CATALYSIS BY NEGATIVE MUONS OF THE NUCLEAR REACTIONS $d\mu + p \to He^3 + \mu^-$ AND $d\mu + d \to t + p + \mu^-$ AND FORMATION OF THE MOLECULES $pd\mu$ AND $dd\mu$ IN GASEOUS HYDROGEN

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We have used diffusion cloud chambers filled with hydrogen and deuterium at pressures of 7-23 atm to measure the yields of the nuclear catalysis reactions (1) and (3). The rate of transfer of a muon from a $d\mu$ atom to atoms of carbon and oxygen was determined on the basis of the experimental distributions in range of $d\mu$ atoms and the yield of events with Auger electrons. It was found that the rates of formation of $pd\mu$ and $dd\mu$ molecules, reduced to the density of liquid hydrogen and deuterium, are

$$\lambda_{pd\mu} = (1.8 \pm 0.6) \cdot 10^6 \text{ sec}^{-1}, \quad \lambda_{dd\mu} = (0.75 \pm 0.11) \cdot 10^6 \text{ sec}^{-1}.$$

An evaluation of the relative yield of reaction (6) shows that $Y(6)/Y(3) < 0.14$ with a probability of 90%. Analysis of the set of experimental data on reactions (1) and (3) leads to the conclusion that a possible cause of the large yield of two-deuteron synthesis reactions under our experimental conditions is a resonance mechanism for $dd\mu$-molecule formation and occurrence of the nuclear reaction in flight.

INTRODUCTION

NUCLEAR reactions synthesizing hydrogen isotopes, catalyzed by negative muons, were first observed by Alvarez et al.[1] in 1957. At the present time the synthesis reactions are studied mainly to investigate the formation of muonic molecules—the preceding phase of the nuclear reactions, and also some features of the nuclear reactions themselves in these molecules. The probability and mechanism of formation of the μ-molecular systems $pd\mu$ and $dd\mu$ are important in connection with the fact that study of muon capture by deuterons can give new information on the spin dependence and interaction constants of μ^- capture.[2]

Recently a number of measurements have been made of the yield of the products of nuclear reactions in μ molecules formed on stopping of muons in liquid hydrogen. Several authors[1, 3-5] have studied the muon yield from the reaction

$$d\mu + p \to pd\mu \to He^3 + \mu^- \tag{1}$$

as a function of the concentration of deuterium added to hydrogen. The time distributions of γ rays from a second reaction channel

$$d\mu + p \to pd\mu \to He^3\mu + \gamma \tag{2}$$

have been measured in liquid hydrogen by Ashmore et al.[6] and recently more accurately by Lederman's group.[7] The latter group measured the absolute rates of a number of μ-atomic processes occurring in liquid hydrogen with small concentrations of deuterium and, in particular, the rate of formation of $pd\mu$ molecules and the rate of the nuclear reaction in this molecule. The rate of formation of $pd\mu$ molecules has also been measured at CERN[8] by measurement of the time distribution of mesic x rays from neon added to liquid hydrogen.

Considerably less experimental information has been obtained on the $dd\mu$ system. Fetkovich et al.[4] and Conforto et al.,[8] working with liquid-deuterium bubble chambers, have measured the proton yield from the reaction

$$d\mu + d \to dd\mu \to p + t + \mu^-. \tag{3}$$

In a previous study we have also obtained data on the yields of reactions (1) and (3) in experiments with a diffusion cloud chamber filled with hydrogen and deuterium.[9] Recently[10] with the same technique we have studied a second reaction channel in the $dd\mu$ molecule:

$$d\mu + d \to dd\mu \to He^3 + n + \mu^-. \tag{4}$$

Here it was established that the ratio of probabilities of channels (3) and (4) is close to unity. The measured yields of all these reactions for the gaseous phase turned out to be different from those expected on the basis of the experimental data obtained in liquid hydrogen and deuterium.

In the present work we have substantially improved the statistics of catalysis-reaction events and have refined the method of determining the rate of transfer of muons from $d\mu$ atoms to the complex nuclei of C and O admixtures (Z admixtures). Possible interpretations of the observed effects are discussed. The principal results of the present work were reported at the XII International Conference on High Energy Physics at Dubna.[11]

I. EXPERIMENTAL SETUP AND RESULTS

1. Experimental Conditions

The negative muons were slowed down in an absorber and stopped in the working region of a high-pressure diffusion cloud chamber operating in a magnetic field of 7000 G.

Data on the yield of reaction (1) were obtained in a series of experiments in which the chamber was filled with hydrogen to a pressure of 23 atm with addition of deuterium, whose concentration (~ 6%) provided almost 100% transfer of muons from protons to deuterons as the result of the exchange reaction

$$p\mu + d \rightarrow d\mu + p, \qquad (5)$$

and subsequent collisions of $d\mu$ atoms with protons led to reaction (1). The statistical data of these experiments on the range distribution of $d\mu$ atoms has already been used by us in a study of elastic scattering of $d\mu$ atoms by protons, deuterons, and complex nuclei,[12] where a detailed description is given of the experimental conditions.

Table I contains data on two series of measurements (designated HDI and HDII) differing in the liquids used in the chamber: in the HDI series methyl alcohol CH_3OH was used, and in the HDII series—normal propyl alcohol C_3H_7OH. Use of propyl alcohol in the experiments provided roughly five times smaller concentration of complex nuclei (C and O) for the same temperature distributions.

Data on the yield of reaction (3) were obtained both in a series of experiments with a deuterium filling to a pressure of 7–17 atm with different concentrations of Z admixtures (experiments D1, D2, and D3, Table I), and in the series of tests described above with 94% H_2 + 6% D_2 (HD, Table I). The degree of purification of the deuterium from tritium was sufficient to provide conditions for reliable identification of the events, without changing the temperature conditions of the chamber (the tritium impurity was 5×10^{-12} at. %, which gives 0.1 tritium β decay per cm^3 per sec). All photographs were doubly scanned. The numbers of muon stoppings found are listed in the last column of Table I. The detection efficiency of muon stoppings and reaction (1) and (3) events of interest in single scanning was 90%, and in double scanning—close to unity.

