TEMPERATURE AND CONCENTRATION OF CHARGED PARTICLES BEHIND THE FRONT OF A STRONG SHOCK WAVE IN AIR

A. I. PETRUKHIN and V. A. PROSKURYAKOV

Earth Physics Institute, Academy of Sciences, U.S.S.R.

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The electron temperatures and the concentrations of the charged particles behind the front of a strong shock wave in air were measured. The shock wave was produced by an electric discharge in a tube at initial pressures of 0.1, 0.2, and 0.5 mm Hg. The temperature was measured using the method of relative and absolute spectral line intensities, while the charged particle concentration was determined from the Stark broadening of the H_β line. The measured values of the temperature and concentration were in good agreement with the equilibrium values of these quantities calculated from the shock wave velocity. The maximum values of the electron temperature were $(60-70) \times 10^3$ °K for charged particle concentrations of the order of $(6-7) \times 10^{16}$ cm^{-3} and of shock wave front velocities of the order of 45–50 km/sec. The oscillator strengths were measured for two nitrogen lines. For the N II line with a wavelength $\lambda = 4026.09$ Å, the measured value was $gf = 0.90$; for the N III line with a wavelength $\lambda = 4510.92$ Å, the measured value was $gf = 1.36$, with an accuracy of 10–15%.

1. INTRODUCTION

A KNOWLEDGE of the temperature and concentration of charged particles behind the front of a shock wave is necessary in the determination of the degree to which a gas heated by a shock wave approximates the state of local thermodynamic equilibrium and of the possibility of the application of the Rankine–Hugoniot relationships to calculations of the state near the front.

It may be regarded as established that in ordinary shock tubes the gas behind the shock wave reaches a state of thermodynamic equilibrium. This has been proved conclusively in the work of Sobolev et al.,[1-3] and of other researchers (cf., for example, the review paper of Kolb and Griem[4]). Such shock tubes are at present used in various physical measurements—in particular, in the determination of the oscillator strengths and other atomic and molecular constants.[4]

There is no agreed view on the state of the gas behind the front of a shock wave in various electric and electromagnetic shock tubes. Thus, for example, McLean et al.[5] and Wiese et al.[6] found that equilibrium was established behind the front of a shock wave in helium and hydrogen in a time of the order of 10^{-7} sec for a plasma density of 10^{17} cm^{-3} and a high-temperature shock lasting 10^{-6} sec. On the other hand, Cloupeau,[7] Makarov and Maksimov[8] carried out investigations at initial gas pressures of 0.1–1.0 mm Hg and rejected the possibility that an equilibrium state might be reached behind the front of a shock wave. It should be mentioned that in these last two investigations the temperature was not measured sufficiently accurately.

To determine the possibility of establishing thermodynamic equilibrium behind the front of a strong shock wave, we undertook a series of preliminary experiments using the apparatus described in [9].

The half-shadow cine photography of a shock wave (Toepler method) made it possible to establish that at initial air pressures in a tube of less than 1 mm Hg and shock wave velocities higher than 6 km/sec, the luminous front coincided with the density discontinuity, within the limits of the experimental error (2–4 mm). In the integrated emission spectra of the plasma, generated by a shock wave, lines were found belonging mainly to nitrogen and oxygen ions in various degrees of ionization, as well as strongly broadened Balmer line series of hydrogen (the half-width of the H_β line reached values of the order of 40 Å).

The qualitative nature of the integrated spectra indicated the presence of a plasma with a temperature and charged-particle concentration close to the equilibrium values. Equilibrium plasma temperatures were found to be $(50-70) \times 10^3$ °K behind the front of a shock wave moving in air, initially at a pressure of 0.1 mm Hg, at velocities of the order of 40–50 km/sec; electron densities were of the order of (2–3) × 10^{17} cm^{-3} and temperatures were calculated by means of thermodynamic tables.[10]

An analysis carried out by Mandel'shtam and Sukhodrev[11] has shown that the processes of excitation and ionization of atoms in a plasma with an electron density ≥ 10^{17} cm^{-3} are mainly due to collisions with electrons and the converse processes are mainly due to collisions of the second type with electrons and recombination in triple collisions. When these conditions apply behind the front of a shock wave, a time of the order of 10^{-7} sec[2] is sufficient to establish local thermodynamic equilibrium of atoms and excited levels of nitrogen and oxygen ions, singly and doubly ionized, with a principal quantum number n ≥ 2. In this state, the steady-state value of the excited atom concentration and the relative distribution of electrons over excited levels of these ions are described by the Boltzmann formula, and the steady-state value of the ionization is given by Saha's formula.

