$

we get

$$\mathscr{H}_s = \sum_{j\mathbf{k}} \varepsilon_{j\mathbf{k}} c_{j\mathbf{k}}^+ c_{j\mathbf{k}}, \quad (4.8)$$

$$\mathscr{H}_{sd} = \mu M_0 \sum_{\mathbf{k}j\mathbf{k}'j'} (\Psi_{\mathbf{kk'}}^{jj'} + \varphi_{\mathbf{kk'}}^{jj'}) c_{\mathbf{k}j}^+ c_{\mathbf{k'}j'} + \text{h. c.} \quad (4.9)$$

here $\epsilon_{1,2} = [\Theta_N^2(ak)^2 + \epsilon_0^2]^{1/2} \pm \mu H$ is the energy of a spin wave, $\Theta_N = (\mu M_0/a)[2\delta(\alpha - \alpha')]^{1/2}$ is a quantity of the order of the Néel temperature, $\epsilon_0 = \mu M_0[2\delta(\beta - \beta')]^{1/2}$ is the activation energy of a spin wave at H = 0, and

$$A_1 = \mu M_0(\alpha k^2 + \delta + \beta - \beta') + \mu H, B = \mu M_0(\alpha' k^2 + \delta).$$

The amplitudes Ψ and φ describe magnetoelastic effects for inhomogeneous and homogeneous magnetization, respectively, and have the form

$$\Psi_{\mathbf{kk'}}^{11} = \Psi_{\mathbf{kk'}}^{22} = (\alpha_1 - \alpha_1')kk' \times [\beta_1 k_i^0 k_k^0 \varepsilon_{ik}(\mathbf{q}) + \beta_2(\mathbf{k}^0\mathbf{k}^{0'})\varepsilon_{ii'}(\mathbf{q})] \times [u^*(k)u(k') + v^*(k)v(k')],$$

$$\Psi_{\mathbf{kk'}}^{12} = \Psi_{\mathbf{kk'}}^{21} = 0; \quad (4.10)$$

$$\varphi_{\mathbf{kk'}}^{11} = \varphi_{\mathbf{kk'}}^{22} = \gamma_1[\varepsilon_{+-}(\mathbf{q}) - \varepsilon_{zz}(\mathbf{q})][u^*(k)u(k') + v^*(k)v(k')] + \gamma_2\{2u^*(k)v(k')\varepsilon_{+-}(\mathbf{q}) + [u^*(k)u(k') + v^*(k)v(k')]\varepsilon_{zz}(\mathbf{q})\} + \frac{1}{2}\delta\gamma_3\varepsilon_{ii}(\mathbf{q})[u^*(k)u(k') + v^*(k)v(k') + 2u^*(k)v(k')],$$

$$\varphi_{\mathbf{kk'}}^{12} = \varphi_{\mathbf{kk'}}^{12} = \varepsilon_{--}(\mathbf{q})\{2\gamma_1 u^*(k)v(k') + \gamma_2[u^*(k)u(k') + v^*(k)v(k')]\}. \quad (4.11)$$

Here $\mathbf{q} = \mathbf{k} - \mathbf{k}'$; the symbols + and − designate the circular components of the tensor ϵ_{ik} ($a_\pm = (a_x \pm ia_y)/\sqrt{2}$).

5. THE COEFFICIENT OF HEAT CONDUCTIVITY

We now compute the spin coefficient of heat conductivity and the mean relaxation times of the magnetic moments of the sublattices. On following the usual procedure for determining the mean relaxation time, we can find the change of the number of spin waves in unit time:

$$n^{\text{st}}{}_{\mathbf{k}j} = L_{\mathbf{k}j}\{n\}, \quad (5.1)$$

$$L_{\mathbf{k}j}\{n\} = 2\pi(\mu M_0)^2 \sum_{\mathbf{k}'j'} \overline{|\Psi_{\mathbf{kk'}}^{jj'} + \varphi_{\mathbf{kk'}}^{jj'}|^2} \times (n_{\mathbf{k}'j'} - n_{\mathbf{k}j})\delta(\varepsilon_{\mathbf{k}j} - \varepsilon_{\mathbf{k}'j'}). \quad (5.2)$$

We consider the case in which $T \gg \mu M_0$; then in formula (5.2), the quantity $\varphi^{jj'}_{\mathbf{kk'}}$ can be neglected, and we get for the lifetime of a spin wave the equation

$$\frac{1}{\tau_{1\mathbf{k}}} = \frac{1}{\tau_{2\mathbf{k}}} = 2\pi(\mu M_0)^2 \sum_{\mathbf{k}'} \overline{|\Psi_{\mathbf{k}\mathbf{k}'}^{11}|^2} \delta(\varepsilon_{\mathbf{k}'1} - \varepsilon_{\mathbf{k}1}). \quad (5.3)$$

On substituting the values of the amplitude $\Psi_{\mathbf{kk}'}^{11}$ in the expression (5.3), we find

$$\frac{1}{\tau_{\mathbf{k}}} = \frac{\pi(\mu M_0)^2}{6}[u(k)^2 + v(k)^2]^2(\alpha_1 - \alpha_1')^2 k^4 \sum_{n=0}^{2} \sum_{\nu} I_n^{(\nu)} B_n, \quad (5.4)$$

where $I_n^{(\nu)}$ is given by

$$I_n^{(\nu)} = \int_0^1 dx \int_{-1}^1 dy\, J_2[2^{3/2} k R^{(\nu)} x (1-y)^{1/2}](1-y)^{n-1}. \quad (5.5)$$

The coefficients B_n are of order unity.

