INTERACTION BETWEEN ELECTROMAGNETIC, PLASMA, AND SPIN WAVES IN ANTIFERROMAGNETIC SEMICONDUCTORS AND METALS

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Coupled electromagnetic, plasma and spin waves in antiferromagnetic semiconductors and metals are investigated. Since, in contrast to ferromagnets, there is not one but two spin waves in antiferromagnets, the spin and electromagnetic (plasma) wave interaction pattern in antiferromagnets turns out to be more complex than in ferromagnets. However, in antiferromagnets the magnetic susceptibility is proportional to a small parameter χ_0 (the static susceptibility), and the spin and electromagnetic oscillation coupling in antiferromagnets is therefore weak. The frequency corrections due to wave coupling are of the order of $\sqrt{\chi_0}$ in the region in which the frequencies of the noninteracting spin and electromagnetic (plasma) branches cross, and of the order of χ_0 far from the intersection region.

1. INTRODUCTION

THE interaction between spin and plasma waves in ferromagnetic semiconductors was investigated in several papers.[1-5] It was shown that the oscillation spectra can be noticeably altered by the interaction, and in particular that oscillations with anomalous dispersion can appear. Similar effects can take place in antiferromagnetic semiconductors and metals.

Since, unlike ferromagnets, antiferromagnets can contain not one but two branches of spin waves, the dispersion law of which can depend strongly on the type of the magnetic anisotropy of the antiferromagnet and also on the magnitude and direction of the magnetic field, it is natural to expect a more complicated picture of the wave-interaction spectra in antiferromagnetic semiconductors and metals. However, because the tensor of the high frequency magnetic susceptibility of antiferromagnets is proportional to the small parameter χ_0 (the static magnetic susceptibility), simple analytic expressions can be obtained for the frequencies of the interacting oscillations. The effective parameter describing the interaction of the spin wave with electromagnetic and plasma waves in the region where the unperturbed branches cross is the quantity $\sqrt{\chi_0}$, while far from the region of crossing of the branches the corrections to the frequencies are proportional to χ_0.

We consider in the paper the spectra of interacting spin, electromagnetic, and plasma waves in antiferromagnetic semiconductors and metals with magnetic anisotropy of the "easy axis" and "easy plane" type, in a wide interval of magnetic-field values.

2. DISPERSION EQUATION

In antiferromagnetic semiconductors and metals, the electromagnetic oscillations are described by Maxwell's equations in which the dielectric tensor ϵ_{ik} and the magnetic susceptibility tensor χ_{ik} are

$$(\varepsilon_{ik}) = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix},$$

$$(\chi_{ik}) = \begin{pmatrix} \chi_1 & i\chi_2 & 0 \\ -i\chi_2 & \chi_1 & 0 \\ 0 & 0 & \chi_3 \end{pmatrix}. \qquad (2.1)$$

(The Z axis is directed along the constant magnetic field in the antiferromagnet.)

Using (2.1), we obtain from Maxwell's equations a dispersion equation for the interacting waves

$$An^4 + Bn^2 + C = 0, \qquad (2.2)$$

where

$$A = A_e A_m, \quad A_e = \varepsilon_1 \sin^2\vartheta + \varepsilon_3 \cos^2\vartheta,$$

$$A_m = 1 + 4\pi[(\chi_1 \cos^2\varphi + \chi_1' \sin^2\varphi)\sin^2\vartheta + \chi_3 \cos^2\vartheta],$$

$$B = B_e - 4\pi\{(\varepsilon_1^2 - \varepsilon_2^2)[\chi_3 + (1 + 4\pi\chi_3)$$
$$\times (\chi_1 \cos^2\varphi + \chi_1' \sin^2\varphi)]\sin^2\vartheta + 2\varepsilon_2\varepsilon_3\chi_2(1 + 4\pi\chi_3)$$

$$\times \cos^2\vartheta + \varepsilon_1\varepsilon_3[(\chi_1+\chi_1')(1+4\pi\chi_3\cos^2\vartheta) + 2\chi_3\cos^2\vartheta + 4\pi(\chi_1\chi_1' - \chi_2^2)\sin^2\vartheta]\},$$

$$B_e = -[(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + \varepsilon_1\varepsilon_3(1+\cos^2\vartheta)],$$

$$C = C_e + 4\pi C_e[\chi_3 + (1+4\pi\chi_3)(\chi_1+\chi_1') + 4\pi(1+4\pi\chi_3)(\chi_1\chi_1' - \chi_2^2)],$$

$$C_e = \varepsilon_3(\varepsilon_1^2 - \varepsilon_2^2), \tag{2.3}$$

where $n = kc/\omega$ is the refractive index, and ϑ and φ are the polar and azimuthal angles in the wave-vector space, reckoned from the Z and X axes respectively.