An estimate of the time taken to establish an approximate equality of electron and ion temperatures, using Landau's formula,[13] gives a value of the order of 10^{-7} sec for electron concentrations of (2–3) × 10^{17} cm^{-3} and equilibrium plasma temperatures (60–70) × 10^3 °K.

The "lifetime" of gas particles in a region with a sufficiently high concentration of particles (close to the equilibrium value) behind the front of a wave can be estimated from the formula

$$\tau = \frac{L}{v} = L/D\left(1 - \frac{2}{\varkappa + 1}\right), \qquad (1)$$

where L is the extent of this region, v is the velocity of motion of the gas with respect to the shock wave, D is the velocity of the shock-wave front, $\kappa = C_p/C_v \approx 1.2$[10] is the ratio of the specific heats of the gas.

To estimate L, we used an instrument of the SFR-2M type, operated as a camera, and obtained a time dependence of the emission of a shock wave. Such a time dependence is shown in Fig. 1. A region of approximately constant brightness, which is next to the shock front, is clearly visible. The extent of this region is 1–2 cm.

Assuming that L in Eq. (1) is 1 cm, we find that for a front velocity of 50 km/sec the time the particles spend in a region of high particle concentration is ≈2 μsec, and for a front velocity of 12–13 km/sec (p_{init} = 0.5 mm Hg) this time amounts to ≈ 8–10 μsec.

Thus, we can expect that behind the front there is a region in which the gas heated by the shock wave is in a state close to equilibrium.

It should be mentioned that the estimates just obtained are made on the assumption that electron temperatures and concentrations behind a front are close to their equilibrium values. The problem of the initial stage of ionization in a shock wave still remains unsolved. The available published data (cf., for example, [4, 14]) indicate that preliminary heating of the gas takes place ahead of a shock wave and that the wave is preceded by considerable concentrations of electrons at a fairly high temperature. All this may alter considerably the duration of the initial ionization stage. However, the problem of local thermodynamic equilibrium behind the front of a shock wave at low initial air pressures and front velocities of 40–50 km/sec has not yet been solved; to solve it, it is necessary to measure the temperature and charged-particle concentration.

2. METHOD OF MEASUREMENT

An electric shock tube was made of steel in the form of four interchangeable sections, giving a total length of 2 m. The internal diameter of the tube was 110 mm. The discharge device was described earlier.[9]

The measurements of the shock-wave velocities, temperatures, and charged-particle concentrations

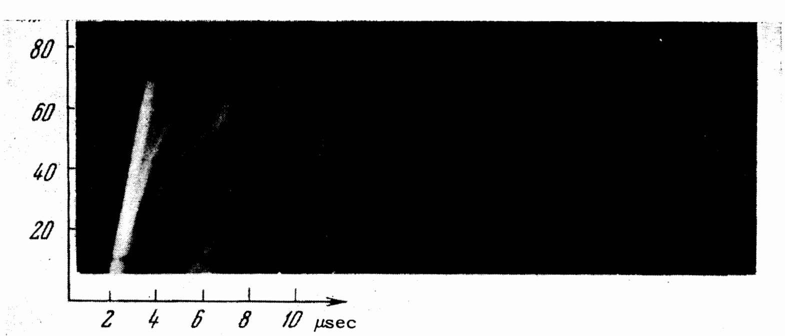

FIG. 1. Recording of a shock wave; p_{init} = 0.1 mm Hg, distance between marks 62 mm, shock-wave velocity 47 km/sec (ordinate gives distance in mm).

in the plasma behind the front were carried out at distances R of 85 and 142 cm from the discharge gap, at initial gas pressures in the tube of 0.1, 0.2, and 0.5 mm Hg. After the passage of each shock, the tube was filled with air at atmospheric pressure and pumped down again to the working pressure. The apparatus used to carry out the measurements is shown in Fig. 2. The velocity of the shock-wave front was determined from the time taken to travel the distance between the two photoelectric probes placed 110 mm apart.