2. Identification of Cases of the Reaction $d\mu + p \rightarrow He^3 + \mu^-$

In the synthesis reaction $p + d \rightarrow He^3$, an energy of 5.5 MeV is released and almost all of it (5.3 MeV) is carried away by the conversion muon. Therefore, identification of reaction (1) events consists of separating events in which a secondary-particle track emerges from the point of stopping of the primary muon with a curvature and ionization corresponding to a muon of 5.3 MeV. Since the concentrations of deuterium in experiments HDI and HDII are small (Table I), $d\mu$ atoms are formed mainly in the process of exchange scattering (5) (only in 6% of the cases are $d\mu$ atoms formed as the result of direct deposition of a muon in an orbit of a $d\mu$ atom). Therefore the $d\mu$ atoms travel a noticeable distance through the chamber gas before formation of a $pd\mu$ molecule.[12] For this reason, in 85% of the reaction (1) events the

Table I

Series of experiments	Total pressure, atm	Deuterium concentration, %	Working liquid	Total number of C and O atoms, $10^{19}/cm^3$	Number of photographs	Number of muon stoppings with $l_\mu > 20$ mm
HDI	23.0	5.6	CH_3OH	0.15 ± 0.05	103 260	26200
HDII	23.0	6.5	C_3H_7OH	0.03 ± 0.01	55 750	11900
D1	7.2	94	C_3H_7OH	0.03 ± 0.01	33 800	3 330
D2	17.3	90	CH_3OH	0.15 ± 0.05	517 000	9 950
D3	16.7	89	C_3H_7OH	0.03 ± 0.01	11 700	2 200
HD	23.0	6.1	{C_3H_7OH, CH_3OH}	0.09 ± 0.03	15 9000	40 600

end of the stopping-muon track and the beginning of the secondary-muon track are separated by a visible displacement whose magnitude lies in the range from the half-width of the muon track up to 8 mm. In addition, the beginning of the secondary-muon track is often accompanied by a visible He^3 recoil-nucleus track about 0.5 mm in length.

An example of a reaction (1) event is shown in Fig. 1. Under the conditions used, reaction (1) events can be imitated by $\pi^- \to \mu^-$ decays in flight occurring shortly before the stopping of the pion, and by $\mu^- \to e^-$ decays in which the curvature and apparent ionization density of the electron are the same as for a secondary muon.

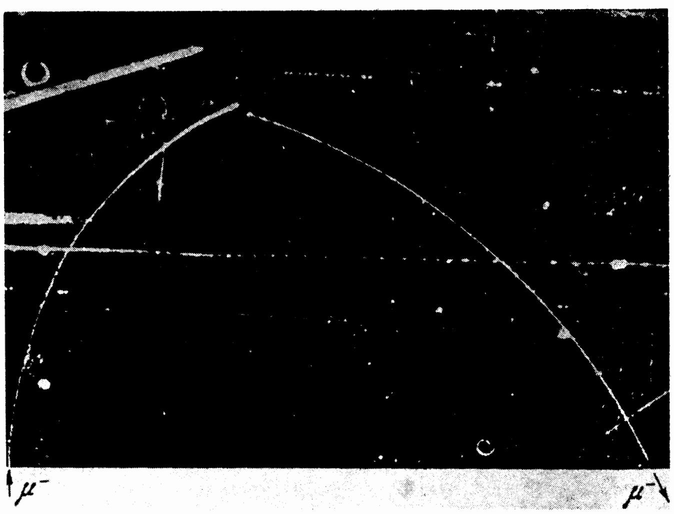

FIG. 1. Photograph of a reaction (1) event.

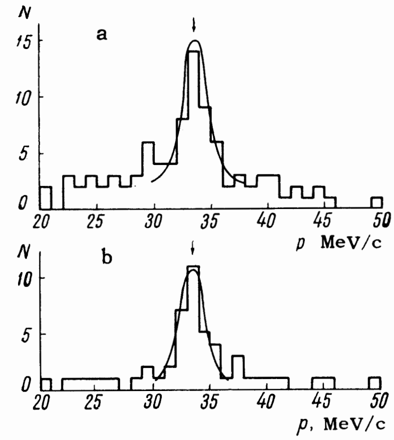

FIG. 2. Distribution in momentum of secondary particles: a—in all events which can be assigned to reaction (1), b—after subtraction of background of $\pi \to \mu$ decays in flight. The arrow indicates the expected value of momentum of the conversion muon in reaction (1).

In order to separate reaction (1) events more reliably from the possible background events described above, we introduced the following limitations on the track lengths of particles: a) the track length of a stopping particle l_μ must be greater than 20 mm; b) the length of the projection on the horizontal plane of the secondary-particle track must be greater than 30 mm. Events found in the scanning and satisfying these criteria were measured in a stereoprojector. In measuring the momenta of the secondary particles we introduced corrections for nonuniformity of the magnetic field over the chamber volume, conic reprojection, and change of curvature along the length of the track as the result of slowing down in the gas. Figure 2a shows the momentum distribution of secondary particles for all events selected. The peak in the momentum region 33.5 MeV/c corresponds to reaction (1) events.

To reduce the background from $\pi \to \mu$ decays we made a separation on the basis of a relative measurement of the masses of the stopping particles for the primary tracks.[13] Figure 2b shows the momentum spectrum of secondary particles after rejection of events assigned to $\pi \to \mu$ decays in flight. After subtraction of this background spectrum, which was assumed independent of momentum, we found that the number of events belonging to reaction (1) was 27 ± 5. This number of events must be corrected for the geometrical detection efficiency associated with the track-length selection criteria chosen.

On the basis of the distribution of points of muon stopping over the sensitive layer of the chamber we calculated that the efficiency for detection of a secondary particle with a track length greater than 3 cm is 59%. To verify the method of calculating the efficiency, we used the distributions of projected lengths of charged particles in one-prong stars in capture of π^- mesons by helium. It was found that the above efficiency is 0.67 ± 0.10, which is in good agreement with that calculated for the present case.

It can be noted also that the distribution of reaction (1) events over the height of the sensitive layer, as was verified experimentally, is close to the distribution of points of muon stopping.

When we take into account the 59% detection efficiency, the number of reaction (1) events in experiments HDI and HDII is 46 ± 9. This number also will be used to determine the yield of reaction (1).

