PERTURBATION OF CHARGED-PARTICLE DENSITY AT LARGE DISTANCES FROM A BODY MOVING RAPIDLY IN A PLASMA IN THE PRESENCE OF A MAGNETIC FIELD

V. V. VAS'KOV

Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 17, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 50, 1124-1132 (April, 1966)

An analytic expression is obtained for the perturbation δN of ion (electron) density at large distances ($r \gg 2\pi\rho_H V_0/v_i$) from a body moving with high velocity V_0 in a collisionless plasma ($V_0 \gg v_i$) in the presence of a constant external magnetic field H_0. The angular dependence of $\delta N(r, \theta)$ is calculated in the (V_0H_0) plane. In the direction normal to this plane, δN decreases rapidly within a distance of the order of the mean Larmor radius ρ_H of the ion if $\rho_H > R_0$ (R_0 is the radius of the body). The result is compared with the perturbation of the plasma due to the body without allowance for the influence of the magnetic or electric field on the ion motion.

We consider the perturbation δN of the ion (electron) density in the case of a rapidly moving body whose velocity V_0 is much larger than the average thermal velocity of the ions, but much smaller than the thermal velocity of the electrons:

$$a_0 = V_0 / v_i \gg 1. \quad (1)$$

We note that in the region considered by us (formula (4)) the plasma is quasineutral and the ion and electron densities are equal[1)]. Since the ions do not move in a direction normal to the magnetic field H_0, the entire perturbation should be concentrated in a layer adjacent to the (V_0H_0) plane, whose thickness is determined by the mean Larmor radius of the ion $\rho_H = v_i/\Omega$ ($\Omega = eH_0/Mc$ is the ion Larmor angular frequency). The problem of determining δN can therefore be reduced in fact to a planar problem, i.e., to the calculation of the perturbation integrated in the direction normal to the (V_0H_0) plane. Since the perturbation propagates along H_0 with velocity of the order of v_i, the angle interval in which all of $\delta N(r, \theta)$ is in fact concentrated is given by the inequality (see Fig. 1)

$$|\theta| \lesssim \theta_0 = a_0^{-1} \sin\alpha. \quad (2)$$

The quantity $\delta N(r, \theta)$, which depends on two variables, denotes the perturbation of the ion (electron) density, averaged in the manner indicated above, α is the angle between the vectors V_0 and H_0, r is a two-dimensional vector in the (V_0H_0) plane, and θ is the angle between $-V_0$ and r (see Fig. 2). The x axis on Fig. 2 is directed along H_0, and the (xy) plane coincides with (V_0H_0). The z axis is perpendicular to the (V_0H_0) plane, and the angle α is chosen such that $\sin\alpha > 0$.

Since the body absorbs SN_0 particles per unit path, we have the following order-of-magnitude relation:

$$\delta N(r, \theta) \approx \begin{cases} -SN_0/2r\theta_0, & |\theta| < \theta_0 \\ 0, & |\theta| > \theta_0 \end{cases}. \quad (3)$$

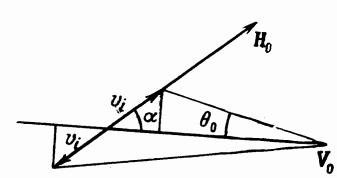

FIG. 1.

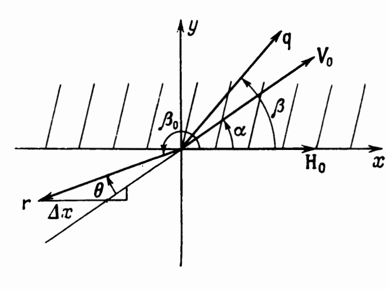

FIG. 2.

[1)]Since the electrons with large velocities have a Boltzmann spatial distribution, it is sufficient to consider the perturbation of the ion density.

Here S is the cross section area of the body in a plane normal to $\mathbf{V}_0$, and N_0 is the unperturbed ion (electron) density.