In a nonmagnetic medium ($\hat{\chi} = 0$, $A = A_e$, $B = B_e$, $C = C_e$), Eq. (2.2) defines the natural frequencies $\omega_j^{(p)}(\mathbf{k})$ and the refractive indices of the electromagnetic and plasma waves, which can propagate in semiconductors and metals. The natural frequencies $\omega_j^{(p)}(\mathbf{k})$ in the case of semiconductors coincide in the approximation considered here with the natural frequencies of the oscillations of a "cold" plasma, which had been investigated in detail (see, for example, [6]). We recall that the number of waves for the "cold" plasma is equal to five in the case when only one species of carrier is present.

If we neglect the spatial dispersion of $\hat{\epsilon}$ and $\hat{\chi}$, then we obtain from (2.2) the following expressions for the refractive indices of waves with frequency ω:

$$n^2 = (-B \pm \sqrt{B^2 - 4AC})/2A. \tag{2.4}$$

The dispersion equation (2.2) allows us also to determine the dependence of the natural frequencies $\omega = \omega_j(\mathbf{k})$ on the wave vector $\mathbf{k}$. To this end it is necessary to know the explicit form of the dielectric-constant and magnetic-susceptibility tensors.

In the case when the Larmor radius r_L of the electrons with thermal velocity (for semiconductors) of electrons with limiting Fermi velocity (for metals) is small compared with the wavelength ($kr_L \ll 1$), and the phase velocity of the wave along the magnetic field is large compared with the thermal velocity or the limiting Fermi velocity, the spatial dispersion of the dielectric tensor can be neglected. The components of the tensor $\hat{\epsilon}$ are then

$$\varepsilon_1 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \omega_{B\alpha}^2}, \quad \varepsilon_2 = -\sum_\alpha \frac{\varkappa_\alpha \omega_{p\alpha}^2 \omega_{B\alpha}}{\omega(\omega^2 - \omega_{B\alpha}^2)},$$

$$\varepsilon_3 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}, \tag{2.5}$$

where

$$\omega_{p\alpha} = (4\pi e_\alpha^2 n_{0\alpha}/m_\alpha^*)^{1/2}, \quad \omega_{B\alpha} = |e_\alpha| B_0 / m_\alpha^* c$$

are the Langmuir frequency and the gyrofrequency, $n_{0\alpha}$ is the equilibrium density of carriers with charge e_α and effective mass m_α^*, and $\kappa_\alpha = |e_\alpha|/e_\alpha$.

We now present expressions for the magnetic-susceptibility tensor. We consider first uniaxial antiferromagnets with magnetic anisotropy of the "easy axis" type.

If a constant sufficiently weak magnetic field is parallel to the anisotropy axis (Z axis), then the magnetic moments of the sublattices are antiparallel, one of the magnetic moments being oriented along the magnetic field and the anisotropy axis. The components of the magnetic-susceptibility tensor are in this case[1)]

$$\chi_1 = \chi_1' = \frac{\chi_0}{2}\left[\frac{\Omega_1(\Omega_1 - \Omega_H)}{\Omega_1^2 - \omega^2} + \frac{\Omega_2(\Omega_2 + \Omega_H)}{\Omega_2^2 - \omega^2}\right],$$

$$\chi_2 = -\chi_0\omega\left[\frac{\Omega_1 - \Omega_H}{\Omega_1^2 - \omega^2} - \frac{\Omega_2 + \Omega_H}{\Omega_2^2 - \omega^2}\right], \quad \chi_3 = 0, \tag{2.6}$$

where

$$\Omega_{1,2} = gH_{AE}[1 + 2\delta(\alpha - \alpha')k^2/(gH_{AE})^2]^{1/2} \pm \Omega_H \equiv \Omega \pm \Omega_H,$$

$$\Omega_H = gH, \quad H_{AE} = (2\delta(\beta - \beta'))^{1/2} M_0, \quad \chi_0 = 1/\delta, \tag{2.7}$$

M_0 is the magnetic moment of the sublattice; $g = e/2mc$ is the gyromagnetic ratio; δ, α and α' are the exchange constants; and β and β' are the magnetic-anisotropy constants. In order of magnitude $\beta \sim \beta' \sim 1$, $\delta \sim T_N/\hbar g M_0$ (T_N is the Neel temperature), and $\alpha \sim \alpha' \sim T_N a^2/\hbar g M_0$, where a is the lattice constant. Formulas (2.6) and (2.7) are valid if $H < H_{AE}$.