3. Identification of Cases of the Reaction $d\mu + d \to p + t + \mu^-$

Cases of this reaction are comparatively easy to identify since, in view of the weak binding of the muon in the $dd\mu$ molecule, the reaction can

Table II

Series of experiments	Number of $\mu \to e$ decays	Detection efficiency for decay electron, %	Number of reaction (3) events	Number of reaction (3) events with complete proton range	Number of reaction (3) events with visible decay electron	Number of muon stars with visible prongs
D1	1050	33	21	5	5	23 (6)
D2	4830	54	27	16	12	127 (9)
D3	240	11	19	8	2	16 (5)
HD	19 000	50	13	10	6	—

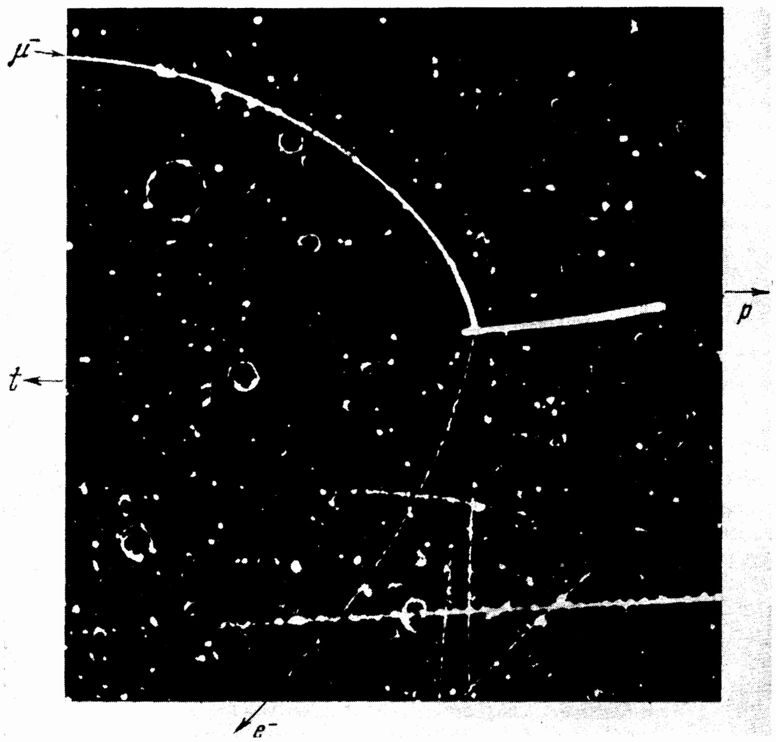

FIG. 3. Photograph of a reaction (3) event.

be considered as two-particle. Here the tritium nucleus and the proton have definite energies (E_t = 1.01 MeV, E_p = 3.02 MeV) and the range values corresponding to these energies are convenient for detection in the chamber. The reaction-product separation angle φ should be 180° within 1° or less. In addition, a μ-decay electron track must emerge from the point of separation of the p and t. However, the electron track often cannot be seen because of the comparatively low detection efficiency for relativistic particles under our experimental conditions (low alcohol-vapor density) or because of nuclear absorption of the muon in carbon and oxygen. Figure 3 shows a photograph of a typical reaction (3) event (experiment D2, Table I).

The results of identification of reaction (3) are listed in Table II. In the last column of the table the numbers in parentheses are the numbers of two-prong stars in each experiment (for the HD experiment the two-prong stars were not analyzed in detail, in view of the substantial background of random proton tracks separated from the point of muon stopping by distances equal to the range of a $d\mu$ atom[1)]). In view of the fact that two-prong stars are observed in a relatively small number of events, and also that the emission of particles in the stars is isotropic and the energies of the particles in a star are completely uncorrelated, the

[1)]In experiments D2 and D3, in view of the high deuterium density and the large elastic-scattering cross section for $d\mu$ atoms by deuterons,[12] we did not observe events with a visible $d\mu$-atom range. In experiment D1 (deuterium pressure 7 atm) events of this type were observed, and here the $d\mu$-atom range was about 1 mm. It must be noted that the nature of this effect in this experiment is related to diffusion of the $d\mu$ atom formed as the result of direct combination of a muon with deuterium (similar to diffusion of $p\mu$ atoms in hydrogen[13,14]), and not as the result of process (5).

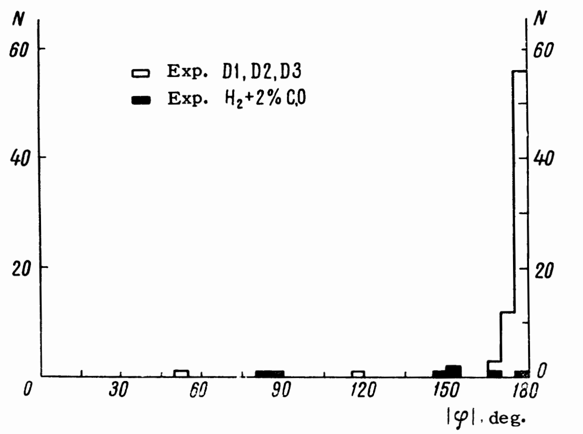

FIG. 4. Distribution in separation angle φ of secondary particles in two-prong stars; the peak near $\varphi = 180°$ is assigned to reaction (3) events.

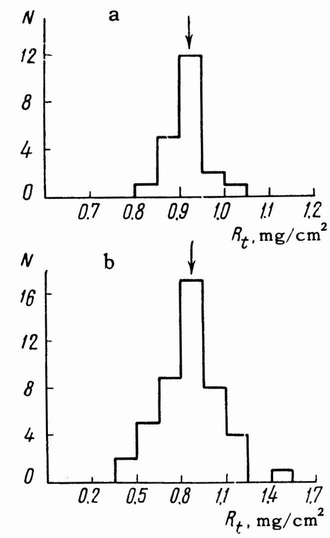

FIG. 5. Distributions in range of the tritium nucleus in reaction (3): a – experiment D1, b – experiments D2 and D3. The arrows indicate the theoretical range values

background from two-prong stars in experiments D1, D2, and D3 was very small.

Figure 4 shows the distribution in angle φ for all cases of reaction (3) in experiments D1, D2, and D3 and for those two-prong stars in which the track lengths or ranges of the secondary particles are close to the track lengths expected for the proton and tritium nucleus in reaction (3). The same figure shows the distribution in angle φ for stars in experiments with hydrogen[13] in which reaction (3) events are not present (the total number of stars in these experiments was 211). From the figure we conclude that the background from stars does not exceed 2–3%.

Figure 5 shows the distribution in range of tritium nuclei in reaction (3) events for experiment D1 (Fig. 5a) and for experiments D2 and D3 (Fig. 5b) (in all cases the range of the tritium nuclei terminated in the sensitive region of the chamber).

In experiment HD the identification of reaction (3) is more difficult as a result of the substantial background noted above. In this case the separation is possible only on the basis of the combined range of the tritium nucleus and proton (when both particles stop in the sensitive volume of the chamber). Figure 6a shows the distribution of events in combined range in experiments D1, D2, and D3. In Fig. 6b we have plotted the one-prong stars from experiment HD whose range terminates in the sensitive layer of the chamber and where the beginning of the track of the star prong is displaced from the point of the muon stopping by a distance of not more than 5 mm. Assuming the background in the range region 6.2–7.2 mm/cm^2 to be linearly varying (broken line in Fig. 6b) and taking into account the efficiency for observation of the tracks, we found that the number of reaction (3) events in experiment HD was 13 ± 5 (Table II).

