PLANE HALL EFFECT IN FERROMAGNETIC METALS

VU-DINH KY

Moscow State University

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A new type of galvanomagnetic effect in ferromagnetic metals is considered. This effect consists of the fact that in a sample which is situated in a magnetic field and which carries a current the resultant transverse electric field lies in the same plane as the direction of the current and of the magnetic field. Therefore it can be named the plane Hall effect. Experimentally the plane Hall effect has been observed in a bulky nickel sample. The value of the constant for this effect in nickel is given. A theory is presented of the plane Hall effect due to the spin-orbit interaction for electrons in ferromagnetic metals. The density matrix method and the effective mass approximation are utilized. A quadratic dependence of the effect on the magnetization is obtained and this is in agreement with the phenomenological theory. It is shown that at low temperatures the plane Hall effect constant is proportional to the specific resistance.

1. INTRODUCTION

In contrast to the well known Hall effect[1)] when the transverse electric field appearing in the sample is perpendicular to the magnetic field and to the current, in the case of the new galvanomagnetic effect discussed in this paper the resultant electric field which is transverse with respect to the current lies in the same plane as the directions of the current and of the magnetic field. In the case when the sample is in the form of a plate lying in the xy plane, when a current is sent through it in the x direction and the resultant transverse field is measured in the y direction the ordinary Hall effect occurs when the magnetic field is perpendicular to the plane of the sample. But if the magnetic field lies in the plane of the sample we have the new galvanomagnetic effect referred to above, which we can call the plane Hall effect.

Starting from the phenomenological theory it can be shown that for an isotropic sample the following relation holds

$$\varepsilon_y = E_y / j_x = \beta B^2 \sin\theta_0 \cos\theta_0, \qquad (1)$$

where ϵ_y is the resultant transverse electric field in the direction of the y-axis expressed per unit current density, θ_0 is the angle between j and B, B is the magnetic induction, β is a constant.

From the theory of galvanomagnetic effects in metals[1-3] it follows that in weak fields $\beta \sim \tau/nm^*$ (τ is the relaxation time, n is the density of conduction electrons, m* is the effective mass). This effect can be easily observed in semiconductors (n is small) and in metals at low temperatures (τ is large).[4-6] At room temperature the constant β is too small in metals and the effect is not observed in practice.

However, it was found that at room temperature in ferromagnetic metals and in alloys the plane effect is sufficiently large—its magnitude is of the same order as for the ordinary Hall effect. In particular, in permalloy alloys the plane Hall effect exceeds the ordinary effect by a factor of several fold. Apparently here, just as in other galvanomagnetic phenomena, in ferromagnetics at temperatures considerably lower than the Curie temperature, the effect is mainly due to the magnetization.

On the basis of the phenomenological theory[8] the following expression was obtained for the plane Hall effect in an isotropic single domain ferromagnetic sample

$$\varepsilon_y = P_s M^2 \sin\theta \cos\theta, \qquad (2)$$

where M is the spontaneous magnetization, θ is the angle between j and M, P_S is a constant.

The plane Hall effect in ferromagnetic metals has received little study so far both theoretically and experimentally. In the present paper experimental results are quoted and a quantum theory of the plane Hall effect in ferromagnetic metals is given.

[1)] Here and subsequently we shall mean by "Hall effect" the well known Hall effect when the magnetic field is perpendicular to the plane of the sample.