If the magnetic field is directed along the anisotropy axis and $H_{AE} < H < H_E$, where $H_E = 2\delta M_0$, then in the ground state the magnetic moments of the sublattices are oriented at an angle θ to the anisotropy axis ($\cos\theta = H/(2\delta - \beta - \beta')M_0$), and in the coordinate system where the magnetic moments lie in the zy plane the components of the magnetic-susceptibility tensor are

$$\chi_1 = \chi_0\Omega_1^2/(\Omega_1^2 - \omega^2), \quad \chi_1' = \chi_0\Omega_H^2/(\Omega_1^2 - \omega^2),$$

$$\chi_2 = -\chi_0\omega\Omega_H/(\Omega_1^2 - \omega^2), \quad \chi_3 = \chi_0\Omega_2^2/(\Omega_2^2 - \omega^2), \tag{2.8}$$

[1)]These formulas can be obtained from the formulas of the paper by Kaganov and Tsukernik[7], in which no account was taken of spatial dispersion, by making the substitution

$$gH_{AE} \to gH_{AE}[1 + 2\delta(\alpha - \alpha')k^2/(gH_{AE})^2]^{1/2}.$$

where

$$\Omega_1^2 = (gM_0)^2[2\delta + (\alpha + \alpha')k^2][H\cos\theta / M_0 + \beta\cos 2\theta + \beta' + (\alpha + \alpha'\cos 2\theta)k^2],$$
$$\Omega_2^2 = (gM_0)^2 2\delta(\alpha - \alpha')k^2\sin^2\theta. \quad (2.9)$$

Expressions (2.8) and (2.9) are valid if $\delta\sin^2\theta \gg (\alpha - \alpha'\cos 2\theta)k^2$.

If the magnetic field **H** is perpendicular to the anisotropy axis (we choose the direction of the magnetic field along the Z axis, and the anisotropy axis is aligned with the X axis), then the components of the tensor $\hat{\chi}$ are

$$\chi_1 = \chi_0\Omega_H^2 / (\Omega_1^2 - \omega^2), \quad \chi_1' = \chi_0\Omega_1^2 / (\Omega_1^2 - \omega^2),$$
$$\chi_2 = -\chi_0\omega\Omega_H / (\Omega_1^2 - \omega^2), \quad \chi_3 = \chi_0\Omega_2^2\sin^2\theta / (\Omega_2^2 - \omega^2), \quad (2.10)$$

where

$$\Omega_1^2 = \Omega_H^2 + (gH_{AE})^2 + 2\delta(\alpha - \alpha')k^2(gM_0)^2,$$
$$\Omega_2^2 = [(gH_{AE})^2 + 2\delta(\alpha - \alpha')k^2(gM_0)^2]\sin^2\theta \quad (2.11)$$

and θ is the angle at which the magnetic moments of the sublattices are oriented to the magnetic field **H**, $\cos\theta = H/H_E$.

We now present expressions for the components of the high-frequency magnetic-susceptibility tensor for uniaxial antiferromagnets with magnetic anisotropy of the "easy plane" type.

If the constant magnetic field **H** is directed along the anisotropy axis (the Z axis) and the magnetic moments lie in the XZ plane, then the components of the tensor $\hat{\chi}$ are

$$\chi_1 = \chi_0\Omega_H^2 / (\Omega_1^2 - \omega^2), \quad \chi_1' = \chi_0\Omega_1^2 / (\Omega_1^2 - \omega^2),$$
$$\chi_2 = \chi_0\omega\Omega_H / (\Omega_1^2 - \omega^2), \quad \chi_3 = \chi_0\Omega_2^2 / (\Omega_2^2 - \omega^2), \quad (2.12)$$

where

$$\Omega_1^2 = \Omega_H^2 + (gH_{AE})^2 + 2\delta(\alpha - \alpha')k^2(gM_0)^2,$$
$$\Omega_2^2 = 2\delta(\alpha - \alpha')k^2\sin^2\theta(gM_0)^2. \quad (2.13)$$