4. Identification of the Reaction $d\mu + d \rightarrow t\mu + p$

After a nuclear reaction in the μ molecule $dd\mu$, the muon either remains free (reactions (3) and (4)) or bound with any of the three charged products of the reactions. Estimates of the probability of attachment of the muon to a proton and to a He^3 nucleus have already been given by Dzhelepov et al.[10] Here we will give an estimate of the probability for attachment of a muon to a tritium nucleus—the reaction

$$d\mu + d \rightarrow dd\mu \rightarrow t\mu + p. \qquad (6)$$

In reaction (6) the $t\mu$ atom is neutral, and cases of this reaction will appear as one-prong stars in

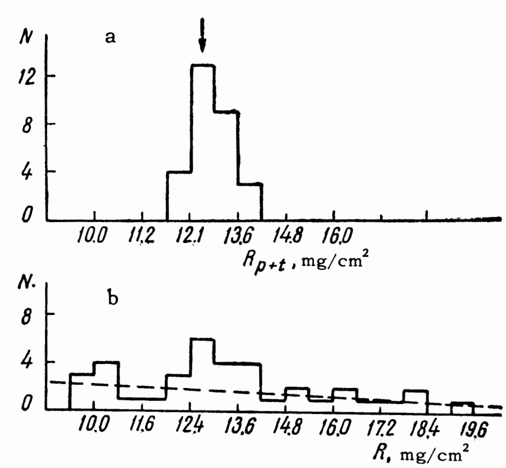

FIG. 6. Distributions: a—in combined range of proton and triton in reaction (3) events for experiments D1, D2, and D3; b—in range of the secondary particle in one-prong stars for experiment HD, reduced to the density of deuterium.

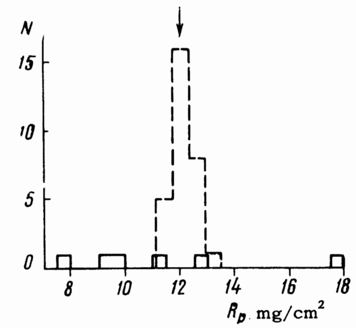

FIG. 7. Distribution in range of secondary particle in one-prong stars for experiments D1, D2, and D3. The broken line shows the distribution in range of protons in reaction (3). The arrow indicates the value of the proton range in reaction (6).

which the proton range is equal to the proton range in reaction (3).

Figure 7 shows the combined distribution of range in one-prong stars for events in experiments D1, D2, and D3. The broken line in the same figure shows the distribution of events in proton range in reaction (3). Assuming that the number of reaction (6) events in the range region near 12 mg/cm^2 does not exceed two, we obtain a ratio of reaction yields

$$W(t\mu) = Y(dd\mu \to t\mu + p) \,/\, Y(dd\mu \to t + p + \mu) < 0.14.$$

with a probability of 90%.

5. Yields of Nuclear Reactions

The yields of reactions (1) and (3) are defined as the ratio of the number of reaction events to the total number of $d\mu$ atoms formed $N_{d\mu}$. For experiments HDI and HDII the average value of $N_{d\mu}$ is $(0.95 \pm 0.01)N_{stop}$, where N_{stop} is the number of muon stoppings. This value of $N_{d\mu}$ was obtained on the basis of the known deuterium concentrations in the present experiments and the previously determined rate of reaction (5).[9, 14]

For experiments D1, D2, and D3, in view of the high rate for reaction (5) and the relatively small hydrogen concentration, $N_{d\mu}$ is almost exactly equal to the number of muon stoppings N_{stop}. The yields of reaction (1) are listed in Table III (first line) and for reaction (3)—in Table IV (second column).

6. Lifetime of the $d\mu$ Atom

To interpret the measured yields of the nuclear reactions it is necessary to know the lifetime of the $d\mu$ atom τ under our experimental conditions, which is determined mainly by the free-muon decay rate λ_0 and the transition rate $\lambda'_Z c_Z$ of a muon from a $d\mu$ atom to atoms of C and O, and also—

Table III

Quantity		Experiment HDI	Experiment HDII
$10^3\, Y_\mu$		0.84 ± 0.24	2.20 ± 0.57
N_{Aug}		123 ± 3	205 ± 10
N_{Aug} with correction*		145 ± 4	221 ± 11
N without Auger electrons		71 ± 2	312 ± 17
Y_{Aug}		$0.67 \pm 0{,}06$	0.41 ± 0.04
η		0.83 ± 0.05	0.83 ± 0.05
$10^{-6}\lambda$, sec^{-1}	From Auger electrons	2.6 ± 0.8	1.0 ± 0.3
	From ranges	2.25 ± 0.30	0.80 ± 0.10
	Average	2.29 ± 0.28	0.82 ± 0.09

*Correction for nuclear capture and δ-electron background.

to a relatively smaller degree—by the rate of formation of mesic molecules:[2)]

$$1/\tau = \lambda = \lambda_0 + \lambda_Z' c_Z + \lambda'_{pd\mu} c_H + \lambda'_{dd\mu} c_D. \qquad (7)$$

A. The quantity λ can be determined from analysis of the range distributions of $d\mu$ atoms. This method has been used by us previously to determine the cross section for elastic scattering of $d\mu$ atoms in hydrogen.[12] It is based on comparison of the experimental range distributions of $d\mu$ atoms, obtained for different concentrations of deuterium in hydrogen and different concentrations of Z admixtures, with distributions calculated by the Monte Carlo method. The values of λ found in this way are listed in Table III.

B. Since the main contribution to λ, in addition to the decay rate λ_0, is from the transition rate $\lambda'_Z c_Z$, λ can also be determined from the observed yield of $\mu \to e$ decay events in which the beginning of the decay-electron track is accompanied by a visible point (Auger-electron track).[12, 14] If we designate the yield of events with Auger electrons as $Y_{Aug} = N_{Aug}/(N + N_{Aug})$, where N_{Aug} and N are the numbers of events with a visible "point" at the beginning of the decay-electron track and without it, then, for example, for experiment HDI we can write

$$Y_{Aug}(\mathrm{HDI}) = \frac{\lambda_Z' c_Z(\mathrm{HDI})}{\lambda_Z' c_Z(\mathrm{HDI}) + \lambda_0 + \lambda'_{pd\mu} c_H}\,\eta, \qquad (8)$$

where η is the average probability of appearance of an Auger electron per transfer to a C or O nucleus. Comparing (8) with the similar expression for experiment HDII and neglecting in the first approximation the term $\lambda_{pd\mu} c_H$, which amounts to 2–6% of the quantity λ, we can find the value of the transition rate $\lambda'_Z c_Z$ and the coefficient η.

The data from which the yields Y_{Aug} were de-

2)The quantities λ' designate the rates under our experimental conditions, i.e., in a gaseous medium of definite density.