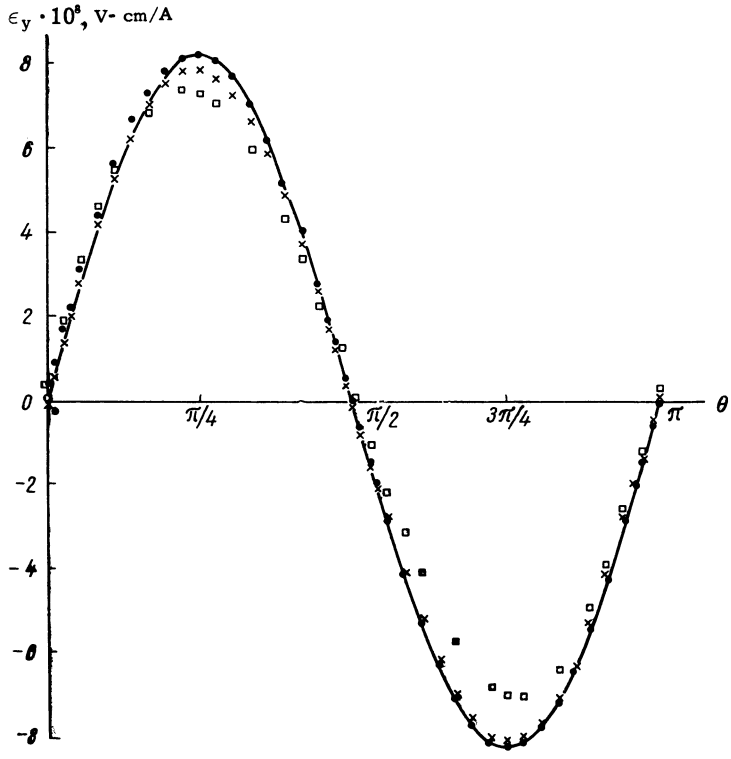
2. EXPERIMENT

The method of making measurements of the ordinary Hall effect and the plane Hall effect is basically the same. But in the latter case the magnetic field is parallel to the plane of the sample and, consequently, owing to the smaller magnetization factor, saturation is attained in smaller fields.

Obviously the plane Hall effect can be easily investigated in thin ferromagnetic films[7-9] in which the vector M lies largely in the plane of the film. However, the plane Hall effect occurs not only in films, but can also be observed in bulky samples. In order to do this in the present work we have prepared samples of pure nickel (99.99%) of dimensions $32 \times 8 \times 0.1$ mm. The probes for determining the transverse potential difference were made by spot welding. With the aid of a special holder the samples were placed into the homogeneous field of an electromagnet in such a manner that it was guaranteed that the field and the plane of the sample were parallel. The holder enabled us to vary the direction of the field in the plane of the sample with the angle of rotation being capable of being determined with an accuracy of up to 0.5°. Measurements were carried out on five samples. The scatter in the experimental data lay within the limits of error for the measurement.

The transverse potential difference was determined by the potentiometer method. With the aid of a photo-optical amplifier it was possible to obtain a sensitivity of 10^{-8} V. Such accuracy was not always needed, since the plane effect in nickel is quite large (by a factor of several fold greater than the ordinary effect). In the present case when the current through the sample was ~ 1 A the maximum resultant transverse potential difference was of the order of tens of microvolts (for $\theta = \pi/4$). It should be noted that in the measurement of the plane Hall effect there is no distortion due to the thermomagnetic Ettingshausen effect which is present in the measurement of the ordinary Hall effect when the field is perpendicular to the plane of the sample.

As regards the $\Delta\rho/\rho$ effect (i.e., the change of resistance in a magnetic field) its influence can be neglected. The point is that the initial difference of potential due to the asymmetry in the position of the transverse probe can vary due to the $\Delta\rho/\rho$ effect. In our experiment this initial difference of potential v is of the order of a microvolt for a current of ~ 1 A, and, therefore, its variation is not greater than a few hundredths of a microvolt, since it is well known that in nickel $\Delta\rho/\rho \approx 2\%$. Moreover, this distortion can be eliminated by measuring for each θ the values of $v(-\theta)$ and $v(+\theta)$, and then calculating by means of the formula $v(\theta) = \frac{1}{2}[v(+\theta) - v(-\theta)]$, since the $\Delta\rho/\rho$ effect is the same for $+\theta$ and $-\theta$ for a given value of the intensity of the field. The current was kept constant with a high degree of accuracy, since its source was a high-capacity bank of storage batteries. The sample was shielded from air currents and measurements were carried out under complete thermal equilibrium.



Dependence of ϵ_y on θ for a bulky nickel sample in the following fields: □ – 500 Oe, × – 2000 Oe, ● – 5000 Oe, solid curve is the theoretical curve.