Finally, if the magnetic field **H** lies in the basal plane, then

$$\chi_1 = \chi_0\Omega_1^2 / (\Omega_1^2 - \omega^2), \quad \chi_1' = \chi_0\Omega_H^2 / (\Omega_1^2 - \omega^2),$$
$$\chi_2 = \chi_0\omega\Omega_H / (\Omega_1^2 - \omega^2), \quad \chi_3 = \chi_0\Omega_2^2 / (\Omega_2^2 - \omega^2), \quad (2.14)$$

where

$$\Omega_1^2 = 2\delta(\alpha - \alpha')k^2(gM_0)^2 + \Omega_H^2,$$
$$\Omega_2^2 = 2\delta(\alpha - \alpha')k^2(gM_0)^2 + (gH_{AE})^2. \quad (2.15)$$

We note that in deriving formulas (2.12)–(2.15) we disregarded in the Hamiltonian of the antiferromagnet the terms that can lead to weak ferromagnetism.

We now consider electromagnetic waves in antiferromagnets with anisotropy of the "easy axis" type for small values of the wave vector k, when the spatial dispersion of the magnetic permeability can be neglected. Then Eq. (2.4) has two roots for n^2 as functions of the frequency, or six branches for $\omega = \omega_j(k)$, if we are interested in the natural frequencies as functions of the wave vector.

Owing to the interaction of plasma waves with spin waves, anomalous dispersion can appear, i.e., in a definite interval of the values of the wave vector some of the natural frequencies can decrease with increasing wave vector (see Figs. a and b).

3. SPECTRA OSCILLATIONS

1. Let us consider waves in antiferromagnets with anisotropy of the "easy axis" type in weak magnetic fields. Recognizing that the tensor of the magnetic susceptibility is proportional to a small parameter χ_0, the dispersion equation (2.2) can be represented in the form

$$D_e D_m + 4\pi\chi_0\eta D_1 = 0, \quad (3.1)$$

where

$$D_e = A_e n^4 + B_e n^2 + C_e, \quad D_m = (\omega^2 - \Omega_1^2)(\omega^2 - \Omega_2^2) / \Omega^4,$$
$$D_1 = A_e\sin^2\vartheta n^4 + B_1 n^2 + 2C_e,$$
$$B_1 = -(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + 2\varepsilon_2\varepsilon_3\cos^2\vartheta\zeta - 2\varepsilon_1\varepsilon_3,$$
$$\eta = (\Omega^2 - \omega^2 - \Omega_H^2) / \Omega^2,$$
$$\zeta = -\chi_2/\chi_1 = 2\Omega_H\omega / (\Omega^2 - \omega^2 - \Omega_H^2).$$

In the zeroth approximation, Eq. (3.1) determines the frequencies of the uncoupled electromagnetic and plasma waves, $\omega = \omega_j^{(p)}(\mathbf{k})$, and of the spin waves, $\omega = \omega_j^{(m)}(\mathbf{k}) = \Omega_{1,2}$. Let us find the changes produced in these frequencies by the interaction between the waves. Assuming that $\omega = \omega_j^{(p)}(\mathbf{k})(1 + \Delta_j^{(p)})$, we find from (3.1) that

$$\Delta_j^{(p)} = -4\pi\chi_0\eta\xi / D_m(\omega_j^{(p)}), \quad (3.2)$$

where

$$\xi = D_1(\omega)(\omega dD_e(\omega)/d\omega)^{-1}|_{\omega=\omega_j^p} \quad (3.3)$$

Similarly we obtain the changes of the frequencies of the magnetic oscillations

$$\omega = \omega_j^{(m)}(k)(1 + \Delta_j^{(m)}),$$

$$\Delta_j^{(m)} = \pi\chi_0(\Omega/\Omega_j)D_1(\omega)/D_e(\omega)|_{\omega=\Omega_j} \qquad (3.4)$$

As seen from these formulas, the changes of the frequencies $\omega_j^{(p)}$ and $\omega_j^{(m)}$, due to the wave interaction, are of the order of χ_0.

