DIFFUSION-DISLOCATION MECHANISM OF CRYSTAL FLOW

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A diffusion mechanism of crystal flow is analyzed, in which the sources and sinks of point defects (vacancies and interstitial atoms) are prismatic dislocation loops within the crystal grain. A uniaxial external load creates conditions leading to the appearance of diffusion flows which transport the substance from one dislocation loop to another. It is shown that these flows may produce a stationary state in the crystal, characterized by a constant plastic-deformation rate. If the number of centers of creation of dislocation loops is not very large, the rate of flow of the material should be proportional to the cross section area of the crystal grain and to the volume density of the creation centers. Under certain conditions the flow velocity increases linearly with increasing external load. In the general case the dependence of the flow velocity on the external load is determined by the nature of the distribution of the dislocation formation centers.

1. INTRODUCTION. FORMULATION OF PROBLEM

THE diffusion mechanism of crystal flow, i.e., the irreversible change in the shape of crystalline samples under the influence of relatively weak static stresses, consists of the occurrence of directed diffusion flows of point defects, which are capable under free motion of transporting a mass (or volume) of the substance under consideration. Such point defects are vacancies and the crystal's own atoms located in interstices of the crystal lattice (interstitial atoms). The character of these flows, and also the intensity of the action of such a mechanism, are essentially determined by the distribution and power of the sources and sinks for the point defects and by the connection between the strength of these sources and the plastic deformation of the crystal. The sources and sinks of the point defects of interest to us may be either some preferred surfaces in the crystal (for example, the grain boundaries or surfaces of pores) or dislocations.[1]

The connection between the strengths of these sources and sinks and plastic deformation of the material is obvious and known. If we confine ourselves to an examination of material without pores, then we can assume that for point defects in each crystalline grain there exist sources and sinks of two types: surface—the external grain boundary, and volume—dislocation loops inside the grain. The diffusion mechanism of crystal flow, using surface sources and sinks of vacancies, was considered by Nabarro[2] and Herring[3] as applied to an isolated grain, and also in a recent paper by I. Lifshitz[4] as applied to a polycrystal. Naturally, the rate of plastic deformation due to such a mechanism is inversely proportional to the square of the average linear dimension of the grain.

In the present paper we propose and analyze a model of diffusion flow of a crystal, constructed for a mechanism which uses volume sources and sinks of point defects—dislocation loops inside the grain. It is perfectly clear that such a mechanism can ensure a certain stationary flow process only in the case when generation (formation) of dislocation loops can occur in the grain. It is also obvious that the distribution of the dislocations and the centers of their production is determined essentially by the prior plastic deformation of the material, and by the conditions of its crystallization and heat treatment.

The center of formation of new dislocations can in principle be any dislocation line segment whose extreme points, by virtue of certain causes, are secured and remain stationary when the shape of this section changes (for example, the elements of a dislocation grid or dislocations in block boundaries). A plastically deformed crystal contains as a rule a large number of such dislocation segments. Their length and spatial-orientation distributions are determined by the prior mechanical and heat

treatment of the crystal and to some degree by the crystallography of the lattice. In the isotropic approximation, which we shall use, the latter circumstance is not taken into account. In the simplest case all the spatial orientations of such dislocation segments and of their Burgers vectors can be regarded as equally probable.

If the external force acting on a dislocation exceeds the linear tension of the curved dislocation, with a curvature radius equal to half the distance between the points where the dislocation is secured, then the center described above is capable of "generating" a dislocation loop under the influence of this force. Under weak stresses, when no dislocation gliding can take place (the "starting" stress for dislocation gliding is large), the formation of dislocations is by diffusion, i.e., by growing of new atomic planes or "dissolution" of certain sections of the crystal planes. In either case we are dealing with formation of prismatic dislocations. The dislocations of the first type will be called interstitial, and those of the second type —vacancy dislocations.

The linear dimensions of the newly formed dislocation will be of the order of the distance between the points of fastening of the dislocation segment. The form of this loop can depend greatly on the mutual arrangement of the dislocation sections between the fastening points. However, inasmuch as the stable shape of an isolated dislocation loop is a circle, we shall assume that the resultant new dislocation loop is flat and circular both at the instant of its creation and during the course of its further evolution. We assume that there is no gliding of the dislocation at all. Then the considered prismatic dislocations of each type (interstitial and vacancy) will be characterized only by their radius R and by the orientations of the Burgers vectors (coinciding in direction with the vectors of the normals to the planes of the dislocation loops).