The ion can be regarded as moving on the average along $\mathbf{H}_0$ only over a distance $r \gg 2\pi\rho_H a_0$. However, as follows from the qualitative formula (3), to satisfy the condition $\delta N/N_0 \ll 1$ it is necessary to satisfy the stronger inequality

$$r \gg \rho_H a_0 / \sin\alpha = V_0 / \Omega \sin\alpha. \quad (4)$$

On the other hand, collisions can be neglected only over distances $r \ll V_0/\nu$ (ν is the effective number of ion collisions with ions and with neutral particles). These inequalities are not contradictory, since $\Omega \gg \nu$ always in the ionosphere.

We now calculate $\delta N(r, \theta)$. We use here the expression for the Fourier component N_q of the ion-density perturbation δN, obtained in [1,2] for a constant external magnetic field $\mathbf{H}_0$ with allowance for the electric field in the quasineutrality region. It was assumed in [1,2] that $v_e \gg v_i$, and scattering of the ions by the electric field near the body was neglected compared with the absorption and neutralization of the ions upon colliding with the body itself.

To find δN it is necessary to evaluate the integral

$$\delta N = \frac{1}{(2\pi)^3} \int N_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} d^3q. \quad (5)$$

For large r, the only points of importance to the integral (5) are those on the real axis at which the continuity of N_q or its derivatives is violated. As follows from [1], N_q has a singularity of this type if $q_{\|} = \mathbf{q}\cdot\mathbf{H}_0/H_0 = 0$ and $\mathbf{q}\cdot\mathbf{V}_0 = 0$. In this case, i.e., under the following condition, which is equivalent to (4),

$$\mathbf{q}\mathbf{V}_0/\Omega \ll 1, \quad q_{\|}v_i/\Omega \ll 1, \quad qR_0 \ll 1, \quad (6)$$

the formula for N_q simplifies and reduces to the form

$$N_{\mathbf{q}} = -\frac{SN_0a_0}{|q_{\|}|}\frac{Q(a)L(b)}{2 + iaQ(a)L(b)},$$

$$Q(a) = \left[\sqrt{\pi} + 2i\int_0 e^{t^2}dt\right]e^{-a^2}, \quad L(b) = e^{-b}I_0(b),$$
$$a = \mathbf{q}\mathbf{V}_0/|q_{\|}|v_i, \quad b = q_\perp^2\rho_H^2/2, \quad q_{\|} = \mathbf{q}\mathbf{H}_0/H_0, \quad (7)$$

$q_\perp$ is the component of the vector q perpendicular to $\mathbf{H}_0$, and I_0 is a Bessel function of imaginary argument.

It follows from (7) that N_q is analytic in q_z (the z axis is normal to the $(\mathbf{V}_0\mathbf{H}_0)$ plane). Consequently, the perturbation δN decreases exponentially in the z direction and actually vanishes over a distance of the order of ρ_H (since q_z is contained in $L(b)$ in the form of the combination $q_z\rho_H$). As already stated, this corresponds physically to the fact that the ions do not move in a direction normal to $\mathbf{H}_0$. Let us integrate δN with respect to dz, i.e., let us put $q_z = 0$ in (7). For small q we obtain

$$N_{\mathbf{q}} = -\frac{SN_0a_0}{q}\left[\frac{1}{|\cos\beta|}B(a)\right]; \quad q_z = 0, \quad L(0) = 1,$$

$$B(a) = \frac{Q(a)}{2 + iaQ(a)}, \quad a = a_0\frac{\cos(\beta - \alpha)}{|\cos\beta|}, \quad (8)$$

β is the angle between $\mathbf{H}_0$ and $\mathbf{q}$ (see Fig. 2), and $Q(a)$ has the following integral representation and asymptotic behavior:

$$Q(a) = \frac{1}{\sqrt{\pi}i}\int_{-\infty}^{+\infty}\frac{e^{-t^2}}{t - a}dt; \quad Q(a) \approx \frac{i}{a},$$