The figure shows the dependence of ϵ_y on θ for a constant field. It can be seen that the plane Hall effect is an even effect. It can be seen from the diagram that relation (2) is the better satisfied the higher the field, i.e., the closer the state of the sample is to saturation. The maximum value of ϵ_y is $\approx 8 \times 10^{-8}$ V-cm/A. It should be noted that for nickel under the same conditions (in the saturated state) in the case of the ordinary Hall effect $\epsilon_y \approx 4 \times 10^{-8}$ V-cm/A, i.e., smaller by a factor of two. From the graph one can determine the constant P_S for nickel. It turns out to be equal to $\sim 7 \times 10^{-13}$ V-cm/AG^2.

3. THEORY

The ferromagnetic Hall effect was studied theoretically in a number of papers.[10-14] It was shown that the spin-orbit interaction for electrons participating in the conduction process is the cause of the ferromagnetic Hall effect. The scattering of electrons by inhomogeneities of various types in the lattice of the ferromagnetic was considered (by impurities, by phonons and by magnetic inhomogeneities).

In the papers cited above the density matrix method was utilized. It is convenient first to evaluate the matrix elements of the operators in the H-representation (cf., below (3)) ($H = H_0 + H^{s.o}$; $H^{s.o}$ is the operator for the spin-orbit interaction for the electrons). Then in order to obtain the dependence of the kinetic coefficients on the magnetization the various matrix elements were expanded in series in terms of $H^{s.o}$. In calculations involving the ferromagnetic Hall effect proportional to the magnetization such expansions were usually restricted to terms of the first order in $H^{s.o}$. It is evident that for the plane Hall effect in ferromagnetic metals which depends quadratically on the magnetization (cf., (2)) one must take into account terms of the second order in $H^{s.o}$ in the expansions indicated above.

We shall not discuss the plane Hall effect due to a magnetic field, but only the ferromagnetic plane Hall effect due to the magnetization. In doing so we shall take into account only the scattering of electrons by impurities.

Following Luttinger[10] we shall take the initial Hamiltonian in the form

$$H^T = H_0 + H^{s.o} + H' + H_E,$$

where

$$H_0 = \frac{p^2}{2m} + U, \quad H^{s.o} = (4m^2c^2)^{-1}([\sigma\nabla U]\mathbf{p}),$$

$$H' = \sum_{i=1}^{N} \varphi(\mathbf{r} - \mathbf{R}_i), \quad H_E = -eE_\alpha r_\alpha, \qquad (3)^*$$

U is the periodic field in the lattice, m is the electron mass, $H^{s.o}$ is the spin-orbit interaction, H′ is the interaction with the impurities.

A system of units is utilized in which $\hbar = 1$. Setting $E_\alpha = E_\alpha^{(0)} e^{st}$ we have the complete density matrix in the form $\rho^T = F + fe^{st}$, where F is the equilibrium density matrix, fe^{st} is the deviation from it of the first order in the electric field, $s > 0$ is the adiabatic parameter (in the final result $s \to 0$), t is the time.

Since the plane Hall effect can be obtained in the lowest order approximation with respect to λ (λ is a dimensionless parameter the smallness of which is determined by the smallness of H′) it is sufficient to take into account the terms in the expansion $f_l^{(-2)}$ and $f_l^{(-1)}$ in the expression for the average velocity of the electrons $\bar{v}_\beta = \mathrm{Sp}(fv_\beta)$ ($f_l^{(-2)}$ and $f_l^{(-1)}$ are respectively proportional to λ^{-2} and λ^{-1}, the subscript l denotes $(n, \mathbf{k})$).

Expanding v_β^l, $f_l^{(-2)}$ and $f_l^{(-1)}$ in series with respect to $H^{s.o}$ and substituting in the expression for $\bar{v}_\beta$ we shall find for the part of $\bar{v}_\beta$ which is of interest to us in the case of a crystal with a center of inversion:

$$\bar{v}_\beta^{(2)} = \bar{v}_\beta^{(-22)} + \bar{v}_\beta^{(-12)}, \qquad (4a)$$

$$\bar{v}_\beta^{(-22)} = \sum_l (f_l^{(-20)} v_\beta^{(2)l} + f_l^{(-22)} v_\beta^{(0)l}), \qquad (4b)$$

$$\bar{v}_\beta^{(-12)} = \sum_l (f_l^{(-10)} v_\beta^{(2)l} + f_l^{(-12)} v_\beta^{(0)l}), \qquad (4c)$$

