# *INDUCED SCATTERING OF LIGHT BY LIGHT*

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The cross sections for the scattering of a photon by a photon of definite frequency in the presence of a third field of the same frequency is calculated. It is shown that for a certain spatial configuration of the field, the scattering cross section increases in proportion to the energy density of the third beam in the region where the first two beams cross. Some numerical results are presented.

## INTRODUCTION

It is well known (see, for example [1-4]) that the cross section predicted in electrodynamics for the scattering of light by light is very small. For photons of wavelength $\sim 1\mu$ the estimates of the cross section lead to a value $\sim 5.5 \times 10^{-65}$ cm$^2$. Thus, even the most powerful laser beams presently available exclude the possibility of experimentally observing the direct photon-photon scattering effect.

One of the methods of facilitating the experimental observation of photon-photon scattering is to increase the energy of the scattered quanta. This method (see [5]) likewise has not yet led to a solution of the problem. All the methods of obtaining intense $\gamma$-quantum beams of high energy do not ensure as yet the possibility of setting up $\gamma$-quantum scattering experiments.

The question of scattering of light by light in the presence of an external field was also considered in several papers. As shown in [6], the fields required to scatter a photon from a laser beam in a constant external field have very large gradients, so that in practice intensification by a factor larger than $10^4$ is impossible. The scattering of a photon in the external field of the nucleus becomes comparable with the cross section for the ordinary Compton effect on the electron only at quantum energies $\sim 10^{10}$ eV.

Thus, the variants considered so far have led to the conclusion that it is impossible to observe experimentally the scattering of light by light. The purpose of the present paper is to show that by using modern laser beams it is possible, in principle, to create conditions under which fourth-order photon interaction processes can have an appreciable probability, sufficient for its registration.

## 1. INDUCED SCATTERING

The process referred to here can be called induced scattering of light by light in vacuum. The experimental conditions proposed for realization of such a scattering reduce to the following.

In addition to two colliding beams of the scattering photons of a certain frequency, a third auxiliary beam of photons of the same frequency is passed through the interaction region of the beams, in a direction perpendicular to them. The probability of scattering of photons from the first two beams is increased in this case in proportion to the density of the photons of the third beam, as is the case for any process induced by a field. The scattered quanta are registered in a direction opposite to that of the third beam.

The nontriviality of the problem lies in the fact that for plane waves the cross section of such a process is equal to zero, owing to the smallness of the phase value admitted by the momentum and energy conservation law $k_1 + k_2 - k_3 = k$. (In practice it is impossible to produce two photon beams directed strictly opposite each other.) However, as shown below, there exists the possibility of circumventing this difficulty. To this end it is sufficient to shape the photon beams in the form of converging (or diverging) waves. The simplest system of shaping such beams can be a system of three lenses that focus three beams in a single point.

## 2. MATRIX ELEMENT OF THE PROCESS OF SCATTERING OF WAVE PACKETS OF DEFINITE FREQUENCY

Let us consider the fourth-order photon-photon interaction process (Fig. 1). We write the regularized fourth-order scattering matrix in the mo-

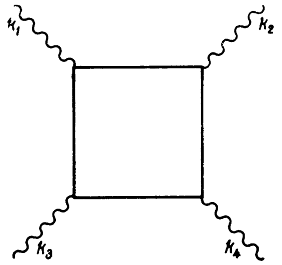

FIG. 1

mentum representation (we put $\hbar = c = 1$, $e^2/4\pi = 1/137$):

$$S^{(4)} = -\frac{i}{12}\left(\frac{e^2}{4\pi}\right)^2 \int\int\int\int (2\pi)^4 \delta(k_1+k_2+k_3+k_4) \times N\left(\frac{A_\mu(k_1)A_\nu(k_2)A_\lambda(k_3)A_\sigma(k_4)}{(2\pi)^{16}}\right) \times I_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4)\, d^4k_1\, d^4k_2\, d^4k_3\, d^4k_4, \tag{1}$$

where $A(k_i)$ are the photon operators of the potential, $k_i$ are the 4-momenta of the photons, N denotes the normal product, and $I_{\mu\nu\lambda\sigma}$ is the regularized value of the scattering tensor (see [3, 4]):

$$I_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4) = G_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4) - G_{\mu\nu\lambda\sigma}(0000).$$

The symmetrized scattering tensor $G_{\mu\nu\lambda\sigma}$ is equal to

$$G_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4) = T_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4) + T_{\mu\nu\sigma\lambda}(k_1k_2k_4k_3) + T_{\mu\lambda\nu\sigma}(k_1k_3k_2k_4),$$

where

$$T_{\mu\nu\lambda\sigma}(k_1k_2k_3k_4) = \frac{1}{i\pi^2}\int \mathrm{Sp}\{\gamma_\mu(i\hat{p}+m)^{-1}\gamma_\nu(i\hat{p}-ik_2+m)^{-1} \times \gamma_\lambda(i\hat{p}-ik_2-ik_3+m) \times{}^{-1}\gamma^\sigma(i\hat{p}-ik_2-ik_3+m)^{-1}\}\, d^4p.$$

The numerical factor preceding the integral (9) takes into account the number of equivalent N-products. On going over to the matrix element it is necessary to take into account 4! more topologically identical diagrams due to the permutation of all the photon lines.