In the analysis of the growth rate of the dislocation loop we shall disregard the direct interaction of the dislocations (strong or diffusion), i.e., we shall consider each loop independently of the others. However, inasmuch as the cause of the growth of the loop is the influx of vacancies or interstitial atoms, and this influx is determined by the gradient of the concentration of the point defect near the dislocation, this unavoidably gives rise to an interaction of the loops via the self-consistent average supersaturation of the point defect in the crystal. Indeed, the concentration gradient of the point defect depends on the difference in the corresponding equilibrium concentration at the dislocation and far from it. The former (at the dislocation) is governed essentially by the orientation of the loop of the given type in the external stress field. The second (far from the dislocation) is connected with the overall balance of point defects, both situated within the crystal (in the solution) and those entering into the prismatic dislocation loop. It is obvious that at a fixed external load the different loops are under different energy conditions, and the diffusion flows will contribute to the growth of the "conveniently located" dislocation loops at the expense of the others. Thus, a "pumping over" of matter will take place from certain atomic planes to others, and will give rise to plastic deformation of the sample. It turns out that the rate of plastic deformation is actually determined by the over-all perimeter of the growing dislocation loops and by the external load.

If the number of dislocation-loop production centers is not very large, then during the loop growth its radius can become so large that the dislocation will emerge to the surface of the grain. When the entire prismatic dislocation is on the surface, then the atomic plane bounded by this loop becomes equivalent to the atomic plane in the crystal, and the dislocation itself ceases to participate in the exchange of point defects. We assume that all the dislocations "drop out of play" if their radii reach the same value R_0, which is of the order of the linear dimension of the grain.[1)] At a fixed density of dislocation formation centers, the rate of plastic deformation increases with increasing R_0. Therefore the discussed diffusion mechanism leads to a flow rate which increases with increasing linear grain dimension.

The rate of plastic deformation turns out to depend essentially on the dislocation segments, which dislocation production centers, and also on the relation between the average lengths of these segments to the critical radius of the loop R_{cr}, starting with which the dislocation-loop growth begins in the specified stress field.

[1)]If the dislocation sources generate dislocation loops that lie only in certain selected crystallographic planes, then at sufficiently large density of the centers of dislocation production it is necessary to take into account processes of coalescence of the loops emitted by different sources in the same plane. These processes can cause the maximum dislocation radius to be determined not by the dimensions of the grain but by the density of the centers for the production of a given type. In the isotropic model, in which all the Burgers-vector directions are equally probable, the processes of direct collisions of dislocations can be neglected for any reasonable density of the dislocation sources.

2. EQUATIONS FOR THE DISLOCATION SIZE DISTRIBUTION FUNCTION

Let us consider a crystal grain with linear dimensions R_0 under a uniform uniaxial load (we denote the corresponding component of the stress tensor by σ), the load being small compared with the elastic limit of the material. The latter condition will be taken in the sense that the load σ does not cause displacement of the dislocations in their glide planes. Assume that circular prismatic dislocation loops of both types (vacancy and interstitial) are uniformly distributed through the volume of this grain. The distribution of the loops with respect to the dimensions and orientations is described by the functions $f^v(R, \varphi)$ and $f^i(R, \varphi)$, where φ is the angle between the Burgers vector (or a vector normal to the plane of the loop) and the axis of the external load. The distribution functions f^v and f^i depend in general also on the time. For the usual normalization of these functions we assume that the integral

$$\int f(R, \varphi) \sin \varphi \, d\varphi \, dR$$

determines the number of loops of a given type per unit volume. The evolution of the distribution function can be described by a continuity equation in dimension space:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial R}(fV) = Q, \qquad (1)$$

where $V = dR/dt$ is the average rate of the change of the radius of the dislocation loop, and $Q = Q(R, \varphi)$ is the strength of the loop-production centers. The production centers (sources) of the loops can be naturally assumed in our mind to be uniformly distributed over the crystal. In this model, a plane dislocation loop does not change its orientation during its growth (we exclude the gliding of the dislocations), and therefore $\dot{\varphi} \equiv d\varphi/dt = 0$ and the angle φ enters Eq. (1) as a parameter.

Equation (1) does not take into account "collisions" of the dislocations, which lead to a mutual deceleration and stoppage of their motion. Stoppages of dislocations as a result of their collisions could decrease the number of loops participating in the diffusion interaction with the point defect. It is clear that the most essential are collisions of dislocations of the same type moving in nearby planes. We therefore formulate the conditions under which we can disregard such collisions.