$v_\beta^{(0)l}$, $f_l^{(-20)}$, $f_l^{(-10)}$ are zero order quantities, $v_\beta^{(2)l}$, $f_l^{(-22)}$, $f_l^{(-12)}$ are quantities of the second order with respect to $H^{s.o}$. Noting that for a crystal with a center of inversion $\epsilon_l = \epsilon_l^{(0)} + \epsilon_l^{(2)}$, and utilizing the well known formula $v_\beta = \partial\epsilon_l / \partial k_\beta$ (we shall assume, as in [10-11], $\omega_{ll'}^2 = (\epsilon_l^{(10)} - \epsilon_{l'}^{(0)})^2 \cong \Delta^2$, where Δ^2 is some average value) we obtain with the aid of standard perturbation theory (under the condition that $H^{s.o}$ is a small perturbation):

$$\varepsilon_l^{(2)} = \frac{1}{2} B_l [(\mathbf{Mk})^2 - M^2 k^2], \qquad (5a)$$

$$v_\beta^{(2)l} = B_l [M_\beta (\mathbf{Mk}) - M^2 k_\beta], \qquad (5b)$$

where

$$B_l = \frac{\pi e^2 q_l'}{3m^5 c^4 M_s^2 \Delta^2},$$

$$q_l' = \int \omega_{n\mathbf{k}}^{(0)*}(\mathbf{r}) \frac{\partial U}{\partial r_\gamma} \frac{\partial q}{\partial r_\gamma} \omega_{n\mathbf{k}}^{(0)}(\mathbf{r})\, d\mathbf{r}_0,$$

M_S is the maximum value of M at $T = 0°\,K$, q is the charge density determining the potential U.

In the lowest approximation the kinetic equation has the form:[10]

$$eE_\alpha^{(0)} \frac{\partial F_l(\varepsilon_l)}{\partial k_\alpha} + \sum_{l'} L_{ll'}^{(0)} (f_l^{(-2)} - f_{l'}^{(-2)}) = 0. \qquad (6)$$

Expanding $\partial F_l(\epsilon_l)/\partial k_\alpha$, $f_l^{(-2)}$, $L_{ll'}^{(0)}$ in series in $H^{s.o}$, we obtain from (5a), (5b), and (6)

*$[\sigma\nabla U] \equiv \sigma \times \nabla U$.

$$eE_\alpha^{(0)}F_l'v_\alpha^{(0)l}+2\pi N\sum_{l'}'|\varphi_{\mathbf{kk}'}|^2\delta(\omega_{ll'})\,(f_l^{(-20)}-f_{l'}^{(-20)})=0, \tag{7}$$

$$eE_\alpha^{(0)}\{F_l'B_l[M_\alpha(\mathbf{Mk})-M^2k_\alpha]-{}^1/_2B_lF_l''([\mathbf{kM}])^2v_\alpha^{(0)l}$$
$$-{}^1/_3k_F^2A_l^2F_l'([\mathbf{kM}])^2v_\alpha^{(0)l}\}+2\pi N\sum_{l'}|\varphi_{\mathbf{kk}'}|^2\delta(\omega_{ll'})$$
$$\times\,(f_l^{(-22)}-f_{l'}^{(-22)})=0, \tag{8}$$

where

$$F_l'=\partial F_l(\varepsilon_l^{(0)})/\partial\varepsilon_l^{(0)},\qquad F_l''=\partial F_l'(\varepsilon_l^{(0)})/\partial\varepsilon_l^{(0)}.$$

In the derivation of (7) and (8) we have utilized the expansion for $\varphi_{ll'}$:[10]

$$\varphi_{ll'}=\varphi_{\mathbf{kk}'}[\delta_{nn'}+(k_\mu'-k_\mu)J_\mu^{nn'}(\mathbf{k})+\ldots]. \tag{9}$$

The expression for $J_\mu^n(\mathbf{k})$ (n = n′) is given in [10-11]:

$$J_\mu^n(\mathbf{k})=iA_l[\mathbf{kM}]_\mu,\quad A_l=\pi e^2q_l/3m^3c^2M_s^2\Delta^2,$$
$$q_l=\int\omega_{n\mathbf{k}}^{(0)*}(\mathbf{r})\,q(\mathbf{r})\,\omega_{n\mathbf{k}}^{(0)}(\mathbf{r})\,d\mathbf{r}_0$$

(here the integral is taken over the volume of an elementary cell).