The 4-momenta of the individual photons of the colliding beams will be denoted by $q_1$ and $q_2$, and the third-beam momenta and the momenta of the scattered photon are denoted respectively by $q_3$ and $q_4$. As a result the summary matrix element is written in the form

$$S_{if} = 2i\left(\frac{e^2}{4\pi}\right)^2 \int\int\int\int (2\pi)^4\delta(-q_1-q_2+q_3+q_4) \times \frac{A_\mu(q_1)A_\nu(q_2)A_\lambda(q_3)A_\sigma(q_4)}{(2\pi)^{16}} \times I_{\mu\nu\lambda\sigma}(-q_1,-q_2,q_3,q_4)\, d^4q_1\, d^4q_2\, d^4q_3\, d^4q_4. \tag{2}$$

We shall assume further that the beams 1, 2, and 3 represent packets of converging waves of frequency $\omega_0$ (the distribution of the waves is only along the direction of wave vectors), and for simplicity we call them sometimes "monochromatic homocentric" packets:

$$A_\mu(q) = \frac{(2\pi)^4}{(2\Omega q_0)^{1/2}} e_\mu{}^\lambda a_\lambda(\mathbf{q})\,\delta(q_0-\omega_0), \tag{3}$$

where $\Omega$ is the normalization volume of the region of the interaction of the beams. The fourth beam (scattered photons) will be fixed in the form of a plane wave k:

$$A_\mu(q_4) = \frac{(2\pi)^4}{(2\Omega\omega)^{1/2}} e_\mu\delta(k-q_4). \tag{3'}$$

The amplitudes of the chosen potentials of the photon field $a(\mathbf{q})$ for beams 1, 2, and 3 are functions only of the direction of the wave vector of the wave, and therefore can be represented in the form

$$a(\mathbf{q}) = a(\mathbf{n})\delta(|\mathbf{q}|-\omega_0)\mathbf{q}^{-2}, \tag{4}$$

where $\mathbf{n}$ are unit vectors directed along the wave vector $\mathbf{q}$. In the general case of non-monochromatic waves, the amplitudes take the form

$$a(\mathbf{q}) = a(\mathbf{n})\rho(|\mathbf{q}|)\mathbf{q}^{-2}. \tag{4'}$$

We find it useful in what follows to relate the potentials $a(\mathbf{q})$ with the energy of the field of the electromagnetic wave or, what is the same, with the number $\nu$ of photons in the packet. We present first the corresponding expressions.

We shall use the concept of the wave function of the photon in the space of the wave vectors $f_\lambda(\mathbf{q})$ defined in terms of the Fermi components of the electric field.[4] From the definition of the wave function, the number of photons in the momentum interval $d\mathbf{q}$ is equal to $|f(\mathbf{q})|^2 d\mathbf{q}$. Expressing the energy of the electromagnetic field in terms of the potential of the photon field (in the absence of longitudinal field components), we can readily show that

$$a_\lambda(\mathbf{q}) = -i(2\pi)^{-3/2}\Omega^{1/2}f_\lambda(\mathbf{q}). \tag{5}$$

If the total number of the photons in the interaction region is $\nu$, then the normalization of the wave function corresponds to the condition

$$\int |f(\mathbf{q})|^2\, d\mathbf{q} = \nu$$

or for the discrete spectrum

$$\sum_k f_k f_k^* \Delta = \nu,$$

where $\Delta k^2\delta\delta_0$ is the unit cell of the phase volume, $\Omega$ is the volume in which the radiation is contained,

$\delta$ is the thickness of the elementary spherical layer in $\mathbf{k}$-space, and $\delta_0$ is the solid-angle cell. We shall assume that the region $\Omega$ is bounded by a sphere of radius R, so that we can put $\delta = \pi/R$.

In the case of discrete normalization in frequencies[1)] the amplitudes (4) and (4′) are written in the form

$$a_k(\mathbf{q}) = a_k(\mathbf{n})\delta_{q\omega}\mathbf{q}^{-2}, \qquad a_k(\mathbf{q}) = a_k(\mathbf{n})\rho(|\mathbf{q}|)\mathbf{q}^{-2}. \quad (4'')$$

Comparing these amplitudes with the wave function in the case of the discrete spectrum

$$f(\mathbf{q}) = f_k(\mathbf{n})\delta_{q\omega} / |\mathbf{q}|\sqrt{\delta}, \qquad (6)$$

We find with the aid of (5) that the chosen notation for the amplitudes of the potentials corresponds to the normalization

$$\int |a(\mathbf{n})|^2 dO \rightarrow \sum_k |a_k(\mathbf{n})|^2 \delta_0 = \frac{\nu}{\delta_0},$$

$$\int \frac{(\rho(|\mathbf{q}|))^2}{\mathbf{q}^2} d|\mathbf{q}| \rightarrow \frac{\delta_0\Omega}{(2\pi)^3}, \qquad \int \rho(|\mathbf{q}|)d\mathbf{q} = 1. \qquad (7)$$

Let us continue the calculation of the matrix element. Substituting the expressions for the potentials given above for the photon-field potentials in the packets into the expression for $S_{if}$ and integrating with respect to $q_4$ and the variable components, we obtain

$$S_{if} = -8i\left(\frac{e^2}{4\pi}\right)^2 \frac{\pi^4}{\Omega^2\omega_0^{3/2}\omega^{1/2}}$$
$$\times \int\int\int \delta(-\mathbf{q}_1 - \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{k})\delta(\omega - \omega_0)$$
$$\times\ a_{1\rho}(\mathbf{q}_1)a_{2\beta}(\mathbf{q}_2)a_{3\gamma}(\mathbf{q}_3)$$
$$\times e_\mu{}^\rho e_\nu{}^\beta e_\lambda{}^\gamma e_\sigma I_{\mu\nu\lambda\sigma}(-q_1, -q_2, q_3, k)\,d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3, \qquad (8)$$

where $\omega_0$ are the frequencies of the photons in the beams 1, 2, 3, and $\omega$ is the frequency of the scattered photons (beam 4). Substituting the expansions for the potentials (4) and integrating with respect to $d|\mathbf{q}_1|$, $d|\mathbf{q}_2|$, $d|\mathbf{q}_3|$, we obtain