$$\operatorname{Im} a > 0, \quad |a| \to \infty. \quad (9)$$

Using (9), we can readily verify that the expression in the square brackets in (8) is continuous and bounded for all β, and consequently $N_q \sim 1/q$ as $q \to 0$, i.e., $\delta N(r, \theta) \sim 1/r$ as $r \to \infty$. Indeed, substituting N_q from (8) in the two-dimensional analog of (5), cutting off the integral at large q by introducing in the exponential a negative increment $-q\delta$ with $\delta > 0$, and integrating with respect to dq, we obtain for large r the asymptotic expression

$$\delta N(r, \theta) = -\frac{i}{r}\frac{SN_0a_0}{(2\pi)^2}\int_0^{2\pi} B(a)\frac{1}{|\cos\beta|}\frac{d\beta}{\cos(\beta - \beta_0) + i\delta},$$
$$\beta_0 = \pi + \alpha - \theta. \quad (10)$$

Let us break up the region of integration with respect to $d\beta$ into two segments, $[-\pi/2, \pi/2]$ and $[\pi/2, 3\pi/2]$, and go over to a new variable $a = a_0 \cos(\beta - \alpha)/\cos\beta$. We obtain

$$\delta N(r, \theta) = \frac{i}{r}\frac{SN_0}{(2\pi)^2}\frac{a_0}{\sin(\alpha - \theta)}$$
$$\times\left\{\int_{-\infty}^{+\infty} B(a)\frac{da}{a + a_1 - i\delta_1} + \int_{-\infty}^{+\infty} B(-a)\frac{da}{a + a_1 + i\delta_1}\right\},$$

$$a_1 = a_0\sin\theta/\sin(\alpha - \theta), \quad \delta_1 = \delta\operatorname{sign}[\sin(\alpha - \theta)]. \quad (11)$$

The asymptotic behavior of $B(a)$ allows us to close the integration contour in (11) in both the upper and lower half-planes of the complex variable a. In the upper half-plane, however, $B(a)$ is analytic in a (see (9)), and in the lower it has an infinite number of simple poles corresponding to the propagation of different branches of ion-sound longitudinal waves [2]. Ultimately, closing the integration contour of the first integral in (11) in the upper half-plane and that of the second in the lower, we obtain

$$\delta N(r,\theta) = -\frac{SN_0}{r} F(a_0,\alpha,\theta),$$

$$F(a_0,\alpha,\theta) = \begin{cases} \dfrac{a_0}{\sin(\alpha-\theta)} F_0(a_1), & \sin(\alpha-\theta) > 0 \\ 0 & \sin(\alpha-\theta) < 0 \end{cases},$$

$$F_0(a_1) = \frac{1}{\pi} \operatorname{Re} B(a_1) = \frac{2}{\sqrt{\pi}} \frac{e^{-a_1^2}}{\pi a_1^2 e^{-2a_1^2} + [2 - a_1 \operatorname{Im} Q(a_1)]^2} \tag{12}$$

with a_1 from (11). In the derivation we used the relation $Q^*(z) = Q(-z^*)$. The function $Q(a) = \sqrt{\pi} w(a)$ which enters in (12) has been tabulated, for example, in [3].

The region $F(\theta) \equiv 0$ is shaded in Fig. 2. It is situated, as it should, in front of the body and is bounded by a straight line parallel to H_0 (the ions propagate only along H_0). The reversal of the magnetic-field direction $H_0 \to -H_0$ corresponds to the substitutions $\theta \to -\theta$ and $\alpha \to \pi - \alpha$, leaving the expression for $F(a_0, \alpha, \theta)$ unchanged. $F = a_0/2\sqrt{\pi} \sin\alpha$ strictly behind the body. The denominator in (12) vanishes nowhere, so that $F_0(a_1) \sim \exp(-a_1^2)$. $F(a_1)$ thus differs from zero only when $|a_1| \lesssim 1$, and we again return to the estimate (2).

Using the smallness of $|\theta|$ compared with α, we can replace $F(\theta)$ in (12) by the approximate expression

$$F(a_0,\alpha,\theta) \approx F_1\left(\frac{a_0}{\sin\alpha}, \theta\right) = \frac{a_0}{\sin\alpha} F_0\left(\frac{a_0}{\sin\alpha}\theta\right). \tag{13}$$

This simplified form discloses most graphically the main features of the angular dependence of the ion (electron) density perturbation $F(\theta)$. When $a_0/\sin\alpha$ varies, the $F_1(\alpha)$ curve simply compresses along the θ axis and stretches along the ordinate axis by the same factor, so that its area remains unchanged. The latter has the simple physical meaning that the particle number is conserved.