The solution of (7) in the effective mass approximation has the well-known form:[10]

$$f_l^{(-20)}=-\tau_0eE_\alpha^{(0)}F_l'v_\alpha^{(0)l}, \tag{10}$$

τ_0 is the relaxation time determined by scattering by impurities. Similarly we obtain $f_l^{(-22)}$ in the effective mass approximation; substituting it into (4b) we obtain with the aid of (5b) and (10) the transverse conductivity

$$\sigma_{yx}^{(-22)}=\frac{ne\bar v_y^{(-22)}}{E_x^{(0)}}=-ne^2\tau_0\Big\{2\sum_l B_lF_l'k_xv_x^{(0)l}$$
$$+\frac{2}{3}\sum_l A_l^2F_l'(k_xv_x^{(0)l})\,(k_yv_y^{(0)l})+\sum_l B_lF_l''(k_xv_x^{(0)l})$$
$$\times\,(k_yv_y^{(0)l})\Big\}M_xM_y. \tag{11}$$

We proceed to the next approximation in λ. The equation for $f_l^{(-1)}$ has the form:[10]

$$\sum_{l'}L_{ll'}^{(1)}(f_l^{(-2)}-f_{l'}^{(-2)})+\sum_{l'}L_{ll'}^{(0)}(f_l^{(-1)}-f_{l'}^{(-1)})=0, \tag{12}$$

$$\times\,L_{ll'}^{(1)}=2\pi N\delta(\omega_{ll'})\sum_{l''}\Big[\frac{\varphi_{ll'}\varphi_{l'l''}\varphi_{l''l}}{d_{ll''}^-}+\text{c.c.}\Big], \tag{13}$$

$d_{ll'}^-=\omega_{ll'}$ —is, where $\omega_{ll'}=\epsilon_l^{(0)}-\epsilon_{l'}^{(0)}$. Expanding $L_{ll'}^{(1)}$ and $f_l^{(-1)}$ in series in terms of $H^{s.o}$ and substituting into (12) we obtain

$$\sum_{l'}L_{ll'}^{(10)}(f_l^{(-20)}-f_{l'}^{(-20)})+\sum_{l'}L_{ll'}^{(00)}(f_l^{(-10)}-f_{l'}^{(-10)})=0, \tag{14a}$$

$$\sum_{l'}L_{ll'}^{(00)}(f_l^{(-11)}-f_{l'}^{(-11)})+\sum_{l'}L_{ll'}^{(11)}(f_l^{(-20)}-f_{l'}^{(-20)})=0, \tag{14b}$$

$$\sum_{l'}L_{ll'}^{(10)}(f_l^{(-22)}-f_{l'}^{(-22)})+\sum_{l'}L_{ll'}^{(00)}(f_l^{(-12)}-f_{l'}^{(-12)})$$
$$+\sum_{l'}L_{ll'}^{(12)}(f_l^{(-20)}-f_{l'}^{(-20)})+\sum_{l'}L_{ll'}^{(02)}(f_l^{(-10)}-f_{l'}^{(-10)})=0, \tag{14c}$$

where $f_l^{(-10)}$ and $L_{ll'}^{(10)}$ are of the first order, $f_l^{(-11)}$ and $L_{ll'}^{(11)}$ are of the first order in terms of $H^{s.o}$, etc.

As is well known, from (14b) one can find the first nonvanishing contribution to the ferromagnetic Hall effect. We have to solve (14a) and (14c). We first note that from (9) and (13) we can easily obtain

$$L_{ll'}^{(10)}=\frac{2n_0}{(2\pi)^2}\frac{\bar\varphi^3}{\Omega}\delta(\omega_{ll'})C_1,$$
$$L_{ll'}^{(12)}=-i\frac{2n_0}{(2\pi)^2}\frac{\bar\varphi^3}{\Omega}\delta(\omega_{ll'})C_2^{\mu\nu}k_\nu J_\mu^n(\mathbf{k}'), \tag{15a}$$

where

$$C_1=\mathrm{P}\int\frac{d\mathbf{k}'}{\omega_{\mathbf{kk}'}},\qquad C_2^{\mu\nu}=\mathrm{P}\int\frac{ik_\mu'J_\nu^n(\mathbf{k}')}{\omega_{\mathbf{kk}'}}d\mathbf{k}',\qquad n_0=\frac{N}{\Omega}, \tag{15b}$$