On taking account of the relation (5.5), we can write the value of $1/\tau_{1\mathbf{k}}$ in the form

$$\frac{1}{\tau_{1\mathbf{k}}} \approx \Theta_N \frac{\Theta_N}{[\Theta_N^2 (ak)^2 + \varepsilon_0^2]^{1/2}} (ak)^3 \xi^{1/2} a. \quad (5.6)$$

For the mean time of scattering of spin waves on dislocations, in the absence of a magnetic field, we have

$$\frac{1}{\tau} \approx \Theta_N \left(\frac{\varepsilon_0}{\Theta_N}\right)^{3/2} \left(\frac{T}{\Theta_N}\right)^{1/2} \xi^{1/2} a \quad \text{for} \quad \mu M_0 \ll T \ll \varepsilon_0; \quad (5.7)$$

$$\frac{1}{\tau} \approx \Theta_N \left(\frac{T}{\Theta_N}\right)^2 \xi^{1/2} a \quad \text{for} \quad \varepsilon_0 \ll T. \quad (5.8)$$

We shall consider the heat conductivity of an antiferromagnet in a constant magnetic field $0 < H < H_1$, where $H_1 = M_0[2\delta(\beta - \beta')]^{1/2}$. Solution of the kinetic equation

$$n_{\mathbf{k}j}^0 (n_{\mathbf{k}j}^0 + 1) \varepsilon_{j\mathbf{k}} T^{-2} (\mathbf{v}_{j\mathbf{k}} \nabla T) = L_{j\mathbf{k}}\{n\} \quad (5.9)$$

leads to the following value of the coefficient of heat conductivity κ_s (see the analogous calculation for a ferromagnet):

$$\varkappa_s = \frac{\Theta_N}{a}\left(\frac{\varepsilon_0}{\Theta_N}\right)^{1/2}\left(\frac{T}{\Theta_N}\right)^{3/2} e^{-\varepsilon_0/T} \operatorname{ch}\left(\frac{\mu H}{T}\right)\frac{1}{\xi a^2}, \quad T \ll \varepsilon_0;$$

$$\varkappa_s = \frac{\Theta_N}{a}\left(\frac{T}{\Theta_N}\right)^2 \frac{1}{\xi a^2}, \quad T \gg \varepsilon_0. \quad (5.10)^*$$

As is seen from the expression (5.10), the heat conductivity of an antiferromagnet increases with increase of the field and reaches a maximum at the phase-transition point of the first kind ($H = H_1$). This is explained by the fact that on one of the energy branches there is a decrease of the activation energy, which leads to an increase of the number of spin waves participating in the transport of heat.

*ch ≡ cosh.

As in the case of a ferromagnet, we shall compare the coefficients of heat conductivity κ_p and κ_s determined by formulas (3.5) and (5.10) with the appropriate coefficients of heat conductivity of the antiferromagnet (see [9, 10]). Scattering of spin waves and phonons on dislocations will make the chief contribution to the heat conductivity if the concentration of dislocations satisfies the following inequalities:

1) if the impurities are diamagnetic and $\epsilon_0 \ll T$, $\Theta_D \gg \Theta_N$, then

$$\xi a^2 \gg \frac{\varepsilon_0}{\Theta_N}\left(\frac{T}{\Theta_N}\right)^2 \zeta_d; \quad (5.11)$$

2) if the impurities are paramagnetic and $\epsilon_0 \ll T$, $\Theta_D \ll \Theta_N$, then

$$\xi a^2 \gg (T/\Theta_D)^3 \zeta_p; \quad (5.12)$$

and if $\epsilon_0 \ll T$, $\Theta_D \gg \Theta_N$, then

$$\xi a^2 \gg \frac{\Theta_D}{\rho c^2}\frac{J_{12}\sigma}{\Theta_N}\left(\frac{T}{\Theta_N}\right)^3 \frac{\zeta_p}{a^3}, \quad (5.13)$$

where σ is the value of the spin of the paramagnetic impurity.

For umklapp processes we have:

a) if $\Theta_N \gg \Theta_D$, then

$$\xi a^2 \gg \frac{\Theta_D}{\rho c^2}\left(\frac{\Theta_D}{T}\right)^4 \exp\left(-\frac{\pi\Theta_D}{T}\right)\frac{1}{a^3}; \quad (5.14)$$

b) if $\Theta_N \ll \Theta_D$ and $T \gg \epsilon_0$, then

$$\xi a^2 \gg \left(\frac{\Theta_D}{T}\right)^3 \exp\left(-\frac{\pi\Theta_N}{T}\right); \quad (5.15)$$

c) if $\Theta_N \ll \Theta_D$, $T \ll \epsilon_0$, and

$$\frac{\Theta_D}{\Theta_N} \gg \left(\frac{T}{\varepsilon_0}\right)^{1/2} \exp\left(\frac{\varepsilon_0 - \mu H}{T}\right)$$

then

$$\xi a^2 \gg \left(\frac{\Theta_D}{\Theta_N}\right)^6 \left(\frac{\varepsilon_0}{\Theta_N}\right)^{1/2} \left(\frac{\Theta_N}{T}\right)^{7/2} \exp\left\{\frac{-\pi\Theta_N + \varepsilon_0 - \mu H}{T}\right\}; \quad (5.16)$$

d) if $\Theta_N \ll \Theta_D$, $T \ll \epsilon_0$, and

$$\frac{\Theta_D}{\Theta_N} \ll \left(\frac{T}{\varepsilon_0}\right)^{1/2} \exp\left(\frac{\varepsilon_0 - \mu H}{T}\right),$$

then

$$\xi a^2 \gg \frac{\Theta_D}{\rho c^2}\left(\frac{\Theta_D}{T}\right)^4 \exp\left(-\frac{\pi\Theta_D}{T}\right)\frac{1}{a^3}. \quad (5.17)$$

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