$$S_{if} = -8i\left(\frac{e^2}{4\pi}\right)^2 \frac{\pi^4\delta(\omega-\omega_0)}{\Omega^2\omega_0^{3/2}\omega^{1/2}} \int\int \frac{\delta(\alpha_1 - \omega)}{\alpha_1^2}$$
$$\times a_{1\rho}\left(\frac{\boldsymbol{\alpha}_1}{\alpha_1}\right) a_{2\beta}(\mathbf{n}_2)a_{3\gamma}(\mathbf{n}_3)$$
$$\times e_\mu{}^\rho e_\nu{}^\beta e_\lambda{}^\gamma e_\sigma I_{\mu\nu\lambda\sigma}(-\boldsymbol{\alpha}_1, -\omega_0\mathbf{n}_2, \omega_0\mathbf{n}_3, \omega_0\mathbf{n})\,dO_1 dO_2, \quad (9)$$

where $\boldsymbol{\alpha}_1 = \omega_0\mathbf{n} + \omega_0\mathbf{n}_3 - \omega\mathbf{n}_2$ and $O_i$ is the solid angle.

[1)] Discrete normalization of the photon wave function and its respective potentials will be necessary whenever we require to find the connection between the squares of f(q) or a(q) with the density or the number of protons in the interaction volume.

The yield of the reaction, i.e., the number of scattered photons produced in the interaction volume $\Omega$ per unit time, is determined by the quantity

$$dW = |S_{if}|^2\Omega d\mathbf{k} / (2\pi)^3 T,$$

where T is the time of the process. Representing, as usual, $S_{if} = M_{if}\delta(\omega - \omega_0)$, we obtain

$$dW / dO = |M_{if}|^2\omega_0\Omega / (2\pi)^4.$$

Taking into account the $\delta$-function $\delta(\omega - \omega_0)$, the expression for $\boldsymbol{\alpha}_1$ should be replaced by $\boldsymbol{\alpha}_0 = \omega_0(\mathbf{n} + \mathbf{n}_3 - \mathbf{n}_2)$. We note also that the $\delta$-function $\delta(\alpha_0 - \omega_0)$ can be represented in the form $\omega_0^{-1}\delta(1 - \mathbf{n}\cdot\mathbf{n}_2 + \mathbf{n}_3\cdot(\mathbf{n} - \mathbf{n}_2))$.

As a result we obtain the following expression for the yield of the reaction under consideration:

$$\frac{dW}{dO} = 4\pi^4\left(\frac{e^2}{4\pi}\right)^4 \frac{L^2}{\Omega^3\omega_0^8}, \qquad (10)$$

where the integral L is determined by the expression

$$L = \int\int \frac{\delta(1 - \mathbf{n}_2\mathbf{n} + \mathbf{n}_3(\mathbf{n} - \mathbf{n}_2))}{(\mathbf{n} + \mathbf{n}_3 - \mathbf{n}_2)^2} a_{1\rho}\left(\frac{\boldsymbol{\alpha}_0}{\alpha_0}\right) a_{2\beta}(\mathbf{n}_2)a_{3\gamma}(\mathbf{n}_3)$$
$$\times e_\mu{}^\rho e_\nu{}^\beta e_\lambda{}^\gamma e_\sigma I_{\mu\nu\lambda\sigma}(-\boldsymbol{\alpha}_0, -\omega_0\mathbf{n}_2, \omega_0\mathbf{n}_3, \omega_0\mathbf{n})\,dO_2 dO_3. \quad (11)$$

The photon-photon scattering tensor in the regularized and symmetrized form, valid for small photon energies ($\omega \ll m_0$), can be represented in the following manner (see, for example, [3,4]):

$$e_{1\mu}e_{2\nu}e_{3\lambda}e_{4\sigma}I_{\mu\nu\lambda\sigma}(-q_1, -q_2, q_3, q_4) = (14R - 5S) / 45m_0^4, \qquad (12)$$

where $m_0$ is the mass of the electron, S and R are the symmetrized tensors which for fixed polarization $\mathbf{e}_i$ can be represented in the form

$$S / 4\omega_0^4 = Q(1234) + Q(1324) + Q(1432), \qquad (13)$$

$$\begin{aligned} Q(1234) = {} & (\mathbf{e}_1\mathbf{n}_2)(\mathbf{e}_2\mathbf{n}_1)(\mathbf{e}_3\mathbf{n}_4)(\mathbf{e}_4\mathbf{n}_3) \\ & - (\mathbf{n}_1\mathbf{n}_2 - 1)(\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_4\mathbf{n}_3)(\mathbf{n}_3\mathbf{e}_4) - (\mathbf{n}_3\mathbf{n}_4 - 1)(\mathbf{e}_3\mathbf{e}_4) \\ & \times (\mathbf{n}_2\mathbf{e}_1)(\mathbf{n}_1\mathbf{e}_2) + (\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_3\mathbf{e}_4)(\mathbf{n}_1\mathbf{n}_2 - 1)(\mathbf{n}_3\mathbf{n}_4 - 1); \end{aligned}$$