The following condition should be satisfied:

$$\int F(\theta)\, d\theta = 1.$$

Using the explicit form of $F(\theta)$ given in (12), we can verify that this relation is satisfied, as it should, accurate to terms $\sim 1/a_0^2$. (The integral of δN along H_0 is precisely equal to $-SN_0/\sin\alpha$. The presence of $\sin\alpha$ is explained in the discussion of formula (22).) Figure 3 shows a universal $F_0(a)$ curve (see (12)), with the aid of which it is easy to determine $F_1(\theta)$ and $F(\theta)$. The same

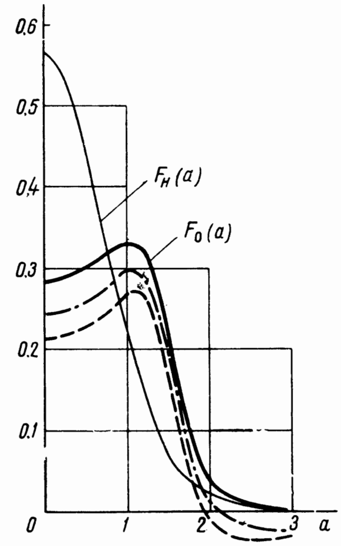

FIG. 3

figure shows for comparison a plot of $F_H(a)$ obtained without allowance for the influence of the electric field on the ion motion (see (20) below). For $a = 0$ we obtain $F_0(0) = \frac{1}{2} F_H(0)$. The curves are given only for $a > 0$, in view of the fact that they are even in a.

Figure 4 shows plots of $F(\alpha, \theta)$ for $a_0 = 8$ and $\alpha = 90, 60, 30$, and $15°$, as calculated from (12). For $\alpha = 15°$ the dashed curve shows a plot of $F_1(\theta)$ corresponding to the approximate formula (13). For $\alpha = 90°$ the function $F_1(\theta)$ coincides in fact with $F(\theta)$. This indicates that formula (13) is a good approximation when $a_0 \sin\alpha \gg 1$. From Figs. 3 and 4 we see that when $|a_1| > 1 (|\theta| > \theta_0)$ the $F(\theta)$ curve falls off rapidly, and when $|a_1| < 1 (|\theta| < \theta_0)$ it experiences insignificant relative changes, forming maxima at $|a_1| = 1$.

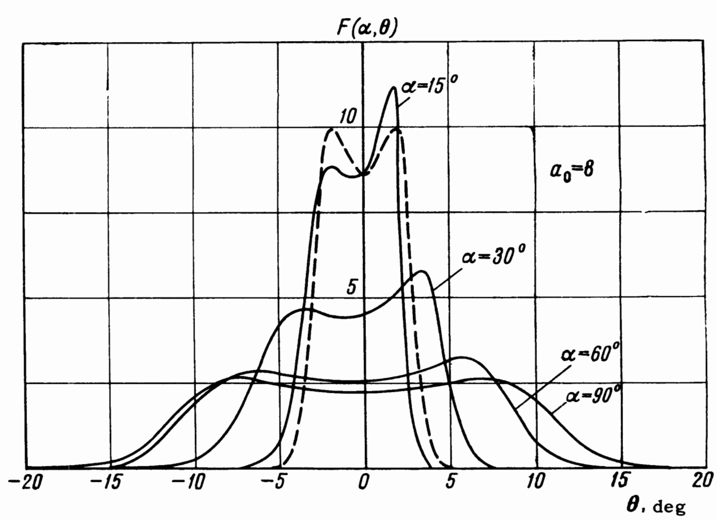

FIG. 4.