P is the principal value of the integral. From (14c) we obtain an expression for $f_l^{(-10)}$. Substituting it into (14c), taking into account (15a) and (10) we obtain $f_l^{(-12)}$ in the effective mass approximation. Proceeding in this manner we obtain

$$\sigma_{yx}^{(-12)}=\frac{ne\bar v_y^{(-12)}}{E_x^{(0)}}=\frac{ne^2\tau_0\bar\varphi}{(2\pi)^3}\Big\{8C_1\sum_l B_l'F_l'k_xv_x^{(0)l}$$
$$+\frac{8C_1}{3}k_F^2\sum_l A_l^2F_l'(k_xv_x^{(0)l})\,(k_yv_y^{(0)l})$$
$$+2C_1\sum_l B_lF_l''(k_xv_x^{(0)l})\,(k_yv_y^{(0)l})$$
$$+\frac{2C_2}{a_0}a_2\sum_l A_l^2F_l'k_xv_x^{(0)l}\Big\}M_xM_y, \tag{16}$$

where

$$C_2=\mathrm{P}\int\frac{(k_x')^2}{\omega_{\mathbf{kk}'}}d\mathbf{k}',\qquad a_0=\int\delta(\omega_{ll'})\,d\mathbf{k}'$$
$$a_2=\int(k_xv_x^{(0)l})\,\delta(\omega_{ll'})\,d\mathbf{k}'.$$

Utilizing relations (11) and (16) we can write

$$\sigma_{yx}=(\sigma_{yx}^{(-22)}+\sigma_{yx}^{(-12)})=Cne^2\tau_0M_xM_y.$$

Since σ_{yx} is very small compared to ρ^{-1} (ρ is the specific resistance), then under the conditions $j_x = j$ and $j_y = 0$ we have $E_y=-\rho^2\sigma_{yx}j$. Taking into account that $M_x = M\cos\theta$ and $M_y = M\sin\theta$, we obtain from (11) and (16) again the phenomenological relation (2), with $P_s=-Cne^2\tau_0\rho^2$.

From (11) and (16) it can be seen that in the expression for C the terms containing A_l are determined by the correction of the second order in $H^{s.o}$ to the second term of the kinetic equation, i.e., to the scattering potential, while terms containing B_l are determined by the correction of the second order in $H^{s.o}$ to the first term of the kinetic equation.

It is well known that the scattering of electrons by impurities plays an essential role for the conduction process only at low temperatures when one can take $\rho \approx \rho_{res}$. This means that the quantity P_s will be proportional to ρ_{res} at low temperatures. Such a result was also obtained for the ferromagnetic Hall constant R_s.[10]

Just as in the case of the ferromagnetic Hall effect[10-14] it is difficult to calculate the quantity P_s from (11) and (16) since the form of the Bloch electron wave functions and the electronic structure in ferromagnetic metals is not accurately known. However, one can compare the values of the plane and the ordinary Hall effect. The point is that the plane Hall effect was obtained in the approximation which is lower by one order in λ and higher by one order in $H^{s.o}$ than the ordinary Hall effect. This can be seen from the expression for the first nonvanishing term for both effects (formula (11) in this paper and formula (4.17) in [10]). Therefore, we can take the ratio of the magnitudes of the plane and the ordinary Hall effect to be approximately of the same order as the ratio of $H^{s.o}$ to $\overline{\varphi}$. We note that for nickel $H^{s.o} \approx 10^{-13}$ erg;[10] if we take for $\overline{\varphi}$ a value of 10^{-14}–10^{-15} erg as in [11], then we can see that the value of the plane Hall effect obtained in this paper exceeds the value of the ordinary effect by approximately a factor of two.

It should be noted that the above calculations can be generalized to the case of scattering both by impurities and by phonons by means of the method described in [12].

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