$$R / \omega_0^4 = P(1234) + P(1324) + P(1432),$$

$$\begin{aligned} P(1234) = {} & (\mathbf{e}_1\mathbf{n}_4)(\mathbf{e}_2\mathbf{n}_1)(\mathbf{e}_3\mathbf{n}_2)(\mathbf{e}_4\mathbf{n}_3) + (\mathbf{n}_3\mathbf{e}_1)(\mathbf{n}_4\mathbf{e}_2)(\mathbf{n}_4\mathbf{e}_3)(\mathbf{n}_2\mathbf{e}_4) \\ & + (\mathbf{e}_1\mathbf{e}_2)\{(\mathbf{n}_3\mathbf{n}_4 - 1)[(\mathbf{n}_2\mathbf{n}_3)(\mathbf{n}_1\mathbf{e}_4) + (\mathbf{n}_1\mathbf{e}_3)(\mathbf{n}_2\mathbf{e}_4)] \\ & - (\mathbf{n}_2\mathbf{e}_3)(\mathbf{n}_3\mathbf{e}_4)(\mathbf{n}_1\mathbf{n}_4 - 1) - (\mathbf{n}_4\mathbf{e}_3)(\mathbf{n}_1\mathbf{e}_4)(\mathbf{n}_2\mathbf{n}_3 - 1) \\ & - (\mathbf{n}_4\mathbf{e}_3)(\mathbf{n}_2\mathbf{e}_4)(\mathbf{n}_1\mathbf{n}_3 - 1) - (\mathbf{n}_1\mathbf{e}_3)(\mathbf{n}_3\mathbf{e}_4)(\mathbf{n}_2\mathbf{n}_4 - 1)\} \\ & + (\mathbf{e}_3\mathbf{e}_4)\{(\mathbf{n}_1\mathbf{n}_2 - 1)[(\mathbf{n}_4\mathbf{e}_1)(\mathbf{n}_3\mathbf{e}_2) + (\mathbf{n}_3\mathbf{e}_1)(\mathbf{n}_4\mathbf{e}_2)] \\ & - (\mathbf{n}_4\mathbf{e}_1)(\mathbf{n}_1\mathbf{e}_2)(\mathbf{n}_2\mathbf{n}_3 - 1) - (\mathbf{n}_2\mathbf{e}_1)(\mathbf{n}_3\mathbf{e}_2)(\mathbf{n}_1\mathbf{n}_4 - 1) \\ & - (\mathbf{n}_3\mathbf{e}_1)(\mathbf{n}_1\mathbf{e}_2)(\mathbf{n}_2\mathbf{n}_4 - 1) - (\mathbf{n}_2\mathbf{e}_1)(\mathbf{n}_4\mathbf{e}_2)(\mathbf{n}_1\mathbf{n}_3 - 1)\} \\ & + (\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_3\mathbf{e}_4)[(\mathbf{n}_1\mathbf{n}_4 - 1)(\mathbf{n}_2\mathbf{n}_3 - 1) \\ & + (\mathbf{n}_1\mathbf{n}_3 - 1)(\mathbf{n}_2\mathbf{n}_4 - 1)]. \end{aligned} \qquad (14)$$

## 3. ACCOUNT OF NONMONOCHROMATICITY OF THE BEAMS

So far we have considered wave packets of photons of definite frequency converging to a single center (focus). We shall show that the obtained result for induced scattering can be extended also to the case when the photons of the beam have a certain frequency scatter.

Let us consider, for simplicity, the scattering when the frequency scatter is small. Let the packets, 1, 2, and 3 be now represented by amplitudes of the type (4′), and let the potential of the fourth wave correspond as before to the plane wave (3′). Substituting the amplitudes $a(\mathbf{q})$ in the form (4′) and (3′) and integrating with respect to the variables $q_4$, $\omega_3$, $|\mathbf{q}_1|$, $|\mathbf{q}_2|$, $|\mathbf{q}_3|$, we obtain the matrix element of induced scattering of nonmonochromatic beams:

$$S_{if} = -8i\left(\frac{e^2}{4\pi}\right)^2 \frac{\pi^4}{\Omega^2\omega^{1/2}} \int \frac{\rho_1(\omega_1)}{\omega_1^{1/2}} \frac{\rho_2(\omega_2)}{\omega_2^{1/2}} \frac{\rho_3(\omega_1+\omega_2-\omega)}{(\omega_1+\omega_2-\omega)^{1/2}}$$
$$\times \delta(-\omega_1\mathbf{n}_1 - \omega_2\mathbf{n}_2 + (\omega_1+\omega_2-\omega)\mathbf{n}_3 + \omega\mathbf{n})$$
$$\times a_{1\rho}(\mathbf{n}_1)a_{2\beta}(\mathbf{n}_2)a_{3\gamma}(\mathbf{n}_3)$$
$$\times I_{\mu\nu\lambda\sigma}(-\omega_1\mathbf{n}_1, -\omega_2\mathbf{n}_2, (\omega_1+\omega_2-\omega)\mathbf{n}_3, \omega\mathbf{n})$$
$$\times e_\mu{}^\rho e_\nu{}^\beta e_\lambda{}^\gamma e_\sigma \, d\omega_1 \, d\omega_2 \, dO_1 \, dO_2 \, dO_3. \tag{15}$$

In view of the fact that the potentials of the field do not contain fourth components, the scattering tensor can be regarded as dependent only on the spatial components of the photon momenta. We use further the fact that the frequency scatter in the beams is small, $\Delta\omega_i \ll \omega_i$, and put for this part of the integrand function which depends on the angular coordinates $\omega_i = \omega_0$, $i = 1, 2, 3$. The subsequent integration with respect to $dO_1$ leads to the expression

$$S_{if} = -8i\left(\frac{e^2}{4\pi}\right)^2 \frac{\pi^4}{\Omega^2\omega^{1/2}\omega_0^3} \int\int \frac{\rho_1(\omega_1)}{\omega_1^{1/2}} \frac{\rho_2(\omega_2)}{\omega_2^{1/2}}$$
$$\times \frac{\rho_3(\omega_1+\omega_2-\omega)}{(\omega_1+\omega_2-\omega)^{1/2}} L_1 \, d\omega_1 \, d\omega_2; \tag{16}$$

$$L_1 = \int\int \delta(1 - \mathbf{n}_2\mathbf{n} + \mathbf{n}_3(\mathbf{n}-\mathbf{n}_2)) a_{1\rho}\left(\frac{\boldsymbol{\alpha}_0}{\alpha_0}\right) a_{2\beta}(\mathbf{n}_2) a_{3\gamma}(\mathbf{n}_3)$$
$$\cdot e_\mu{}^\rho e_\nu{}^\beta e_\lambda{}^\gamma e_\sigma I_{\mu\nu\lambda\sigma}(-\boldsymbol{\alpha}_0, -\omega_0\mathbf{n}_2, \omega_0\mathbf{n}_3, \omega_0\mathbf{n}) \, dO_3 \, dO_2,$$
$$\boldsymbol{\alpha}_0 = \omega_0(\mathbf{n} + \mathbf{n}_3 - \mathbf{n}_2). \tag{17}$$