So far we have considered the averaged value of δN, calculated for $q_z = 0$. Formula (7) can be turned around in the (V_0H_0) plane also when $q_z = \text{const} \neq 0$. We take account here of the fact that $L(b) \leq 1$ and therefore the denominator $[2 + iaQ(a)L(b)]$ in (7) has certainly no zeroes in the upper half of the complex a plane when $L(b) < 1$. Using transformations similar to the preceding ones, we again obtain formula (12), in which $F_0(a)$ must be replaced by $F_{q_z}(a)$:

$$F_{q_z}(a) = \frac{1}{\pi}\operatorname{Re}\frac{Q(a)L(q_z^2\rho_H^2/2)}{2 + iaQ(a)L(q_z^2\rho_H^2/2)}. \tag{14}$$

It must be remembered, however, that the expression (7) itself, together with (14), is valid only when $q_zR_0 \ll 1$.

We can turn (7) around also when $\alpha = 0\,(V_0 \parallel H_0)$[2]. In this case the perturbation forms a tube behind the body, with radius $\sim \rho_H$. When a_0 is large we obtain for N_g directly from (7)

$$N_q = -\frac{SN_0}{q_\parallel}\frac{iL(b)}{2 - L(b)}. \tag{15}$$

when $q_\parallel = 0$, (15) has a pole singularity. A more detailed calculation allows us to establish a rule for circling around this pole and leads to the substitution $1/q_\parallel \to 1/(q_\parallel + i\nu)$, $\nu \to +0$ in (15). Turning (15) around in the V_0 direction and evaluating the integral by residues, we obtain the following asymptotic expression for δN when $|x|$ is large:

$$\delta N_{q_\perp} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} N_q e^{iq_\parallel x}\, dq_\parallel = \begin{cases} -SN_0\dfrac{L(b)}{2 - L(b)}, & x < 0 \\ 0 & x > 0 \end{cases} \tag{16}$$

accurate to terms $\sim 1/a_0^2$ for $x < 0$ and with exponential accuracy for $x > 0$. If $q_\perp = 0$ and $x < 0$, then $\delta N = -SN_0$ as before.

No special calculations are necessary to determine the perturbation of the ion (electron) concentration due to the motion of a cylinder in a magnetic field. (Naturally, it is meaningful to consider only the cylinder velocity component normal to the cylinder axis.) The body can be regarded as cylindrical if its length l satisfies the condition $l\cos\delta > \rho_H$, and its width h changes little over the distance ρ_H in the z direction normal to the (V_0H_0) plane. Here δ is the angle between the cylinder axis, which is directed along z', and the normal z axis (see Fig. 5). In this case formula (12) for the perturbation δN remains valid provided the cross section S is replaced by $h/\cos\delta$:

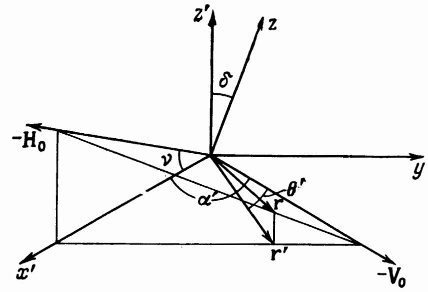

FIG. 5.

$$\delta N(r,\theta) = -\frac{hN_0}{r\cos\delta}\left[\frac{a_0}{\sin(\alpha-\theta)}F_0\left(a_0\frac{\sin\theta}{\sin(\alpha-\theta)}\right)\right]. \tag{17}$$

This analogy with a spherical body is due to the fact that the perturbations caused by different sections of the cylinder, which are separated in the z direction by distances larger than ρ_H, do not interfere. We note that in the case of a cylinder $\delta N(r,\theta)$ stands for the non-averaged value of the particle-density perturbation. An expression for δN can also be obtained by rigorous mathematical means by turning (7) around not in the (V_0H_0) plane with $q_z = 0$, as before, but in the plane $(x'y')$ normal to the cylinder axis with $q_z' = 0$ (see Fig. 5). We obtain

$$\delta N(r',\theta') = -\frac{hN_0}{r'}\left[\frac{a_0}{\cos\gamma\sin(\alpha'-\theta')} \times F_0\left(a_0\frac{\sin\theta'}{\cos\gamma\sin(\alpha'-\theta')}\right)\right], \tag{18}$$

γ is the angle between H_0 and the $(x'y')$ plane; r', α', and θ' are the analogs of r, α, and θ but measured in the $(x'y')$ plane (see Fig. 5).

