ABSORPTION OF A HIGH FREQUENCY FIELD IN PURE SUPERCONDUCTING FILMS

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Submitted to JETP editor December 1, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 50, 1364-1368 (May, 1966)

The dependence of the ordering parameter Δ on the magnetic field strength in thin superconducting films is found. A comparison with the experimental values of Δ obtained in experiments on absorption of a high-frequency field is carried out.

ALL the quantities characterizing the behavior of thin pure superconducting films in a magnetic field can be expressed in terms of Green's functions. The Green's functions are determined from the Gor'kov equations [1] and depend on H directly; in addition, they depend on it via the ordering parameter Δ. If the field satisfies the condition

$$d/\xi_0 \ll \rho = eHd^2/4 \ll 1 \tag{1}$$

(d—thickness of film, $\xi_0 = v/T_c$—correlation parameter), the direct field dependence of the physical quantities can be neglected for fixed Δ. Then in the zeroth approximation the dependence on the field will enter only via Δ. For example, the gap in the excitation spectrum differs from Δ by only a quantity of the order of $\rho^2\Delta$ almost everywhere except in a narrow region of angles of the order of Δ[2]. As will be shown later, the dependence of Δ on the field cannot be neglected even when $\rho \ll 1$.

In Sec. 1 we find the value of Δ for pure thin films in a magnetic field. We consider only fields directed along the film and satisfying the condition $d/\xi_0 \ll \rho \ll 1$. Owing to quantum effects, the field is bounded from below by $\rho \gg (p_0d)^{-1}$. However, this limitation is weak and applies to almost all fields up to critical. The equations have been obtained for both diffuse and specular reflection from the walls. The case of specular reflection was considered by Nambu and Tuan [3], who obtained the dependence of Δ on the field for the case of weak fields ($\rho \ll (p_0d)^{-1}$). However, even for such fields the answer is true only in order of magnitude, in view of the linear divergence of each term of the perturbation-theory series.

In Sec. 2 we present a comparison with experimental values of Δ, obtained from experiments on the absorption of a high frequency field.

1. EQUATION FOR THE ORDERING PARAMETER Δ.

In the case in question ($\rho \ll 1$), Δ depends little on the coordinates. For diffuse reflection and for the critical point this was demonstrated by Shapoval [4] (for the specular case this is shown in the Appendix). Therefore the assumption that Δ has a weak dependence on the coordinates is natural when $H < H_c$.

In Gor'kov's equations [1] the ordering parameter $\Delta(\mathbf{r})$ is determined from the supplementary condition

$$\Delta(\mathbf{r}) = |\lambda| T \sum_{\omega_n} \mathcal{F}_{\omega_n}(\mathbf{r}, \mathbf{r}). \tag{2}$$

The Green's Function $\mathfrak{F}_{\omega_n}(\mathbf{r}, \mathbf{r}')$ is determined from the Gor'kov equations for specified $\Delta(\mathbf{r})$, and depends on the magnetic field H as a parameter. In very weak fields ($\rho \ll (p_0d)^{-1}$) the correction to Δ is quadratic in the field. In order of magnitude, this correction was calculated by Nambu and Tuan [3].

In stronger fields ($d/\xi_0 \ll \rho \ll 1$), a linear term appears in the expansion of the function $\mathfrak{F}_{\omega_n}(\mathbf{r}, \mathbf{r}')$ in powers of ρ (for fixed Δ). In first approximation we obtain

$$\Delta = |\lambda| T \left\{ \sum_{\omega_n} \mathfrak{F}_{\omega_n}{}^0(\Delta) - \frac{\rho m p_0}{2\pi} \sum_{\omega_n} \mathfrak{F}_1(\omega_n, \Delta) \right\}. \tag{3}$$

The function $\mathfrak{F}^0_{\omega_n}(\Delta)$ is obtained from Gor'kov's equations without a magnetic field, and does not depend on the thickness of the film:

$$\mathfrak{F}_{\omega_n}{}^0(\Delta) = \frac{\Delta m p_0}{2\pi(\omega_n{}^2 + \Delta^2)^{1/2}}. \tag{4}$$

In determining the sum of $\mathfrak{F}_1$ over ω_n, we confine ourselves to logarithmic accuracy. For this purpose it is sufficient to know $\mathfrak{F}_1$ in the frequency region $|\omega_n| \gg \Delta$. For such frequencies, however, $\mathfrak{F}_1$ can be expanded in powers of Δ:

$$\mathfrak{F}_1(\omega_n, \Delta) = C_1\frac{\Delta}{|\omega_n|} + C_2\left(\frac{\Delta}{|\omega_n|}\right)^3 + \ldots \quad (5)$$

With logarithmic accuracy we can retain in (5) only the first term. Then $\mathfrak{F}_1$ is proportional to Δ and can therefore be obtained from the equation for Δ at the critical point.