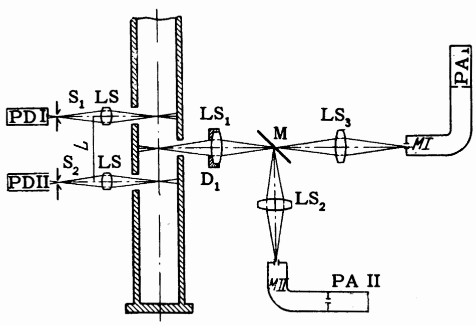

FIG. 2. Apparatus used to measure the intensities of the spectral lines and the shock-wave velocities: M I and M II are, respectively, monochromators UM-2 and ZMR-3; PA I and PA II are photoelectric attachments; M is a semitransparent mirror; LS are lenses; D_1 is the entry diaphragm of the system; S_1 and S_2 are slits; PD I and PD II are photoelectric detectors; L = 110 mm is the distance between the entry slits of the photoelectric detectors.

To measure the temperature and concentration of charged particles, we used a system of two monochromators, in which a beam of light was split by means of a semitransparent mirror. The signals from the outputs of the photoelectric attachments were recorded with a two-beam oscillograph. Such a system made it possible to show simultaneously the behavior of the same element of volume at various times in two different parts of the spectrum. In the temperature measurements, the monochromators were tuned to two spectral lines with known oscillator strengths. The widths of the exit slits of the monochromators were such that the whole selected spectral line reached the photocathode of a photomultiplier. To measure the concentration of charged particles, the monochromators were tuned to different parts of the H_β line and one of the monochromators, tuned to the center of this line, was used as the monitor, while the second monochromator was shifted along the spectrum by intervals of 5 Å in successive experiments, in which the widths of the selected spectral intervals were of the order of 1.5 Å. In this way, we obtained the H_β line contours as a function of time averaged out over many experiments.

In all these measurements, the entry slits of the monochromators were 0.02 mm wide. The whole system was calibrated in absolute units with an accuracy of 6%, using a ribbon-filament lamp. The amplitude characteristics of the photomultipliers used in the measurements were recorded using a pulse-discharge lamp and calibrated light filters. During the measurements, the light filters were introduced into the optical channels so that the signal levels at the photomultiplier were within the limits of the linear part of the characteristics. The electric circuits were calibrated with a special electronic circuit producing a calibrated current pulse. The electric circuits were calibrated after each experiment.

Typical oscillograms showing the time dependence of the spectral lines are given in Fig. 3. The characteristic features of all these oscillograms was a sharply rising front of $\approx 0.1-0.3$ μsec duration and a region of approximately constant intensity of $0.2-0.5$ μsec duration. The time constant of the recording channel was about 0.1 μsec.

Thus, behind the front of a shock wave, we observed an approximately steady-state region of 1–2 cm length. From the signal rise-time, we could estimate the time taken to reach the steady state. In the worst case, this time did not exceed 1–2 μsec in a reference system fixed to a particle. If we also allowed for the fact that the rise-time of a signal could be affected by the shock-wave front not being parallel to the line of observation, the time for the establishment of the steady state was obviously less than the maximum value given above.

3. RESULTS OF THE MEASUREMENTS OF TEMPERATURE AND OF CHARGED-PARTICLE CONCENTRATION

In the case of the Boltzmann distribution of electrons in excited levels of atoms and ions and in the absence of reabsorption, the intensity of a spectral line is described by the formula

$$I_{mn} = \frac{2\pi e^2 h}{mc^3} \nu_{mn}{}^3 \frac{g_n}{g_0} f_{nm} N_0 l \exp\left\{-\frac{E_m}{kT_e}\right\} \quad [\mathrm{erg \cdot cm^{-2} \cdot sec^{-1} \cdot sr^{-1}}] \qquad (2)$$

where f_{nm} is the oscillator strength in absorption, g_n is the statistical weight of the lower level in a transition, g_0 is the statistical weight of the ground state, E_m is the excitation energy of the upper level, N_0 is the number of atoms or ions in the ground state, and l is the depth of the emitting