It is easy to verify, by using expression (2.5) for the tensor $\epsilon_{ik}(\omega)$, that the quantity $\Delta_j^{(m)}$ (even in the case of one species of carriers) can either increase or decrease with decreasing wave vector k (in the latter case the oscillations can have anomalous dispersion).

It follows from (3.2) and (3.4) that the coupling between the oscillations increases when one of the frequencies of the plasma oscillations $\omega_j^{(p)} \to \Omega_{1,2}$ (the expressions for D_m and D_e in the denominators of (3.2) and (3.4) tend to zero). Then, however, the expressions (3.2) and (3.4) are no longer valid.

Let us now consider the changes occurring in the frequencies near the region of intersection of the unperturbed plasma and magnetic branches ($\omega_j^{(p)} \to \Omega_{1,2}$). Putting in this case $\omega = \omega_j^{(p)}(1+\Delta)$ and assuming that $|\omega_j^{(p)}/\Omega_{1,2} - 1| \ll 1$, we find from the dispersion equation (3.1) that

$$\Delta = {}^1/_2\{1 - \omega_j^{(p)}/\Omega_{1,2} \pm [(1-\omega_j^{(p)}/\Omega_{1,2})^2 + 4\pi\chi_0\xi\Omega/\Omega_{1,2}]^{1/2}\}, \qquad (3.5)$$

where ξ is determined by formula (3.3).

When $(1-\omega_j^{(p)}/\Omega_{1,2})^2 \gg \chi_0$, formulas (3.5) go over into the formulas (3.2) and (3.4), provided we neglect in the latter the terms of order $|1-\omega_j^{(p)}/\Omega_{1,2}|$ compared with unity. It follows therefore that near the region where the plasma and magnetic branches crossed, when $(1-\omega_j^{(p)}/\Omega_{1,2})^2 \lesssim \chi_0$, the changes in the frequencies are of the order of $\sqrt{\chi_0}$.

In a high-density plasma ($\omega_p \gg \omega_B$) at angles ϑ not close to $\pi/2$, propagation of low-frequency waves ($\omega \ll \omega_B$) (helicons) is possible, with a dispersion law

$$\omega = \omega^{(p)} = c^2\omega_B k^2/\omega_p^2. \qquad (3.6)$$

In this case, the quantity ξ in formulas (3.2) and (3.5) is $\xi = \sin^2\vartheta/2$. Formula (3.5), which determines in this case the frequencies of the interacting spin waves and helicons, has the simple form

$$\Delta = {}^1/_2\{1 - \omega/\Omega_{1,2} \pm [(1-\omega/\Omega_{1,2})^2 + 2\pi\chi_0\sin^2\vartheta\Omega/\Omega_{1,2}]^{1/2}\}. \qquad (3.6')$$

We note that expressions (3.6) and (3.6′), derived for a "cold" plasma, are valid also in the presence of spatial dispersion, when the phase velocity of the helicons along the magnetic field $\omega/k\cos\vartheta$ is of the same order as or smaller than the thermal velocity of the electrons (or the limiting Fermi velocity), under the condition that the wavelength is large compared with the Larmor radius of the electrons.

2. We now proceed to consider the interaction of spin waves of plasma waves in antiferromagnets in sufficiently strong magnetic fields $H_{AE} < H < H_E$, when the magnetic field is directed along the anisotropy axis. In the region of magnetic fields not close to H_{AE}, the frequency Ω_1 is close to Ω_H, and therefore when $\omega \gg \Omega_2$ we assume that $\chi_1 \approx \chi_1'$ and $\chi_3 \approx 0$. In this case the dispersion equation (2.2) will take the form, accurate to terms $\sim\chi_0$,

$$D_e(1-\omega^2/\Omega_1^2) + 4\pi\chi_0 D_1 = 0, \qquad (3.7)$$

where ζ in the coefficient B_1 or D_1 is now $\zeta = \omega/\Omega_1$. From this we find that the changes in the plasma frequencies are

$$\Delta_j^{(p)} = -4\pi\chi_0\xi\Omega_1^2/(\Omega_1^2 - \omega_j^{(p)2}), \qquad (3.8)$$

where ξ is given by (3.3) with $\zeta = \omega/\Omega_1$. The change in the spin-wave frequency $\omega \approx \Omega_1$ is equal to

$$\Delta_j^{(m)} = 2\pi\chi_0 D_1(\Omega_1)/D_e(\Omega_1). \qquad (3.9)$$

In the region where the plasma and spin branches cross, $\omega_j^{(p)} \approx \Omega_1$, it is easy to find that $\omega = \omega_{(j)}^{(p)}(1+\Delta)$, where

$$\Delta = {}^1/_2\{1 - \omega_j^{(p)}/\Omega_1 \pm [(1-\omega_j^{(p)}/\Omega_1)^2 + 8\pi\chi_0\xi]^{1/2}\}. \qquad (3.10)$$