With the aid of the relations

$$\cos\delta = \cos\gamma\frac{\sin\alpha'}{\sin\alpha}, \quad \cos\alpha = \cos\alpha'\cos\gamma, \quad r'\cos\theta' = r\cos\theta \tag{19}$$

we can readily verify that formulas (17) and (18) are identical.

We now calculate for comparison $\delta N(r,\theta)$ in a magnetic field without allowance for the influence of the electric field on the ion motion. This case was considered in detail in [4]. However, an expression for $\delta N(r,\theta)$ can be easily obtained also by the method described above, since N_q without allowance for the electric field differs from (7) only in that the denominator is missing[2]. After the customary transformations we obtain, using the notation of (12),

[2]Such a calculation for the case $\alpha = 0$ is only of methodological interest, since, as indicated by A. V. Gurevich, at low values of α the electrons cease to have a Boltzmann distribution even when $v_e \gg V_0$.

$$\delta N_H(r,\theta) = -\frac{SN_0}{r}\left[\frac{a_0}{\sin(\alpha-\theta)} F_H(a_1)\right], \quad \sin(\alpha-\theta) > 0,$$

$$F_H(a_1) = \frac{1}{\sqrt{\pi}} e^{-a_1^2}, \quad a_1 = a_0 \frac{\sin\theta}{\sin(\alpha-\theta)}, \tag{20}$$

which, naturally, coincides with the expression obtained in [4].

It must be borne in mind, however, that if no account is taken of the electric field then N_q has a singularity not only when $q_{||} = 0$ and $\mathbf{q}\cdot\mathbf{V}_0 = 0$, but also when $q_{||} = 0$ and $\mathbf{q}\cdot\mathbf{V}_0 = p\Omega$ ($q_x = 0$, $q_y = p\Omega/V_0 \sin\alpha$), where p is an arbitrary integer [2]:

$$N_{pq} = -\frac{SN_0 a_0}{|q_{||}|} Q(a_p) L_p(b),$$

$$a_p = \frac{\mathbf{q}\mathbf{V}_0 - p\Omega}{|q_{||}| v_i}, \quad L_p(b) = e^{-b} I_p(b). \tag{21}$$

This leads to the appearance of oscillations of δN relative to the variable y (see Fig. 2) with frequency $\Omega/V_0 \sin\alpha$ (see [4]). Upon averaging in the z direction, however, the oscillating parts tend to zero when

$$a_0 \sin\alpha \gg 1, \tag{22}$$

or else when $a_0 \sin\alpha \ll 1$, since

$$L_1\left(\frac{\rho_H^2}{2}\left(\frac{\Omega}{V_0 \sin\alpha}\right)^2\right) \sim \frac{1}{a_0^2 \sin^2\alpha}$$

when $a_0 \sin\alpha \gg 1$ and $L_1 \sim a_0 \sin\alpha$ when $a_0 \sin\alpha \ll 1$.

Formula 20 and the condition (22) have a simple physical meaning. It follows from Fig. 2 that $\delta N(r,\theta)$ is produced at the point with coordinates r and θ as a result of particles that have traversed a path $\Delta x = r\sin\theta/\sin(\alpha-\theta)$ within a time $t = (r/V_0)\sin(\alpha-\theta)/\sin\alpha$, i.e., moving with a velocity $v = V_0 \sin\theta/\sin(\alpha-\theta)$. Recognizing that $dv = dx/t$ and that the body sweeps out per unit length, in the direction y normal to H_0, a total of $N_0 S/\sin\alpha$ particles having a Maxwellian velocity distribution in the direction $x \parallel H_0$:

$$f(v_x)\,dv_x = \frac{N_0}{\sqrt{\pi}\, v_i} e^{-v_x^2/v_i^2} dv_x,$$

we obtain directly formula (20). In addition to the inequality (4), we used here the fact that the ions separated by distances of the order of ρ_H in the y direction oscillate in phase (the phase of the Larmor oscillations is $\varphi = t\Omega = y\Omega/V_0 \sin\alpha$, i.e., $\rho_H\Omega/V_0 \sin\alpha \ll 1$, which is equivalent to the first inequality of (22).