We see that the integral (17) coincides with the expression (11) and is completely determined by the geometry of the beams and by the polarization of the photon. Essentially, even expression (16) makes it possible to estimate the probability of the reaction of induced scattering for arbitrary energy spectrum of the photons.

In order to demonstrate that the scattering process occurs when the beams are not monochromatic, let us consider the case when beams 1 and 2 are monochromatic with frequencies $\omega_{10}$ and $\omega_{20}$, respectively, and beam 3 has a finite width of frequency interval $\Delta\omega_3$.

Thus, we put

$$\rho_1(\omega_1) = \delta(\omega_1 - \omega_{10}), \quad \rho_2(\omega_2) = \delta(\omega_2 - \omega_{20})$$

and integrate (16) with respect to $\omega_1$ and $\omega_2$, using the condition (7) for the normalization of the functions $\rho$. The probability of the scattering reaction with formation of the scattered quantum in a unit solid-angle interval is given by

$$\frac{dW}{dO} = \int |S_{if}|^2 \frac{\Omega\omega^2 d\omega}{(2\pi)^3 T} = 8\left(\frac{e^2}{4\pi}\right)^4 \frac{\pi^5}{\Omega^3 T \omega_{01}\omega_{02}\omega_0^6}$$
$$\times \int [\rho_3(\omega_{10}+\omega_{20}-\omega)]^2 \frac{L^2\omega d\omega}{\omega_{10}+\omega_{20}-\omega}. \tag{18}$$

Inasmuch as under our chosen conditions all the frequencies under consideration are close to $\omega_0$, we can write

$$\int \frac{[\rho_3(\omega_{10}+\omega_{20}-\omega)]^2 \omega d\omega}{\omega_{10}+\omega_{20}-\omega} = \frac{R}{\pi}.$$

Assuming further that the linear dimension of the wave packet (3), interacting over a time T, is determined by the distance $2R = T$, we obtain

$$\frac{dW}{dO} = 4\left(\frac{e^2}{4\pi}\right)^4 \frac{\pi^4}{\Omega^3\omega_0^8} L^2.$$

We see that the total yield of the reaction for a nonmonochromatic beam 3 coincides with expression (10) for the yield of the reaction for monochromatic beams (under the condition $\Delta\omega_3 \ll \omega_3$).

So far we have considered photon wave packets converging to a single center. In the notation employed for the wave packets, the amplitudes $a(\mathbf{q})$ do not contain phase factors that depend on $\mathbf{q}$. This means that all the plane waves which up the homocentric beam have the same phase when the front of the wave crosses the focal point. However, to ensure the possibility of the reaction it is sufficient to stipulate that the difference of phases between the individual components of the packet not exceed a certain fraction of $\pi$. This condition imposes definite requirements on the optical system shaping the beams. (Using the usual diffraction theory, we can show that the spherical operation should be sufficiently small and the Abbe sine condition should be satisfied.) For lack of space we omit a discussion of these questions.

## 4. DETERMINATION OF THE MATRIX ELEMENT IN EXPLICIT FORM

Calculation of the integral (11) in general form, for arbitrary functions $a_i(\mathbf{n}_i)$ and for arbitrary photon polarizations is rather cumbersome. It is of interest to calculate the yield of the reaction for a real experimental geometry, for example, one in which the beams of plane monochromatic waves are focused in a single point by lenses. In this case the functions $a_i(\mathbf{n}_i)$ will differ from zero within the limits of a certain finite solid angle, and the polarizations of the wave packets will be determined by the polarization of the plane waves striking the lenses.

Let us turn to calculate expression (11). We choose a coordinate system x, y, and z and direct the axis of beam 1 in the positive z direction, the axis of beam 2 in the negative z direction, and the axis of beam 3 in the positive y direction (Fig. 2). We shall also use the spherical coordinates $\theta$ and $\Phi$, measuring the angle $\theta$ from the positive z axis, and the angle $\Phi$ from the positive y axis.

The $\delta$ function contained in (11) contains as an argument an expression which can be represented in the form

$$A + B\cos\theta_3 + C\cos\theta_3;$$
$$A = 1 - \sin\theta\sin\theta_2\cos(\Phi - \Phi_2) - \cos\theta\cos\theta_2,$$
$$B = \sin\theta\cos(\Phi - \Phi_3) - \sin\theta\sin(\Phi_2 - \Phi_3),$$
$$C = \cos\theta - \cos\theta_2.$$