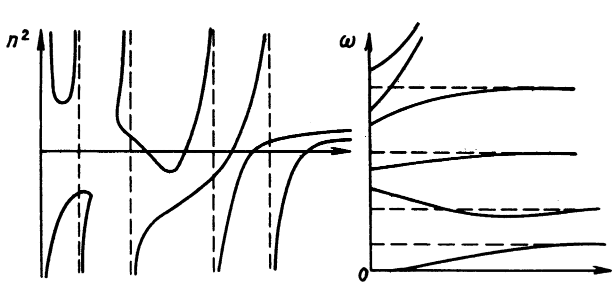
In the frequency region $\omega \sim \Omega_2 \ll \Omega_1$, the components of the magnetic-susceptibility tensor are approximately equal to

$$\chi_1 = \chi_0, \quad \chi_1' = \chi_0\Omega_H^2/\Omega_1^2, \quad \chi_2 = 0,$$

$$\chi_3 = \chi_0\Omega_2^2/(\Omega_2^2 - \omega^2).$$

In this case the dispersion equation takes the form

$$D_e(1-\omega^2/\Omega_2^2) + 4\pi\chi_0 D_2(\omega) = 0, \qquad (3.11)$$



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where

$$D_2(\omega) = A_e \cos^2\vartheta n^4 - [(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + 2\varepsilon_1\varepsilon_3\cos^2\vartheta]n^2 + C_e + (1 - \omega^2/\Omega_2^2)\{A_e n^4(\cos^2\varphi + \sin^2\varphi\Omega_H^2/\Omega_1^2) - [(\varepsilon_1^2 - \varepsilon_2^2)(\cos^2\varphi + \sin^2\varphi\Omega_H^2/\Omega_1^2)\sin^2\vartheta + \varepsilon_1\varepsilon_3(1 + \Omega_H^2/\Omega_1^2)]n^2 + C_e(1 + \Omega_H^2/\Omega_1^2)\}. \qquad (3.11')$$

From this equation we find that

$$\Delta_j^{(p)} = -4\pi\chi_0\xi'\Omega_2^2/(\Omega_2^2 - \omega_j^{(p)2}), \qquad (3.12)$$
$$\Delta_2^{(m)} = 2\pi\chi_0 D_2(\Omega_2)/D_e(\Omega_2), \qquad (3.13)$$

where

$$\xi' = D_2(\omega)(\omega dD_e(\omega)/d\omega)^{-1}|_{\omega=\omega_j^{(p)}}.$$

In the region where the frequencies cross, $\omega_j^{(p)} \approx \Omega_2$, we find that $\omega = \omega_j^{(p)}(1 + \Delta)$, where

$$\Delta = {}^1/_2\{1 - \omega_j^{(p)}/\Omega_2 \pm [(1 - \omega_j^{(p)}/\Omega_2)^2 + 8\pi\chi_0\xi']^{1/2}\}. \qquad (3.14)$$

3. If the magnetic field is perpendicular to the anisotropy axis, then when $\omega \gg \Omega_2$ it follows from (2.10) that $\chi_3 \cong 0$; on the other hand, if $\omega \sim \Omega_2 \ll \Omega_2$, then

$$\chi_1 = \chi_0\Omega_H^2/\Omega_1^2, \quad \chi_1' = \chi_0, \quad \chi_2 = 0,$$

$$\chi_3 = \chi_0\Omega_2^2\sin^2\theta/(\Omega_2^2 - \omega^2).$$

In this case, when $\omega \gg \Omega_2$ the dispersion equation coincides with (3.7), and when $\omega \ll \Omega_2$ it coincides with (3.11), where it is necessary to put in these formulas