Using the results of Shapoval[4] for diffuse reflection, and formula (19) of the Appendix, we obtain

$$C_{1\,\mathrm{dif}} = {}^4/_3, \quad C_{1\,\mathrm{spec}} = {}^5/_6 \text{ for } |\omega_n| \ll v\rho / d,$$
$$C_1 = 0 \quad \text{for} \quad |\omega_n| \gg v\rho / d. \quad (6)$$

From (3) and (6) we get an equation for Δ:

$$\ln\frac{\Delta_0}{\Delta} = 2\sum_1^\infty (-)^{n+1} K_0\left(\frac{n\Delta}{T}\right) + C_1\rho\ln\left(\frac{\xi_0\rho}{d}\right). \quad (7)$$

Here

$$\Delta_0 = \Delta \quad (T = 0, \ H = 0).$$

Comparison with the formulas for the critical fields in contaminated films shows that in order for (7) to be applicable it is necessary that the mean free path l satisfy the condition

$$l \gg 2\rho v / 3\pi T. \quad (8)$$

In the vicinity of the critical field Δ is small, and expanding $\mathfrak{F}^0_{\omega_n}(\Delta)$ in powers of $(\Delta/T)^2$, we obtain

$$\Delta^2 = \frac{2(\pi T)^2 ed^2}{7\zeta(3)} C_1(H_c - H)\ln\left(\frac{\rho\xi_0}{d}\right). \quad (9)$$

If the correction to Δ, due to turning on the field, is small, for which purpose it is necessary that the second term in (7) be small, then

$$\Delta^2 = \Delta^2(T)\left\{1 - \frac{N}{N_s}\rho C_1\ln\left(\frac{\rho\xi_0}{d}\right)\right\}. \quad (10)$$

Here N_s is the number of superconducting electrons:

$$\frac{N_s}{N} = \pi T\Delta^2(T)\sum_{\omega_n}[\omega_n^2 + \Delta^2(T)]^{-3/2}. \quad (11)$$

Formulas (7), (9), and (10) are valid only under the condition

$$\ln(\xi_0\rho / d) \gg 1. \quad (12)$$

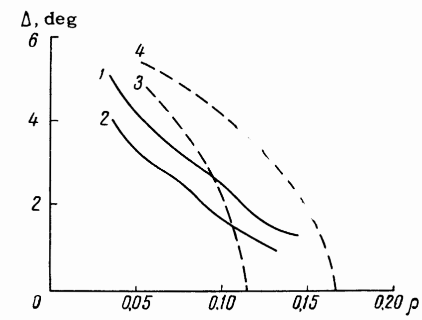
This means that the fields should be sufficiently strong. For T close to T_c, formulas (9) and (10) coincide. Condition (12) leads to a limitation on the closeness of T to T_c:

$$(d / \xi_0) \ll \rho \ll 1 - T / T_c. \quad (13)$$

2. ABSORPTION OF HIGH FREQUENCY FIELD

The coefficient k for absorption of a high-frequency field by a superconducting film depends on H in two ways, directly and via Δ. When $\rho \ll 1$ the direct dependence on H can be neglected. We then obtain for k the usual expression in the absence of the field, but with $\Delta = \Delta(H)$. One cannot neglect the dependence of Δ on H even when $\rho \ll 1$, since the quantity ρ is multiplied in Eq. (7) for Δ by a large logarithm.

White and Tinkham[5] measured the absorption of the high-frequency field in superconducting films of indium, tin, and lead. This yielded, on the basis of Miller's calculations[6], a relation $\Delta = \Delta(H)$. The theoretical and experimental values for tin are shown in the figure. We see that a decrease in frequency leads to a decrease of Δ_{exp}.



Dependence of the ordering parameter Δ on the magnetic field: $1 = \omega = 4.5 \times 10^{11}$ sec^{-1}, 2 – $\omega = 2.1 \times 10^{11}$ sec^{-1} 3 – diffuse reflection, 4 – specular reflection; the continuous curves – experiment, dashed curves – calculation.

This result can be understood by assuming that the absorption is in fact determined not by Δ, but by a certain effective gap in the spectrum. In a magnetic field Δ and the gap in the spectrum do not coincide and, as shown in an earlier paper[2], in the case of specular reflection the spectrum of the single-particle excitations is strongly altered even when $\rho \ll 1$. There exists a region of angles of the order of ρ, for which the gap in the spectrum is small. A decrease in the frequency leads to a relative increase in the contribution to the absorption of the angles with small gap, and this is manifest as a decrease of Δ_{exp}.

When T, $\omega \ll \Delta$, the absorption is determined just by the angles with small gap. Therefore in this case the value obtained for D_{exp} will be strongly underestimated.