We introduce further the notation $u = \cos\theta_3$, $v = \sin\theta_3$, and transform the $\delta$ function into

$$\delta(A + B(1-u^2)^{1/2} + Cu) = \sum_{i=1}^{2}\delta(u - u_i)\frac{v_i}{Cv_i - Bu_i},$$

where $u_{1,2}$ are the roots of the equation $A + B(1-x^2)^{1/2} + Cx = 0$. As a result the integration of expression (11) with respect to d $\cos\theta_3$ entails no difficulty, and we obtain in spherical coordinates

$$L = \sum_{i=1}^{2}\int\int a_{1\rho}(\cos\theta_1, \Phi_1)a_{2\beta}(\cos\theta_2, \Phi_2)$$
$$\times a_{3\gamma}(u_i, \Phi_3)\frac{v_i}{Cv_i - Bu_i}$$
$$\times e_\mu{}^\nu e_\nu{}^\rho e_\lambda{}^\gamma e_\sigma I_{\mu\nu\lambda\sigma}(-\omega_0\boldsymbol{\beta}_0, -\omega_0\mathbf{n}_2, \omega_0\boldsymbol{\gamma}_3, \omega_0\mathbf{n})\,d\cos\theta_2\,d\Phi_2\,d\Phi_3, \tag{19}$$

where the unit vectors $\boldsymbol{\beta}_0$ and $\boldsymbol{\gamma}_0$ are defined in the following manner:

$$\beta_x = \sin\theta\sin\Phi + v_i\sin\Phi_3 - \sin\Phi_2\sin\theta_2,$$
$$\beta_y = \sin\theta\cos\Phi + v_i\cos\Phi_3 - \sin\theta_2\cos\Phi_2,$$
$$\beta_z = \cos\theta - \cos\theta_2 + u_i,$$
$$\gamma_x = v_i\sin\Phi_3, \quad \gamma_y = v_i\cos\Phi_3, \quad \gamma_z = u_i.$$

Further integration was carried out by numerical means. It was assumed for concreteness that prior to the shaping of the converging packets all three beams were linearly polarized plane waves. Let us consider by way of an example the case

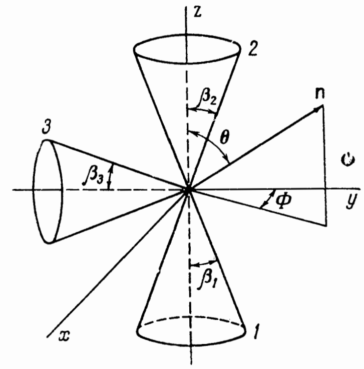


FIG. 2

when the polarization vectors are directed along the x axis in beam 1 and along the z axis in beam 3. We note that as a result of refraction of wave 1 in the lens, there appears a polarization-vector component along the z axis, but the polarization vector remains in the xz plane. Similarly, the polarization vector of beam 3 remains in the yz plane. As a result we find that the polarization vectors in the packets 1 and 3 respectively can be written in the form:

$$e_{1x} = \cos\theta_1(1 - \sin^2\theta_1\cos^2\Phi_1)^{-1/2},$$
$$e_{1y} = 0, \quad e_{1z} = -\sin\theta_1\sin\Phi_1(1 - \sin^2\theta_1\cos^2\Phi_1)^{-1/2},$$
$$e_{3x} = 0, \quad e_{3y} = -\cos\theta_3(1 - \sin^2\theta_3\sin^2\Phi_3)^{-1/2},$$
$$e_{3z} = \sin\theta_3\sin\Phi_3(1 - \sin^2\theta_3\sin^2\Phi_3)^{-1/2}.$$

We can specify in similar form the polarizations of the scattered wave and beam 2.

Let us assume that the energy density of the radiation in the initial beams is uniformly distributed over the area in a cross section perpendicular to the direction of the beam propagation. We determine under these conditions the form of the amplitudes of the wave packet produced after the fraction of the beams in the focusing lenses. Generally speaking, it would also be necessary to take into account the dependence of the Fresnel light-transmission coefficients on the angle of diffraction of the light ray in the lens, which would lead to a slight change in the angular distribution of the light density in the beam. This dependence, however, is small and can be neglected in first approximation.

Recalling that the connection between $a(\mathbf{n})$ and the photon-number density per unit solid angle is given by expression (7), we get*

$$\frac{d\nu}{dO} = \frac{\nu}{\pi\,\mathrm{tg}^2\beta}\frac{1}{\cos^3\theta},$$

where $\beta$ is the angular dimension of the packet $\theta_{max}$. In the numerical integration of (19), we used

*tg ≡ tan.

the values of the amplitudes $a_i'(\mathbf{n}) = a_i(\mathbf{n})(\delta/\nu)^{1/2}$, i.e., the values

$$a_1' = (-\pi^{1/2}\,\mathrm{tg}\,\beta_1 \cos^{3/2}\theta_1)^{-1},\quad a_2' = (\pi^{1/2}\,\mathrm{tg}\,\beta_2 \cos^{3/2}\theta_2)^{-1}$$
$$a_3' = (-\pi^{1/2}\,\mathrm{tg}\,\beta_3 \sin\theta_3 \cos\Phi_3)^{-1}. \tag{20}$$

The amplitudes (20) correspond to the condition when the interaction volume contains one quantum from each beam (the energy of the electromagnetic wave of each of the beams in the volume $\Omega$ corresponds to the energy of one quantum of frequency $\omega_0$). The results of the calculations are represented in Tables I–II. The angular dimensions of the interacting beams are specified by the angles $\beta_1$, $\beta_2$, and $\beta_3$. The values of the angles $\theta$ and $\Phi$ determine the direction of emission of the scattered quantum. The form of the polarization of the interacting beams is indicated in the tables. The number of the integral (19), corresponding to the interaction of the three photons (one each in beams 1, 2, and 3) is connected with the value of $V_0$ listed in the table in the following manner:

$$L_0 = \frac{\omega_0^4}{m_0^4}\delta_0^{-3/2} V_0.$$

If the energy of the electromagnetic wave of beams 1, 2, and 3 in the volume $\Omega$ exceeds the energy of the single photon by a factor $\nu_1$, $\nu_2$, and $\nu_3$ respectively, then the interaction will be given by

$$L = \frac{\omega_0^4}{m_0^4}\frac{(\nu_1\nu_2\nu_3)^{1/2}}{\delta_0^{3/2}} V_0. \tag{21}$$

The calculations were made with an electronic computer. The method of integration ensured an accuracy $\sim 0.001$. The maximum error of the values of $V_0$ given in the table do not exceed 1–2 units in the third decimal place.