The distance ρ_H is the characteristic distance over which Larmor rotation causes mixing of particles in the y direction. In the opposite limiting case $\rho_H\Omega/V_0 \sin\alpha \gg 1$ the ions arriving at the given point have with a phase shift $\varphi \gg 2\pi$, which leads to the vanishing of the periodicity of δN in y at large values of r. Satisfaction of the first condition in (22) is necessary also for the results of [4] to be valid, since the time t, which enters as a parameter in the expression for the ion velocity at a given point in terms of its initial velocity, has been determined in that reference accurate to $\Delta t = \rho_H \sin\alpha V_0$, and this quantity must be smaller than $1/\Omega$.

We now compare the obtained formulas (12) and (18) with the perturbation of the plasma behind a cylindrical body when $H_0 = 0$[3)].

In the case of a cylindrical body we have for N_q [1]

$$N_{Eq} = -\frac{hN_0 a_0}{q} B(a'), \quad a' = \frac{\mathbf{q}\mathbf{V}_0}{q v_i} = -a_0 \cos\varkappa, \tag{23}$$

where $\mathbf{q}$ is a two-dimensional vector lying in a plane normal to the cylinder axis, κ is the angle between $-\mathbf{V}_0$ and $\mathbf{q}$, and $\mathbf{V}_0$ is perpendicular to the cylinder axis. In analogy with (10), we obtain an asymptotic expression for $\delta N_E(r,\theta)$ for large r:

$$\delta N_E(r,\theta) = -\frac{i}{r}\frac{hN_0 a_0}{(2\pi)^2}\int_0^{2\pi} B(-a_0\cos\varkappa)\frac{d\varkappa}{\cos(\varkappa-\theta)+i\delta},$$

$$\delta \to +0, \tag{24}$$

θ is the angle between $-\mathbf{V}_0$ and $\mathbf{r}$, and $\mathbf{r}$ is a two-dimensional vector.

Breaking up the region of integration with respect to $d\kappa$ into two segments $[0, \pi]$ and $[\pi, 2\pi]$, and changing to a new variable $a = a_0 \times \cos\kappa$, we can readily reduce δN_E to the form

$$\delta N_E(r,\theta) = \frac{hN_0 a_0}{(2\pi)^2}\frac{i}{r}(a_0\cos\theta)$$

$$\times \int_{-a_0}^{+a_0} \frac{2B(a)}{\sqrt{a_0^2 - a^2}} \frac{a\,da}{(a^2 - a_0^2\sin^2\theta) - i\delta_1} \tag{25}$$

Choosing the integration contour with $\cos\theta > 0$, in accordance with Fig. 6, we obtain, accurate to exponentially small terms, an expression which is close in form to (12) or (18):

$$\delta N_E(r,\theta) = \begin{cases} -hN_0 r^{-1}[a_0 F_0(a_0\sin\theta)], & \cos\theta > 0 \\ 0, & \cos\theta < 0 \end{cases}. \tag{26}$$

Thus, allowance for the magnetic field leads in (26) to the change $a_0 \to a_0/\cos\gamma \sin(\alpha' - \theta')$,

3)The perturbation caused by a cylindrical body is equivalent to the perturbation behind a spherical body, integrated along the cylinder axis.