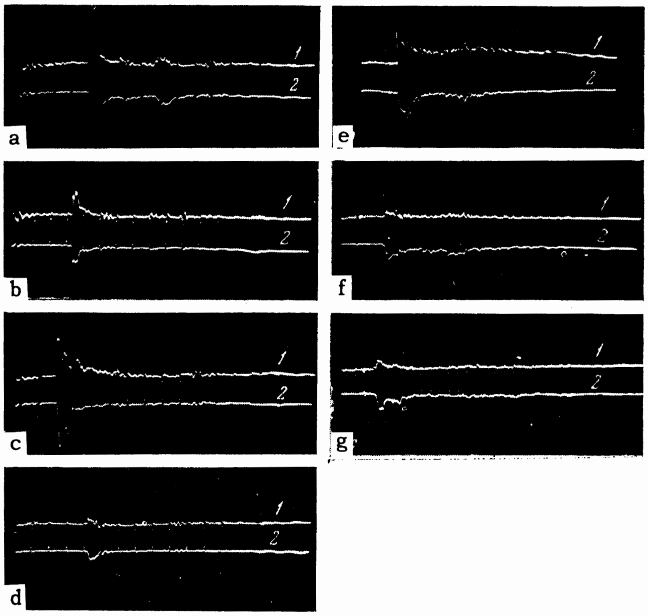

FIG. 3. Oscillograms of the development with time of the spectral lines. R = 85 cm; time marks every 2 μsec; frames a – e – p_{init} = 0.1 mm Hg, frames f – g – p_{init} = 0.5 mm Hg (oscillograms obtained by attenuating the intensity of the spectral lines by a factor of 3 and – in the case of frame d – by a factor of 30).

Frame	Line	λ_1, Å	λ_2, Å
a	NII	5045	5179
b	NII	3995	4026
c	NIII	4515	4379
d	{ NIV NIII	4058	4379
e	H_β	4861*	4870
f	NI	4935	4100
g	NI	4137.6	4114.9

*Line center.

layer. The other symbols have their usual meanings.

The degree of reabsorption can be estimated by determining the optical thickness of the emitting layer. To make this estimate, it is necessary to know the spectral line profile. The Doppler broadening of the nitrogen lines at temperatures of the order of $(50-70) \times 10^3$ °K is $\approx 0.1-0.2$ Å, and the broadening due to the quadratic Stark effect, calculated using Griem's tables,[12] is 0.5–1 Å. For such spectral line widths, the product of the linear absorption coefficient and the depth of the emitting layer, i.e., the quantity $a_\nu l$, is less than 0.1. In this case, the absorption of radiation in the spectral line itself is not very strong and its intensity is described sufficiently accurately by Eq. (2).

According to Griem,[15] an equilibrium population of all the energy levels, beginning with the ground state, cannot be expected at the temperatures and electron densities obtained in our experiments. However, the populations of highly excited levels, beginning from the level with the principal quantum number $n \geq 2$, are in local thermodynamic equilibrium with one another in the case of singly- and doubly-ionized atoms, i.e., they are described by the Boltzmann formula, and free electrons have the equilibrium distribution of velocities. Thus, the electron temperature can be determined from the relative intensities of these lines exactly in the same way as in the case of an equilibrium population of all the energy levels. These conditions were observed for the lines selected by us, with the exception of the 4058 Å line, which belonged to triply ionized nitrogen.

The concentrations of charged particles were estimated in order to be able to use the absolute intensities of the spectral lines to estimate the degree of departure of the population of energy levels of nitrogen ions from its equilibrium value. The values obtained from the intensities of the lines of singly ionized nitrogen were close to the equilibrium values at temperatures of $(60-70) \times 10^3$ °K; the concentrations determined from the intensities of the lines of doubly and triply ionized atoms were, at the same temperatures, about two to four times lower than the equilibrium values and the difference increased when the temperature was reduced.

The temperatures of the shock-heated gas were determined, for an initial pressure of 0.5 mm Hg (front velocities 10–12 km/sec), from the absolute intensities of the spectral lines of atomic nitrogen because the differences between the excitation energies of the upper levels of the lines with known oscillator strengths was small (ΔE_m = 0.4 eV). The exponential term in the calculation formula used to determine temperature, $\exp(\Delta E_m/kT)$, was, for kT = 1 eV, not very sensitive to temperature variation. On the other hand, the absolute intensities of the lines, which were proportional to $\exp(-E_m/kT)$, varied with temperature much more strongly (E_m = 13 eV).