Table IV

Experiment	$10^2 Y_p(dd)$	λ, 10^6 sec^{-1}	$10^2 Y^0_p(dd)$	ρ_0/ρ_D	$10^{-6}\lambda_{dd\mu}$, sec^{-1}	$\bar{E}_{d\mu}$, eV	$10^{22}\sigma_{dd\mu}$, cm^2
D1	0.63 ± 0.14	$0{,}74\pm0.10$	1.03 ± 0.26	110	1.03 ± 0.26	0.043	1.3 ± 0.4
D2	0.27 ± 0.05	2.2 ± 0.3	1.3 ± 0.3	49	0.59 ± 0.14	0.046	0.73 ± 0.17
D3	0.86 ± 0.20	0.74 ± 0.10	1.4 ± 0.4	50	0.63 ± 0.18	0.042	0.82 ± 0.23
HD	0.032 ± 0.012	1.47 ± 0.20	0.10 ± 0.04	540	0.50 ± 0.20	0.170	0.32 ± 0.13
Ref. 1	$0.18^{+0.12}_{-0.08}$	5.8 ± 0.3	$2.3^{+1.5}_{-1.0}$	27	$0.76^{+0.50}_{-0.33}$	0.012	$1.8^{+1.2}_{-0.8}$
Ref. 4	4.0 ± 0.8	0.88	6.4 ± 1.0	1.05	$0.072\pm0{,}014$	0.0039	0.30 ± 0.05
Ref. 5	7.96 ± 0.31	0.61	8.85 ± 0.35	1.01	0.098 ± 0.004	0.0039	0.412 ± 0.017

termined in experiments HDI and HDII are listed in Table III. In selection of the events N_{Aug} and N we used a series of selection criteria: we selected only events with a visible displacement of the beginning of the decay-electron track with respect to the point of stopping of the muon by at least 1.5 mm; the angle $\theta_{d\mu}$, formed by the direction of the $d\mu$-atom path and the tangent to the muon track at the point of stopping, was required to be within the range $120° > \theta_{d\mu} > 60°$; we used only part of the data, in which the conditions for observation were particularly favorable. The observed number N_{Aug} must be corrected for nuclear absorption of the muon from an orbit of a $C\mu$ or $O\mu$ atom, and also for the accidental appearance of a δ electron at the beginning of the decay-electron track (see Table III).

The value of λ determined in this way (including a small correction for formation of $pd\mu$ molecules) also is listed in Table III. It is evident that methods A and B for determining λ give nearly the same results.

Table III lists the value of the coefficient η, which agrees with the value determined previously by another method.[14]

C. For experiments D1, D2, and D3 the total rate of "inelastic" processes λ can also be determined from comparison of the yield of stars with visible prongs for experiments with different concentrations of C and O atoms, i.e., by the method which was used to determine λ for processes involving $p\mu$ atoms.[13]

The finally accepted averaged values of λ for experiments HDI and HDII are listed in the last line of Table III, and for experiments D1, D2, and D3, including a small correction for formation of $dd\mu$ molecules,—in Table IV.

II. INTERPRETATION OF EXPERIMENTAL DATA ON YIELD OF THE REACTION $d\mu + d \rightarrow p + t + \mu^-$

Table IV lists the complete set of existing data for the yields $Y_p(dd)$ for reaction (3), obtained both in our experiments (D1, D2, D3, and HD) and in other experiments.

Alvarez et al.[1] observed several cases of reaction (3) in a liquid-hydrogen chamber with an admixture of 4.3% deuterium. The yield shown in the table was calculated on the assumption that three events were observed. Fetkovich et al.[4] measured the yield of reaction (3) in a liquid-deuterium bubble chamber; here the hydrogen concentration was 5%. Doede[5] used a similar apparatus to obtain considerably better statistics for this reaction (the hydrogen concentration in deuterium in his experiment was 1%.)

The experimental yield of reaction (3) can be represented by the following expression:

$$Y_p(dd) = \frac{\lambda_{dd\mu}\rho_D}{\lambda\rho_0}\cdot\frac{1}{2}F(dd), \tag{9}$$

where $\lambda_{dd\mu}$ is the rate of formation of $dd\mu$ molecules for a liquid-deuterium density ρ_0 corresponding to a number of deuterons $N_0 = 4 \times 10^{22}$ cm^{-3}; λ is the total rate of "inelastic" processes:

$$\lambda = \lambda_0 + \lambda_{dd\mu}\rho_D/\rho_0 + \lambda_{pd\mu}\rho_H/\rho_0 + \lambda_Z\rho_Z/\rho_0,$$

where ρ_D, ρ_H, and ρ_Z are the densities of deuterium, hydrogen, and Z atoms; F(dd) is the probability of the nuclear reaction (3) in a $dd\mu$ molecule; the factor $^1/_2$ takes into account a second equally probable reaction channel in the $dd\mu$ molecule (reaction (4)).

For convenience in comparing the data of different experiments, we can convert from the yield $Y_p(dd)$ to the yield $Y_p^0(dd)$, which depends only on the deuterium density (i.e., the experimental yield is reduced to the conditions $\rho_H = 0$, $\rho_Z = 0$) and is defined by the following expression:

$$Y_p{}^0(dd) = \frac{\lambda_{dd\mu}\rho_D/\rho_0}{\lambda_0 + \lambda_{dd\mu}\rho_D/\rho_0}\cdot\frac{1}{2}F(dd). \tag{10}$$

Then the yield $Y_p^0(dd)$ defined in this way can be expressed in terms of $Y_p(dd)$:

$$Y_p{}^0(dd) = Y_p(dd)\frac{\lambda}{\lambda_0 + \lambda_{dd\mu}\rho_D/\rho_0}. \quad (11)$$

As we will see later, $\lambda_{dd\mu}$ is of the order of 10^5–10^6 sec^{-1}, and therefore for the conditions of the experiments with low deuterium density (the first five experiments in Table IV) we can write with good accuracy (of the order of several per cent, which is considerably less than the experimental errors)

$$Y_p{}^0(dd) = Y_p(dd)\lambda/\lambda_0. \quad (12)$$

For the experiments of Fetkovich et al.[4] and Doede,[5] in view of the low concentration of hydrogen and the nearly complete absence of Z admixtures, the yield $Y_p^0(dd)$ is very close to $Y_p(dd)$. For these experiments Table IV lists the maximum possible value of $Y_p^0(dd)$, which corresponds to $\lambda_{dd\mu} = 1 \times 10^5$ sec^{-1}.

We will discuss later whether all of the existing data on the yield $Y_p^0(dd)$ for reaction (3) can be reconciled.