## 5. YIELD OF INDUCED SCATTERING REACTION

Using the results of the calculations of the matrix elements, let us determine the final expres-

**Table I.** Values of $10V_0$ for polarization of beams 1, 2, 3, and 4 respectively in the planes xz, xz, yz, yz.

| β, deg | Φ, deg (θ = π/2) | | | | | | | π/2 − θ, deg (Φ = 0) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 3 | 5 | 10 | 20 | 30 | 50 | 1 | 3 | 5 | 10 | 20 | 30 |
| | $\beta_1=\beta_2=\beta_3=\beta$ | | | | | | | | | | | | |
| 1 | 0.747 | | | | | | | 0.006 | | | | | |
| 3 | 0.691 | 0.431 | 0.166 | | | | | 0.670 | 0.019 | | | | |
| 5 | 0.695 | 0.605 | 0.442 | 0.090 | | | | 0.682 | 0.613 | 0.035 | | | |
| 10 | 0.704 | 0.692 | 0.660 | 0.501 | 0.125 | | | 0.702 | 0.672 | 0.607 | 0.054 | | |
| 20 | 0.747 | 0.748 | 0.754 | 0.777 | 0.711 | 0.472 | | 0.745 | 0.728 | 0.699 | 0.562 | 0.050 | |
| 30 | 0.797 | 0.806 | 0.814 | 0.875 | 1.041 | 1.056 | 0.349 | 0.797 | 0.781 | 0.759 | 0.646 | 0.226 | 0.054 |
| | $\beta_1 = 30°,\ \beta_2 = 20°,\ \beta_3 = \beta$ | | | | | | | | | | | | |
| 1 | 0.725 | 0.545 | 0.299 | 0.142 | 0.086 | 0.072 | | 0.135 | | | | | |
| 10 | 0.767 | 0.768 | 0.768 | 0.768 | 0.659 | 0.442 | | 0.771 | 0.755 | 0.690 | | | |
| 20 | 0.755 | 0.754 | 0.756 | 0.751 | 0.734 | 0.634 | 0.041 | 0.761 | 0.756 | 0.733 | 0.611 | 0.116 | |
| 30 | 0.670 | 0.673 | 0.683 | 0.714 | 0.785 | 0.799 | 0.206 | 0.673 | 0.669 | 0.655 | 0.572 | 0.236 | 0.043 |
| 40 | 0.551 | 0.555 | 0.566 | 0.624 | 0.811 | 0.942 | 0.491 | 0.553 | 0.550 | 0.540 | 0.485 | 0.231 | 0.211 |

**Table II.** Values of $V_0$ for polarization of beams 1, 2, 3, 4 in the planes xz, yz, yz, and xy respectively.

| β, deg | Φ, deg (θ = π/2) | | | | | | | π/2 − θ, deg (Φ = 0) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 3 | 5 | 10 | 20 | 30 | 60 | 1 | 3 | 5 | 10 | 20 | 30 |
| | $\beta_1=\beta_2=\beta_3=\beta$ | | | | | | | | | | | | |
| 1 | 1.070 | | | | | | | 0.009 | | | | | |
| 3 | 1.069 | 0.657 | 0.248 | | | | | 1.025 | 0.029 | | | | |
| 5 | 1.070 | 0.917 | 0.654 | 0.121 | | | | 1.040 | 0.928 | 0.053 | | | |
| 10 | 1.062 | 1.029 | 0.957 | 0.657 | 0.121 | | | 1.049 | 0.997 | 0.910 | 0.862 | | |
| 20 | 1.044 | 1.033 | 1.015 | 0.943 | 0.638 | 0.311 | | 1.031 | 1.001 | 0.968 | 0.867 | 0.147 | |
| 30 | 1.001 | 0.998 | 0.988 | 0.957 | 0.838 | 0.622 | 0.100 | 0.989 | 0.961 | 0.940 | 0.875 | 0.712 | 0.201 |
| | $\beta_1 = 30°,\ \beta_2 = 20°,\ \beta_3 = \beta$ | | | | | | | | | | | | |
| 1 | 0.997 | 0.816 | 0.534 | 0.262 | 0.121 | 0.071 | | 0.149 | | | | | |
| 10 | 1.007 | 1.002 | 0.990 | 0.933 | 0.683 | 0.370 | | 0.998 | 0.957 | 0.872 | 0.169 | | |
| 20 | 0.947 | 0.940 | 0.926 | 0.864 | 0.663 | 0.420 | 0.012 | 0.942 | 0.892 | 0.892 | 0.798 | 0.187 | |
| 30 | 0.806 | 0.799 | 0.793 | 0.755 | 0.613 | 0.441 | 0.053 | 0.798 | 0.781 | 0.763 | 0.714 | 0.604 | 0.184 |
| 40 | 0.636 | 0.626 | 0.622 | 0.613 | 0.558 | 0.452 | 0.110 | 0.630 | 0.614 | 0.601 | 0.572 | 0.532 | 0.492 |

sion for the yield of the induced-scattering of converging beams.