Figure 4 shows the results of calculations of the temperature dependences of the absolute intensities of six atomic nitrogen lines under conditions corresponding to the observed shock-wave velocity for an initial pressure of 0.5 mm Hg. It is evident that at temperatures lower than 16×10^3 °K the intensity varies very rapidly with temperature. Since, according to [15], the local thermodynamic equilibrium for nitrogen atoms at temperatures of $\approx 10 \times 10^3$ °K and electron concentrations $\geq 10^{17}$ cm^{-3} is established in a time shorter than 10^{-6} sec, the determination of temperature from

Table I

Line pair and λ, Å	E_m, eV	$g_n f_{nm}$	p_{init} =0.1 mm Hg: $\log \frac{I_1}{I_2}$	T_e, 10^3 °K	D, km/sec	T_{equil} 10^3 °K	p_{init} =0.2 mm Hg: $\log \frac{I_1}{I_2}$	T_e, 10^3 °K	D, km/sec	T_{equil} 10^3 °K
$R = 85$ cm										
N II {5045 / 5179	20.85 / 30.23	0.57 / 8.0	−0.37	62±3	49.0	66.4	−0.05	43±3	32.4	42.6
N II {3995 / 4026	21.6 / 26.2	1.5 / 0.90	0.55	70±5	51.5	70.0	0.70	50±2	38.6	52.0
N III {4514.89 / 4379	38.24 / 42.53	1.7 / 6.3	−0.29	66.5±2.5	48.0	64.8	−0.19	51±2.5	35.0	46.8
N III {4510.89 / 4379	38.23 / 42.53	1.36 / 6.3	−0.39	65±3	48.0	64.8				
N IV 4057.88 / N III 4379	53.24 / 42.53	1.0 / 6.3	0.40	66±3	48.0	64.8				
$R = 142$ cm										
N II {5045 / 5179	20.85 / 30.23	0.57 / 8.0	0.28	33±3	29.5	35.0	1.25	20±2	18.0	18.0
N II {3995 / 4026	21.6 / 26.2	1.5 / 0.90	0.97	31.6±2	26.2	30.8	1.44	19±1.5	18.0	18.0

NOTE: Oscillator strengths f_{nm} taken from [12, 16–18].

the absolute intensities of spectral lines should be sufficiently accurate.

The results of the temperature measurements immediately behind a shock-wave front (in the region of approximately constant spectral line intensity) are given in Tables I and II. These tables include also the temperatures calculated from the front velocity using the Rankine–Hugionot relationships for equilibrium conditions behind the front of a shock wave.

Figure 5 shows the H_β line profiles plotted from the ratios of the intensities of the selected spectral parts of this line and the intensity at its center, while Table III gives concentrations of charged particles behind the front of a shock wave calculated from these profiles, using the well-known formulas.[20] The values of the temperatures necessary in these calculations were taken from the experiments, and the effective charge Z

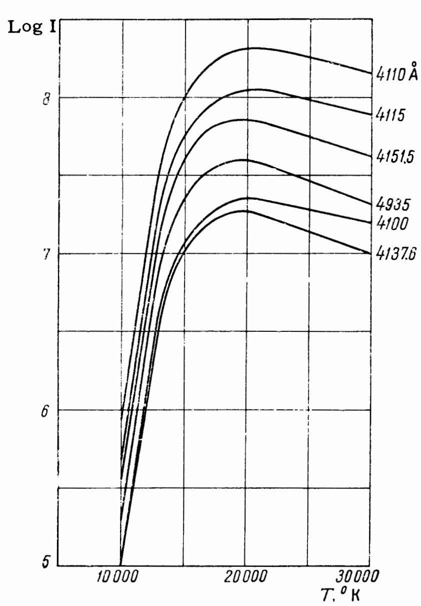

FIG. 4. Dependence of the intensities of the spectral lines of atomic nitrogen (N I); N_{tot} = 5.4×10^{17} cm^{-3}.

Table II

Line Pairs NI, λ, Å	$10^3 g_n f_n$	E_m, eV	log I	T_e, 10^3 °K	D, km/sec	T_{equil} 10^3 °K
p_{init} = 0.5 mm Hg, R = 85 cm						
4109.93	41	13.7	6.62	11.1	}12.5	13.2
4151.46	9.6	13.3	6.55	11.9		
4935.03	8.7	13.2	6.39	12.1	}12.1	12.8
4099.94	4.6	13.7	6.51	13.0		
4114.90	22.7	13.7	6.52	11.5	}11.5	12.2
4137.63	2.5	13.2	6.30	12.7		

NOTE: Oscillator strengths taken from Motschmann's paper[19].

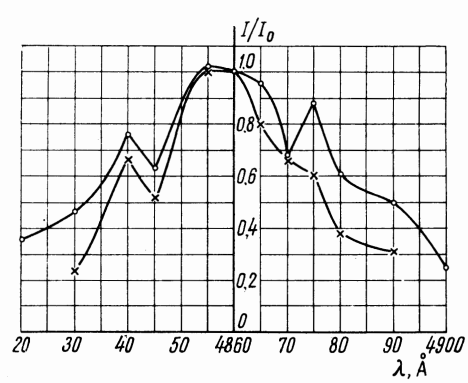

FIG. 5. Profiles of the H_β line; p_{init} = 0.1 mm Hg; o – τ = 0, × – τ = 1 μsec; time is measured from the first maximum of the line.