1. Determination of $\lambda_{dd\mu}$ and F(dd) from Comparison of Yields for Different Densities

If we do not make any special assumptions regarding the dependence of $\lambda_{dd\mu}$ and F(dd) on deuterium density and the velocity of the dμ atom, we can find the values of $\lambda_{dd\mu}$ and F(dd) from Eq. (10). Approximation of all of the experimental data by this expression by the method of least squares gives

$$\lambda_{dd\mu} = (3.5 \pm 0.7) \cdot 10^6 \text{ sec}^{-1}, \quad F(dd) = 0.20 \pm 0.01. \quad (13)$$

Figure 8 shows the quantity $2Y_p^0(dd)\rho_0/\rho_D$ as a function of the relative density ρ_0/ρ_D with the parameters (13). As can be seen from the figure, by this formal means we can explain rather well the yields $Y_p^0(dd)$ obtained in all the experiments considered. However, this interpretation is extremely unsatisfactory for the following reasons.

First, according to the theory,[15, 16] the nuclear-reaction rate $\lambda_F(dd)$ in the ddμ molecule should be $\sim 10^{11}$ sec^{-1} (i.e., F(dd) = 1), if formation of μ molecules occurs with orbital momentum K = 0 in the excited vibrational state $\nu = 1$. In the case where the ddμ molecule is formed as the result of an E1 electric-dipole transition in a state with the momentum of the molecule K = 1, the nuclear-reaction rate will be smaller as the result of the existence of the centrifugal barrier. However, as Zel'dovich and Gershteĭn[15] have shown, even in this case $\lambda_F(dd)$ should be at least 10^8 sec^{-1}. Since F(dd) and $\lambda_F(dd)$ are connected by the relation (without taking into account hyperfine splitting in the ddμ system)

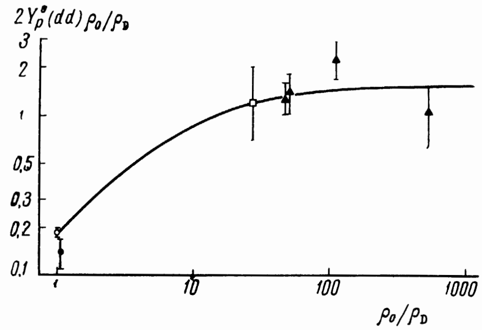

FIG. 8. The quantity $2Y_p^0(dd)\rho^0/\rho_D$ as a function of the relative density of deuterium ρ_0/ρ: ▲—present work, □—ref. 1, o—ref. 5, ●—ref 4. The smooth curve was obtained from Eq. (10) with the parameters (13).

$$F(dd) = \lambda_F(dd) / [\lambda_F(dd) + \lambda_0], \quad (14)$$

we can use the value of F(dd) from (13) to obtain the nuclear reaction rate $\lambda_F(dd) = (1.13 \pm 0.06) \times 10^5$ sec^{-1}, which differs from the theoretically expected value by three to six orders of magnitude.

In the second place, the value $\lambda_{dd\mu} = 3.5 \times 10^6$ sec^{-1} is excluded by the rather large experimental yield of reaction (1) observed under the conditions of liquid deuterium with small hydrogen admixtures.

Using the reaction (1) yields found experimentally by Fetkovich et al.[4] and Doede[5] and taking into account the effect of hyperfine structure of the pdμ molecule,[17] we can determine that under the conditions of liquid deuterium, $\lambda_{dd\mu} < 0.34 \times 10^6$ sec^{-1} with a probability of 90%.

2. Possibility of a Resonance Dependence of $\lambda_{dd\mu}$ on dμ-Atom Energy

In view of the fact that direct comparison of the reaction yields for different densities does not give reasonable values of the parameters $\lambda_{dd\mu}$ and F(dd), in the future we will assume, in accordance with theory, F(dd) = 1. The values of $\lambda_{dd\mu}$ found in this case from (9), converted to liquid-deuterium density ρ_0, are listed in Table IV. The average value of $\lambda_{dd\mu}$ for our experiments (first four lines in Table IV) in this case is

$$\lambda_{dd\mu} = (0.75 \pm 0.11) \cdot 10^6 \text{ sec}^{-1}.$$

At the same time the average value of $\lambda_{dd\mu}$ found from experiments with liquid deuterium (see the same table) is less by almost an order of magnitude, and for the ordinary mechanism of formation of μ molecules by an E1 electric-dipole transition (with transfer of the binding energy to the electron), according to the theoretical calculations of Zel'dovich and Gershteĭn[15] and Cohen et al.,[18] it is 0.04×10^6 sec^{-1}. To explain this difference let us consider the question of the possibility of a resonance dependence of the cross section for formation of μ molecules on dμ-atom energy.

Calculations of the levels of the ddμ molecule[15] lead to the conclusion that excited levels can exist in this system with a low binding energy (the level $K = 0$, $\nu = 1$, $E_{bind} = 40$ eV; and the level $K = 1$, $\nu = 1$, $E_{bind} = 7$ eV). As Zel'dovich[19] showed, existence in the μ molecule of a bound real or virtual level with energy near zero leads, in a collision of a low-energy dμ atom with a deuteron, to a resonance which can substantially increase the probability of formation of a ddμ molecule and of a nuclear reaction in flight. Rough estimates[19] show that this increase, in comparison with the ordinary rate of formation of μ molecules as the result of electric-dipole or electric-monopole transitions, can amount to a factor of U/E_{bind}, where U is the well depth of the ddμ molecule, 600 eV. In this case the rates of formation of μ molecules and of the nuclear reaction in flight can depend on the relative energy of the dμ atom.

For experiments D1, D2, and D3, and also for experiments with liquid deuterium, the energy of the dμ is simply the thermal energy at temperatures of 240 and 120°K, respectively. This is due to the fact that if a dμ atom is formed with an initial energy greater than thermal, and equal to ~ 1 eV,[13] the large cross section for elastic scattering of dμ atoms by deuterons results in rapid slowing down on the dμ atom to thermal energy. For experiments with low deuterium concentration (experiment HD and that of Alvarez et al.[1]) dμ atoms are formed mainly as the result of process (5) with an initial energy of 45 eV. Monte Carlo calculations with the known values of elastic-scattering cross section[12] give average dμ-atom energies for these experiments which exceed the thermal energies.