We note, first, that the quantity $\delta_0$ (the unit cell of a solid angle occurring during discrete normalization of the wave function of the photon) can be readily eliminated in the final result, since we can put

$$\delta_0^3 = \frac{(2\pi)^9}{\Omega^3 \omega_0^6} \frac{R^3}{\pi^3}.$$

We note further that expression (21) corresponds to the yield of the reaction per second when the flux, say, of beam 2 is equal to $\nu_2/\Omega$ ($c = 1$), at the time when the volume $\Omega$ contains $\nu_1$ and $\nu_2$ photons of the remaining two beams. The total yield due to the passage of $N_2$ through the region of intersection of beams 1 and 3 is obtained by replacing the coefficient $\nu_2/\Omega$ by $N_2 t/\Omega = N_2/s$, where s is the cross section of beam 2.

As a result we find on the basis of (10) and (21) that the total number of photons scattered at an angle $\theta$ per unit solid angle is

$$\frac{d\nu}{dO} = \frac{1}{96\pi}\left(\frac{e^2}{4\pi}\right)^4 \frac{1}{m_0^2}\left(\frac{\omega_0}{m_0}\right)^6 \nu_1\nu_2 \frac{N_2}{s} |V_0|^2, \qquad (22)$$

where $\nu_1$ is the average number of the photons of beam 1 situated in the region of intersection of the three beams (interaction volume); $\nu_3$ is the average number of photons of beam 3 in the interaction volume, $N_2$ is the total number of photons of beam 2 passing through the interaction region during the considered time interval; s is the dimension of the transverse cross section of the interaction region (relative to the direction of propagation of beam 3); $V_2$ is a geometrical factor that depends on the angular dimensions of the beams, on the considered scattering angle $\theta$, and on the polarization of the beams (see the tables).

In order to understand more clearly the orders of magnitude of the quantities characterizing this induced scattering process, let us consider a concrete example of the proposed installation and let us estimate the number of scattered quanta.

Let us assume thus that the installation is shown schematically by Fig. 3, where A, B, and C are reflecting mirrors, I and II are optical light generators (lasers), D is a semi-transparent mirror, $F_1$, $F_2$, and $F_3$ are focusing lenses, and P is a photomultiplier registering the scattered quanta. The light from generator 1 circulates between the mirrors A and B. Light from generator II passes through the setup once (one half of the beam goes through the interaction zone, the other is reflected by the mirror D).

Let us specify for concreteness the parameters of the installation and of the experiment in the following manner: $l$ = 100 cm—distance between mirrors A and B; $\lambda = 10^{-4}$ cm—length of the radiation wave; $\Omega \sim \lambda^3$—volume of the region of intersection of the three beams; $s \approx \lambda^2$ cm$^2$—transverse dimension of beams 1 and 2 in the zone of intersection with beam 3; $N = 5 \times 10^{21}$p—number of photons generated in each of the lasers I and II with total energy radiation p (kJ); $\nu_i = N_i\lambda/l$—number of quanta of frequency $\omega_0$, the total energy of which corresponds to the energy of the electromagnetic field of the wave formed by the beam i in the interaction volume $\Omega$.

Noting further that

$$\left(\frac{e^2}{4\pi}\right)\frac{1}{m_0} = 2.82\cdot 10^{-13}\,\mathrm{cm},$$

$$\frac{\omega_0}{m_0} = 2\cdot 10^{-34}, \qquad \frac{e^2}{4\pi} = \frac{1}{137},$$

we obtain from (22)

$$d\nu\,/\,dO = 1.8\cdot 10^{-5} p^3 V_0^2.$$

Let us assume that the geometrical boundaries of the beams are determined by the angles $\beta_1$, $\beta_2 = \beta_3 = 30°$, and the conditions of polarization correspond to the case shown in Table II. Then for one of the two possible polarizations of the quanta scattered at the angle $\theta = 0$ we have in accordance with Table II

$$d\nu\,/\,dO = 2\cdot 10^{-5} p^3.$$

Thus, in 50,000 discharges of the lasers, with light intensity of 1 kJ each, there will be scattered on the average one quantum per unit solid angle. At a laser discharge power of 10 kJ each, the yield increases to 0.02 quantum/sr per discharge. Such intensities are in principle measurable with mod-

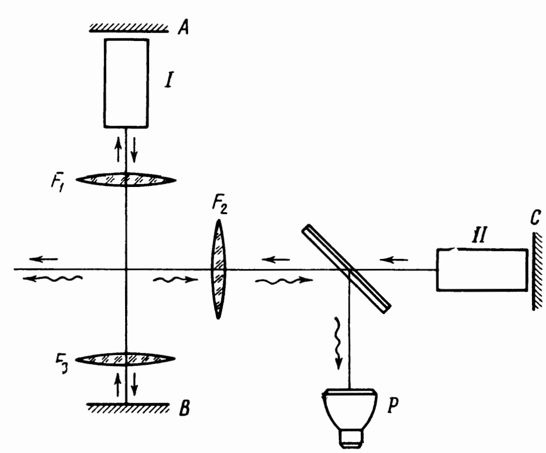


FIG. 3

ern quantum-counting means. They entail, however, great technical difficulties. Before answering finally the question whether such an experiment is realizable in practice, it is necessary to analyze many questions of both general and purely technical character. Thus, one of the complicated problems would be the appropriate focusing of the beams, and also the elimination of the background due to scattering by the residual gas in the optical system. If it is necessary to employ several crystals, synchronization of the radiation must be provided for. The lasers must have large power and at the same time small dimensions. The cooling system should be very efficient to ensure large repetition rate of the discharges. All these questions pertaining to the setup of the experiment call for a separate analysis.

We note, finally, that one can raise the question of increasing the yield of the scattered reaction without further increase of the laser power. One such method, for example, is to use the microstructure of the beams (the time variation of the intensity). The reaction yield is proportional to the densities of all three beams, so that it is more convenient to have a beam not of constant intensity, but of intensity in the form of individual peaks.

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