Table III

	p_{init}, mm Hg ($R = 85$cm)		
	0.1	0.2	0.5
$\Delta\lambda_{1/2}$, Å	57.6	52.6	38.2
$10^{-13}\Delta\omega_{1/2}$, sec^{-1}	4.6	4.2	3
$10^{-17}N_i$, cm^{-3}	0.68	1.5	1.1
$10^{-17}N_e$, cm^{-3}	2	2	1.1
$10^{-17}N_{i,\,equil}$, cm^{-3}	0.95	1.9	1.1

was calculated from Saha's formula for equilibrium conditions. The problem as to whether the emitting hydrogen atoms at the wave front belonged to a hot plasma and not to a wall layer was discussed earlier.[9]

The tables show good agreement between the measured and equilibrium values of the electron temperature and the charged-particle concentration. This indicates that the relaxation time is short for the electron–ion temperature and for excitation and ionization processes behind the front of a shock wave. The difference between the populations of atoms and ions in various degrees of excitation and the equilibrium values has little effect on the electron–ion temperature. An estimate of the fraction of the excitation energy in the total internal energy of a gas behind the front of a shock wave gives a value of the order of 1%. Therefore, in spite of some departure from the equilibrium values of the populations of the excited levels, particularly in the case of highly ionized ions, the temperatures and gas densities behind the front of a shock wave calculated from the Rankine-Hugionot relationships for equilibrium conditions are in good agreement with experiment.

4. MEASUREMENT OF OSCILLATOR STRENGTHS

The good agreement between the measured and calculated temperatures and densities allowed us to use a gas heated by a shock wave to determine the oscillator strengths of some spectral lines.

We measured the oscillator strengths for two optical transitions corresponding to the spectral lines of singly and doubly ionized nitrogen atoms with the wavelengths of 4026.09 and 4510.9 Å. The oscillator strengths were determined relative to the well-known strengths of the lines $\lambda_1 = 3994.99$ Å and $\lambda_2 = 4514.89$ Å, using the formula

$$g_2 f_2 = g_1 f_1 \frac{I_2}{I_1}\left(\frac{\lambda_2}{\lambda_1}\right)^3 \exp\left\{\frac{E_2 - E_1}{kT_e}\right\},$$

Table IV

Quantity	Lines λ, Å	
	NII 4026.09	NIII 4510,92
Relative oscillator strength $g_2 f_2$	0.60 ± 0.02	0.80 ± 0.05
$g_2 f_2$	0.90	1.36
$g_2 f_2$[12]	—	1.26

where f is the oscillator strength, g is the statistical weight of the lower level, E is the excitation energy of the upper level, and I is the intensity of the spectral line (the values with the subscript "1" refer to the line with a known oscillator strength, and the values with the subscript "2" refer to the line whose oscillator strength has to be found).

We measured experimentally, at the same time, the absolute intensities of the spectral lines I_1 and I_2; the value of the electron temperature T_e, necessary in the calculation of the oscillator strength, was assumed to be equal to the equilibrium temperature of a plasma heated by a shock wave at the velocity of a shock-wave front measured in the same experiment.

The measured values of the oscillator strengths are listed in Table IV. This table includes also a comparison of the measured oscillator strength of the double line $\lambda = 4510.9$ Å with the strength calculated in the Coulomb approximation;[12] the difference did not exceed 10%, which lay within the limits of the error of the measurement method, estimated to be 10–15%.

The lines with the measured oscillator strengths were then used in the measurements of the electron temperature of the plasma, as indicated in Table I.

5. CONCLUSIONS

The measurements of the electron temperature and of the charged-particle concentration have indicated that a hot plasma is generated behind the front of a shock wave produced by an electric spark discharge at low initial air pressures; the plasma parameters are close to the equilibrium values with temperatures of the order of $(60-70) \times 10^3$ °K and particle concentrations of the order of 10^{17} cm^{-3}; this plasma can be used as a source of radiation in the measurement of the oscillator strengths and other physical quantities. The state of the shock-heated gas near the front can be calculated with sufficient accuracy by means of the Rankine-Hugionot equations.

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