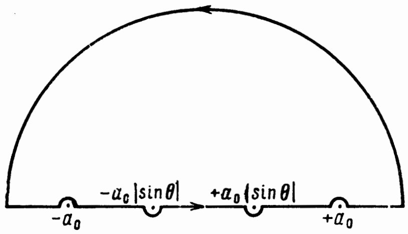

FIG. 6.

and if $a_0 \cos\gamma \sin\alpha' \gg 1$ simply to an effective replacement of a_0 by $a_0' = a_0/\cos\gamma\sin\alpha'$, which does not depend on θ' (cf. Fig. 1). A numerical calculation of δN_E behind a cylinder, with $H_0 = 0$, is given in[5]. Calculation of the perturbation of charged-particle density behind a spherical body for $H_0 = 0$, given in[5-7], leads of course to a marked difference from (12), i.e, in this case $\delta N_E \sim 1/r^2$ at large distances from the body.

Thus, the universal function $F_0(a)$ describes all three cases: large body and cylinder in a magnetic field, and also a cylinder with $H_0 = 0$. (The magnetic field can be disregarded at distances $r \ll \rho_H a_0$.)

We note in conclusion that the perturbation δN given in (12) can be represented as a sum of contributions due to the propagation of different branches of ion-sound waves, in analogy with the procedure used by Bud'ko[6] in the case of $H_0 = 0$. To this end it is sufficient to close the integration contour in formula (11) in the lower and upper half-planes of the variable α, respectively. As a result we obtain for $F_0(a)$ the following expression:

$$F_0(a) = \frac{1}{\pi}\operatorname{Re} B(a),$$

$$B(a) = \frac{Q(a)}{2 + iaQ(a)} \equiv \sum_n \frac{i}{z_n^2 - 1}\,\frac{1}{a - z_n}, \qquad (27)$$

which follows already from the converges of the series used at the asymptotic values of z_n. Here z_n is the n-th pole of B(a), $i/(z_n^2 - 1) = \operatorname{Res}_n B(a)$.

The first three poles with $\operatorname{Re} z_n > 0$ and the asymptotic expression for z_n at large values of $|z_n|$ are given in[6]:

$$z_0 = 1.48 - i0.58;\quad z_1 = 2.36 - i1.85;\quad z_2 = 3.00 - i2.49;$$
$$z_n \approx \rho_n e^{i\varphi_n},\quad \rho_n^2 = \frac{3\pi}{4} + 2\pi n,\quad \varphi_n = -\frac{\pi}{4} + \frac{\ln 4\pi\rho_n^2}{4\rho_n^2}. \qquad (28)$$

The dashed curve in Fig. 3 shows the contribution of the first pole to $F_0(a)$, and the dash-dot curve shows the sum of the contributions of the first three poles. We see that the one-pole approximation represents $F_0(a)$ quite well in the main region. At large values of a, each pole makes a negative contribution, but the sum of the contributions of all the poles, as follows from (12), is always positive. A similar effect was observed by Bud'ko[6]. We note that such a pole treatment of formula (12) can be obtained by examining the manner in which the perturbation produced in the plasma by the moving body is dissipated along H_0 when account is taken of the influence of the electric field on the ion motion.

In conclusion, I thank Ya. L. Al'pert for valuable remarks and consultations and L. P. Pitaevskiĭ for a useful discussion of the results.

[1] L. P. Pitaevskiĭ and V. Z. Kresin, JETP 40, 271 (1961), Soviet Phys. JETP 13, 185 (1961); L. P. Pitaevskiĭ, Geomagnetizm i aéronomiya 1, 194 (1961).

[2] Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoĭ plazme (Artificial Satellites in a Rarefied Plasma), Nauka, 1964.

[3] V. N. Faddeeva and N. M. Terent'ev, Tablitsy znacheniĭ integrala veroyatnosteĭ ot kompleksnogo argumenta (Tables of the Probability Integral of Complex Argument), Gostekhizdat, 1954.

[44] A. V. Gurevich, Iskusstvennye sputniki (Artificial Satellites) 7, 101 (1961).

[5] Yu. M. Panchenko, Issledovaniya Kosmicheskogo prostranstva (Investigations of Outer Space, Trans. All-union Conf. on Outer-space Physics), Nauka, 1965.

[6] N. I. Bud'ko, Geomagnetizm i aéronomiya 6, No. 4, (1966).

[7] Yu. M. Panchenko and L. P. Pitaevskiĭ, ibid. 4, 256 (1964).

Translated by J. G. Adashko
138