Table IV shows the energies of the dμ atom $\overline{E}_{d\mu}$ under the conditions of each experiment, averaged over a Maxwellian distribution and taken with respect to a stationary deuteron. In view of the very high nuclear reaction rate in the ddμ molecule, the ddμ-molecule formation rate and the rate of the nuclear reaction in flight (like the cross sections for these processes) cannot be determined individually in our experiments) If we understand $\lambda_{dd\mu}$ and $\sigma_{dd\mu}$ to mean the combined values of these quantities, we can write

$$\lambda_{dd\mu} = N_d \sigma_{dd\mu} \bar{v}_{d\mu}. \tag{15}$$

On the basis of the $\lambda_{dd\mu}$ values listed in Table IV and the known values of N_d and $\bar{v}_{d\mu} = (2\overline{E}_{d\mu}/M_d)^{1/2}$, we have determined the cross sections $\sigma_{dd\mu}$ from Eq. (15) (Table IV). Figure 9 shows these cross sections for different experiments as a function of dμ-atom energy. It can be seen from the figure that the simple law $\sigma \sim 1/E^{1/2}$ (broken line) is not satisfied; on the other hand, a resonance behavior of the cross section on dμ-atom energy is not excluded.

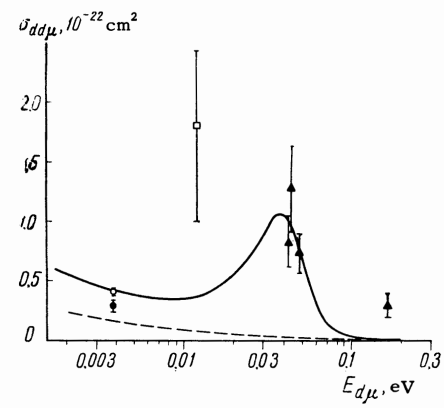

FIG. 9. Plot of $\sigma_{dd\mu}$ (combined cross section for formation of ddμ molecules and cross section for nuclear reaction in flight) as a function of the relative energy of the dμ atom: ▲–present work, □–ref. 1, ○–ref. 5, ●–ref. 4. The solid curve was calculated from formula (16), and the broken curve represents a $1/E^{1/2}$ law normalized to the theoretical value $\lambda_{dd\mu} = 0.04 \times 10^6$ sec^{-1}.

The experimental data can be roughly approximated by the following Breit-Wigner formula, which is valid for the case when the resonance-level is close to zero:[20]

$$\sigma_{dd\mu} = \frac{A}{(\overline{E}_{d\mu})^{1/2}} \frac{\gamma_e \Gamma_r}{(\overline{E}_{d\mu} - \varepsilon_0)^2 + {}^1/_4(\Gamma_r + \gamma_e (\overline{E}_{d\mu})^{1/2})^2}, \tag{16}$$

where Γ_r is the "inelastic" width of the level; γ_e and ϵ_0 are constants related to the "elastic" width and the resonance energy, respectively. The solid curve in Fig. 9 shows the dependence given by expression (16) with parameters $\Gamma_r = 0.012 \pm 0.009$ eV, $\epsilon_0 = 0.040 \pm 0.007$ eV, $\gamma_e = 0.073 \pm 0.050$ eV$^{1/2}$, which were found by the method of least squares ($\chi^2_{min} = 13.6$ for $\overline{\chi}^2 = 3$).

In view of the qualitative nature of the analysis carried out to confirm the resonance nature of the dd reaction catalysis and the parameters of the excited level, further experimental and theoretical work on this question is necessary.

In conclusion it is also necessary to clarify whether or not the difference in yields of reaction (3) for sharply differing values of deuterium density is the consequence of any other factors, related to the technique of investigation. The efficiency of observation of cases of this reaction in a diffusion chamber, of course, is considerably higher than in bubble chambers, since in the latter case one observes only the track of the proton, whose range is roughly 1 mm. However, the low background of extraneous radiation in bubble chambers and the practically complete agreement of the yields found by Fetkovich et al.[4] and Doede[5] allow us to conclude that the yields of this reaction have been rather accurately determined. The presence of Z admixtures in the liquid deuterium which have not been taken into account may decrease the true yield of reaction (3), but this would simultaneously lead to a decrease of the yield of reaction (1) under the experimental conditions employed in refs. 4 and 5, which is not observed.

A specific condition of experiments using liquid-deuterium bubble chambers is the presence near the point of stopping of the muon of gas bubbles with roughly ten times less deuterium density. As the result of the relatively low rate of μ-atomic processes ($\sim 10^6$ sec^{-1}), a fraction of the dμ atoms formed may enter a region of already grown gas bubbles, which will lead to a reduction of the yield of reaction (3). However, estimates which have been made show that this effect does not exceed a few per cent.

III. INTERPRETATION OF EXPERIMENTAL DATA OF THE REACTION $d\mu + p \rightarrow He^3 + \mu^-$

We determined the reaction (1) yield mainly for the purpose of obtaining information on the rate of formation of pdμ molecules and the nuclear-reaction rate in this μ molecule. Since the formation of μ molecules does not give directly observable effects, we have attempted to find the quantities of interest from comparison of the yields of reaction (1) for two sharply differing values of hydrogen density. More specifically, if we designate by Y_μ and Y'_μ the respective reaction (1) yields for hydrogen densities ρ_0 and ρ under the conditions of saturation of the exchange process (5), then

$$Y_\mu = \frac{\lambda_{pd\mu}}{\lambda_{pd\mu} + \lambda_0} F_\mu(pd), \tag{17}$$

$$Y_\mu' = \frac{\lambda_{pd\mu}\rho/\rho_0}{\lambda_{pd\mu}\rho/\rho_0 + \lambda_0} F_\mu(pd). \tag{18}$$

Here $\lambda_{pd\mu}$ is the rate of formation of μ molecules for density ρ_0, and F_μ (pd) is the probability of reaction (1) in a pdμ molecule. From Eqs. (17) and (18) we can find $\lambda_{pd\mu}$ and F_μ(pd), knowing the experimental yields Y_μ and Y'_μ. This means of determination assumes that the rate $\lambda_{pd\mu}$ is proportional to the hydrogen density and does not depend on the dμ-atom energy, and that the nuclear-reaction probability F_μ(pd) does not depend on hydrogen density.

1. Rate of Formation of pdμ Molecules

From Eqs. (17) and (18), taking into account the fact that in our experiments the quantity $\lambda_{pd\mu}\rho/\rho_0 + \lambda_0$ is equal to the combined rate λ of inelastic processes (see (7)), we obtain

$$\lambda_{pd\mu} = \lambda \frac{\rho_0}{\rho} \frac{Y_\mu'}{Y_\mu} - \lambda_0, \tag{19}$$

where ρ_0 is the liquid-hydrogen density corresponding to a number of protons $N_0 = 4.2 \times 10^{22}$ cm^{-3}; ρ is the hydrogen density in experiments HDI and HDII ($N = 1.2 \times 10^{21}$ cm^{-3}).