Experiments with tin at T = 2°, $\omega_1 = 2.1 \times 10^{11}$ sec^{-1}, $\omega_2 = 4.5 \times 10^{11}$ sec^{-1}, and d = 325 Å[5] yield relatively satisfactory agreement with theory. The relatively small deviations of D_{exp} for different frequencies serve as an experimental confirmation that it is impossible to neglect the direct dependence of the absorption coefficient on the field.

In experiments on lead ρ was not small, $\rho_c \approx 0.8$. This should have led to strong differences of Δ for different frequencies. Unfortunately, the value of Δ given in[5] was obtained from measurements at only one frequency. The mean free path can be determined from comparison of the critical fields with the theoretical formulas. Such a comparison shows that the films used in[5] were not very pure. In particular, for samples of tin at $T = 2°$ we have $3\pi T l / 2\rho_c v \approx 3$, that is, condition (8) is rather poorly satisfied.

For samples of indium, the value of the critical field obtained from (7) is much higher than the experimental one. This is apparently connected with the fact that the mean free path in indium is $l \ll \xi_0$.

In conclusion I am grateful to A. I. Larkin for directing the work and E. A. Shapoval for valuable remarks.

APPENDIX

CRITICAL FIELDS OF THIN SUPERCONDUCTING FILMS (SPECULAR REFLECTION)

At the transition point ($\Delta \to 0$) Gor'kov's equations can be expanded in powers of Δ. In the approximation linear in Δ we obtain

$$\Delta^*(\mathbf{r}) = |\lambda| T \sum_{\omega_n} \mathfrak{F}^+_{\omega_n}(\mathbf{r}, \mathbf{r})$$

$$= |\lambda| T \sum_{\omega_n} \int \widetilde{\mathfrak{G}}_{-\omega}(\mathbf{r}_1, \mathbf{r}) \Delta^*(\mathbf{r}_1) \widetilde{\mathfrak{G}}_{\omega}(\mathbf{r}_1, \mathbf{r}) \, d\mathbf{r}_1. \tag{14}$$

If the film thickness d satisfies the condition $d \gg p_0^{-1}$, then quantization in the transverse direction is insignificant and the film problem becomes equivalent to the problem of a bulky superconductor with periodic potential

$$\mathbf{A}(z + 2d) = \mathbf{A}(z), \quad A_y = A_z = 0,$$

$$A_x(z) = H(-1/2 d + |z|), \quad -d \leqslant z \leqslant d. \tag{15}$$

In the quasiclassical approximation

$$\widetilde{\mathfrak{G}}_\omega(\mathbf{r}, \mathbf{r}') = \mathfrak{G}^0_\omega(\mathbf{r} - \mathbf{r}') \exp\left[ie \int_{\mathbf{r}'}^{\mathbf{r}} \mathbf{A}\, d\mathbf{l}\right]. \tag{16}$$

From (14), (16) we obtain ($R = |\mathbf{r} - \mathbf{r}_1|$)

$$\Delta^*(\mathbf{r}) = |\lambda| T \sum_\omega \int \left(\frac{m}{2\pi R}\right)^2 \Delta^*(\mathbf{r}_1)$$

$$\times \exp\left[-\frac{2|\omega|}{v} R - 2i \int_{\mathbf{r}_1}^{\mathbf{r}} \mathbf{A}\, d\mathbf{l}\right] d\mathbf{r}_1. \tag{17}$$

When $|\omega| \gg v\rho/d$, the region of small R is essential, and the two last factors in (17) can be taken at $\mathbf{r} = \mathbf{r}_1$. When $|\omega| \ll v\rho/d$, we can average the integrand in (17) over the distances r_1 of order of d. Carrying out the averaging, we obtain

$$\Delta(z) \ln \frac{2\gamma\xi_0\rho}{\pi d} = \frac{\pi T}{4} \sum_{\omega < v\rho/d} |\omega|^{-1} \int_{-1}^{1} [\exp\{-\rho|t^2 - t_1^2|\}$$

$$+ \exp\{\rho(t^2 + t_1^2 - 2)\}] \Delta(t_1) \, dt_1, \tag{18}$$

where $t = 2z/d$ and $\ln \gamma = 0.577$. For $\rho \ll 1$ it follows from (18) that

$$C_1 = 5/6 \quad \text{for} \quad |\omega| \ll v\rho/d, \tag{19}$$

$$C_1 = 0 \quad \text{for} \quad |\omega| \gg v\rho/d,$$

$$\Delta(t) = 1 + \rho t^2 (1 - 2/3 |t|),$$

$$\rho_c = \frac{6}{5} \ln \frac{T_c}{T} \Big/ \ln\left(\frac{v\rho}{d}\right) \tag{20}$$

We note that integral equation (18) can be reduced to a differential equation.

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Translated by J. G. Adashko
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