$$D_1(\omega) = A_e n^4(\cos^2\varphi\Omega_H^2/\Omega_1^2 + \sin^2\varphi)\sin^2\vartheta - n^2[(\varepsilon_1^2 - \varepsilon_2^2)(\cos^2\varphi\Omega_H^2/\Omega_1^2 + \sin^2\varphi)\sin^2\vartheta - 2\varepsilon_2\varepsilon_3\cos^2\vartheta\omega\Omega_H/\Omega_1^2 + \varepsilon_1\varepsilon_3(1 + \Omega_H^2/\Omega_1^2)] + C_e(1 + \Omega_H^2/\Omega_1^2), \qquad (3.15)$$

$$D_2(\omega) = \sin^2\theta\{A_e n^4\cos^2\vartheta - n^2[(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + 2\varepsilon_1\varepsilon_3\cos^2\vartheta] + C_e\} + (1 - \omega^2/\Omega_2^2)D_1(\omega). \qquad (3.16)$$

Therefore the corrections to the frequencies $\omega_j^{(p)}$, Ω_1, and Ω_2 are determined in this case by formulas (3.8)–(3.10) with $\omega \approx \omega_j^{(p)} \gg \Omega_2$ and $\omega \approx \Omega_1 \gg \Omega_2$ and by formulas (3.12)–(3.14) with $\omega \approx \omega_j^{(p)} \ll \Omega_1$ and $\omega \approx \Omega_2$.

4. We now proceed to study the interaction between the plasma and spin waves in antiferromagnets with anisotropy of the "easy plane" type. If the constant magnetic field is directed along the anisotropy axis, then for $\omega \gg \Omega_2$ we find from (2.12) that $\chi_3 = 0$ when $\omega \sim \Omega_2 \gg \Omega_1$

$$\chi_1 = \chi_0\Omega_H^2/\Omega_1^2, \quad \chi_1' = \chi_0, \quad \chi_2 = 0,$$

$$\chi_3 = \chi_0\Omega_2^2/(\Omega_2^2 - \omega^2).$$

Therefore in this case the dispersion equation coincides with (3.7) or (3.11), where $D_1(\omega)$ and $D_2(\omega)$ are determined by formulas (3.15) and (3.16) (it is merely necessary to replace Ω_H in the expression for $D_1(\omega)$ by $-\Omega_H$, and replace $\sin^2\theta$ in (3.16) by unity).

5. If the magnetic field lies in the basal plane, then for $\omega \gg \Omega_2$ we have $\chi_3 = 0$ and the dispersion equation coincides with (3.7), with

$$D_1(\omega) = A_e n^4(\cos^2\varphi + \sin^2\varphi\Omega_H^2/\Omega_1^2)\sin^2\vartheta - n^2[(\varepsilon_1^2 - \varepsilon_2^2)(\cos^2\varphi + \sin^2\varphi\Omega_H^2/\Omega_1^2)\sin^2\vartheta + 2\varepsilon_2\varepsilon_3\cos^2\vartheta\omega\Omega_H/\Omega_1^2 + \varepsilon_1\varepsilon_3(1 + \Omega_H^2/\Omega_1^2)] + C_e(1 + \Omega_H^2/\Omega_1^2). \qquad (3.17)$$

When $\omega \sim \Omega_2 \ll \Omega_1$ the dispersion equation takes the form (3.11), where $D_2(\omega)$ coincides with (3.11′).

Thus, far from the point of crossing of the frequencies of the noninteracting electromagnetic (plasma) and spin waves, the changes in the frequencies, due to the coupling of the waves, are of the order of χ_0 in all the cases considered above; near the crossing point the coupling of the oscillations was much stronger, and the changes in the frequencies are of the order of $\chi_0^{1/2}$.

We note that the interaction between spin waves and electromagnetic or plasma waves turns out to be significant only when the damping of the waves is small, when $\Gamma/\omega \ll \chi_0^{1/2}$, where Γ is the coefficient of damping due to all possible dissipative processes.

Antiferromagnetic semiconductors with relatively large mobility, in which interaction between spin waves and electromagnetic or plasma waves can probably be observed, were discovered recently. These include $CuFeS_2$ ($n_0 \sim 10^{18}$ cm^{-3}, mobility $\mu \sim 30$ cm^2/V-sec)[8] and UTe_2 ($n_0 < 10^{19}$ cm, $\mu \sim 30$ cm^2/V-sec.[9]

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