The yield of reaction (1) for liquid hydrogen was determined by Schiff,[3] who found that $Y_\mu = (2.64 \pm 0.35) \times 10^{-2}$. As noted by the author, the greatest uncertainty in this result is introduced by our inaccurate knowledge of the deuterium concentration in natural hydrogen. Doede[5] has measured the reaction (1) yield for a deuterium concentration in a liquid-hydrogen chamber of 2.2×10^{-3}%. Using the results of these two studies, we have obtained $Y_\mu = (2.84 \pm 0.25) \times 10^{-2}$. If we take for the value of Y'_μ the number in the first line of Table III and for λ the value in the last line of the same table, and average over experiments HDI and HDII, we find[3)] $\lambda_{pd\mu} = (1.8 \pm 0.06) \times 10^6$ sec^{-1} (for $N_0 = 4.2 \times 10^{22}$ cm^{-3}), $\lambda'_{pd\mu} = (0.052 \pm 0.015) \times 10^6$ sec^{-1} (for $N = 1.2 \times \times 10^{21}$ cm^{-3}).

3)The rate of formation of pdμ molecules can also be estimated directly, without recourse to determination of the muon yield in reaction (1), from determination of the total rate of inelastic processes λ in experiment HDII. Proceeding from the yield of events with Auger electrons (Table III), we can show that the quantity $\lambda_Z' c_Z$ amounts to at least 33% of the total rate λ, with a probability of 90%. Then we have, with a probability of 90%, $\lambda'_{pd\mu} < 0.2 \times 10^6$ sec^{-1} and $\lambda_{pd\mu} < 7 \times 10^6$ sec^{-1}, which is consistent with the determination of $\lambda_{pd\mu}$ from the yield of conversion muons.

Table V

Source	$10^{-6}\ \lambda_{pd\mu}$, sec^{-1}	Number of deuterons per cm^3	$10^{-6}\ \lambda$, sec^{-1}	$10^{-6}\ \bar{v}_{d\mu}$, cm/sec
Ref. 7	5.8±0.3	$3.1 \cdot 10^{20}$	6.3	3.0
Ref. 8	6.82±0.25	$2.2 \cdot 10^{20}$	10	4.3
Present work	1.8±0.6; 0.052±0.015 *	$0.7 \cdot 10^{20}$	1.5	4.0

*Data for $N = 1.2 \times 10^{21}$ cm^{-3}, the remainder for $N_0 = 4.2 \times 10^{22}$ cm^{-3}.

The result obtained, as we can see from Table V, is roughly two to three times lower than the experimental results for liquid hydrogen.[7, 8] This difference can be explained by the fact that the rate of formation of molecules is not strictly proportional to the density. This conclusion is reached by considering the mechanism of formation of pdμ molecules.[15, 18] For example, if we take into account that the binding energy of the μ molecule (95 eV) is transferred to an electron bound to an H_2 molecule, and not to an atomic electron, then the rate of formation of pdμ molecules, which is proportional to the cube of the effective charge of the electron, is increased by 1.7 times, since $Z_{eff} = 1.19$. In this connection, for high densities of matter, states are possible in which two or more molecules are at a distance comparable with the distance between the nuclei in the H_2 molecule. This can lead to a large increase in the effective charge. At the same time, the Coulomb screening of the proton field is increased, which also increases the rate of formation of pdμ molecules (complete screening of the Coulomb field leads to an increase in the rate by roughly three times[15]).

Another possible cause of the difference in the rates of formation of pdμ molecules in liquid and gaseous hydrogen could be the strongly differing relative velocities of the dμ atoms (in analogy with the formation of ddμ molecules). However, calculations made by the Monte Carlo method and utilizing the results of our earlier work,[12] have shown that the average relative velocities of dμ atoms in the experiments being considered are practically the same (see Table V).

2. Nuclear-Reaction Rate in the pdμ Molecule

The nuclear factor F_μ(pd), found from Eqs. (17) and (18), is $F_\mu(pd) = (3.5 \pm 0.8) \times 10^{-2}$. On the basis of this nuclear factor, and assuming the population of the pdμ-molecule hyperfine structure sublevels[15, 21] to be statistical, can determine the nuclear reaction rate λ_F(pd) in this molecule, (see, for example, formula 16.10 from Gershteĭn's dissertation[21]). If we take into account the muon conversion coefficient,[4)] this rate is

$$\lambda_F(pd) = \lambda_\gamma + \lambda_\mu = (0.6 \pm 0.3) \cdot 10^6 \text{ sec}^{-1}.$$

This value agrees with the directly measured value of λ_F(pd),[7] which was determined from the time distribution of γ rays in reaction (2) and turned out to be

$$\lambda_F(pd) = (0.305 \pm 0.010) \cdot 10^6 \text{ sec}^{-1}.$$

It must be noted that, according to the theory,[15, 18] nuclear reaction (1) in the region of the hydrogen and deuterium densities being considered, should occur from the S state of the pdμ molecule with the same population of the hyperfine-structure sublevels. In this case the nuclear reaction rate λ_F(pd) should not depend on the density.

CONCLUSION

The experimental studies which have been performed at the present time of the muon catalysis of nuclear reactions confirm the main features of current theoretical ideas of the mechanisms of μ-atom processes and nuclear reactions in μ molecules of hydrogen. This refers primarily to reaction in the pdμ molecule. In particular, the data of the present study agree with the already established fact that the nuclear reaction rate in a pdμ molecule is less than the rate of formation of this molecule.

The combination of all the experimental data relating to muon catalysis of dd reactions allows us to conclude that the theoretical deduction of a high nuclear reaction rate in the ddμ molecule in comparison with the muon decay rate is consistent with the experiments. The theoretical predictions for the relative yields of the different reaction channels in the dd molecule are also well fulfilled. However, the rate of formation of the ddμ molecule found in the present study exceeds by more than an order of magnitude the theoretical value calculated on the assumption of the ordinary mechanism of formation of μ molecules by an electric-dipole transition. As the analysis shows, a possible explanation of this fact is the idea of a

4)Since the γ-ray yield from reaction (2) in liquid hydrogen is $Y_\gamma = 0.14 \pm 0.02$,[7] and $Y_\mu = (2.84 \pm 0.25) \times 10^{-2}$ (see above), the muon conversion coefficient is $Y_\mu / Y_\gamma = \lambda_\mu / \lambda_\gamma = 0.20 \pm 0.04$, where λ_μ and λ_γ are the respective rates of reactions (1) and (2).

resonance nature of the dependence of the cross section for formation of ddμ molecules on dμ-atom energy, in connection with the existence in the ddμ system of energy levels close to zero. Further experimental and theoretical study of this question is necessary to confirm the resonance nature of dd-reaction catalysis.

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