Let us consider some growing dislocation loop and denote by r the distance at which the stress field of the dislocation becomes equal to the external stresses ($r \sim a(G/\sigma)$, where a is the lattice constant and G the shear modulus). During its development, the separated dislocation will be hindered by other dislocations which are created by loop sources situated in a strip parallel to the plane of the loop in question and having a width r (the volume of this strip is of the order of rR_0^2). We can neglect collisions of oppositely moving dislocations of the same type if

$$(r/R_0)^2 n(rR_0^2) = nr^3 \ll 1,$$

where n is the number of dislocation sources of all types per unit volume, and the factor $(r/R_0)^2 \ll 1$ determines the fraction of the dislocations whose orientations are such that they remain during their entire growth in the strip indicated above.

The required condition has a simple physical meaning: at a distance on the order of the average distance between the sources of the dislocations, the elastic field of each individual dislocation can be smaller than the external field. If we write down the resultant inequality in the form $\sigma^3 \gg G^3(na^3)$, then it becomes clear that for a specified dislocation-source density this inequality limits those stresses at which the assumption proposed is valid.[2)] We shall henceforth assume this condition to be satisfied.

For a specified intensity of the loop sources, i.e., for a specified right side, Eq. (1) for $f(R, \varphi, t)$ can be solved if we know the function $V = V(R, \varphi, t)$. The function $V(R, \varphi, t)$ is determined, naturally, not only by the state of the individual dislocation with parameters R and φ, but also by the interaction of the dislocations. It is easily understood that in the case of a uniform dislocation flux the average elastic field of the dislocation loops does not change the velocity V. As to the diffusion interaction of the dislocation, we shall take it into account only via the self-consistant supersaturation of the point defects. The rate of diffusion growth of each individual dislocation loop, due to the supersaturation of point defects in the crystal, was calculated by us earlier.[5] The influence of

[2)] The fact that the obtained inequality does not depend on the grain dimension R_0 is the consequence of the isotropy of the assumed model. In a single crystal the Burgers vector of the dislocations can have only certain chosen directions, and therefore there exist only several independent systems of parallel planes in which prismatic dislocations can develop. Collisions of dislocations in precisely such a system of planes are "dangerous" in the sense discussed, and the condition for their small probability is of the form $rR_0^2 n_\alpha \ll 1$, where n_α is the density of the sources of dislocations belonging to the given system (the discrete index α numbers different dislocation systems).

the external load on this rate was taken into account in [6].

Let c_0^v and c_0^i be the equilibrium translations of the point defects of the two types (vacancies and interstitial atoms), $\Delta^v = (\bar{c}_v - v_0^c)/c_0^v$ and $\Delta^i = (\bar{c}_i - c_0^i)/c_0^i$—their average relative supersaturations in the crystal, while D^v and D^i their respective diffusion coefficients. Then the number of point defects of each sort, entering per unit time and per unit length of the vacancy loop in an isotropic medium, is

$$\left(\frac{\delta N^v}{\delta t}\right)_v = c_0^v D^v A(R)\left[\Delta^v - \frac{\sigma\omega}{kT}\cos^2\varphi - \frac{B(R)}{R}\right],$$

$$\left(\frac{\delta N^i}{\delta t}\right)_v = c_0^i D^i A(R)\left[\Delta^i + \frac{\sigma\omega}{kT}\cos^2\varphi + \frac{B(R)}{R}\right]. \quad (2)$$

Accordingly, we obtain in lieu of (2) for the influx of point defects per unit length of interstitial dislocation

$$\left(\frac{\delta N^v}{\delta t}\right)_i = c_0^v D^v A(R)\left[\Delta^v + \frac{\sigma\omega}{kT}\cos^2\varphi + \frac{B(R)}{R}\right],$$

$$\left(\frac{\delta N^i}{\delta t}\right)_i = c_0^i D^i A(R)\left[\Delta^i + \frac{\sigma\omega}{kT}\cos^2\varphi - \frac{B(R)}{R}\right], \quad (3)$$

where

$$A(R) = \frac{2\pi}{\omega \ln(8R/r_0)},$$

$$B(R) = \frac{a}{4\pi(1-\nu)}\frac{\omega G}{kT}\left(\ln\frac{R}{r_0} + \alpha\right),$$

a—magnitude of the Burgers vector, equal to the lattice constant, r_0—characteristic radius of the "dislocation tube" ($r_0 \sim a$), ω—atomic volume ($\omega = a^3$), G—shear modulus, ν—Poisson coefficient, α some constant quantity determining the linear energy of the crystal-structure defect along the dislocation axis, and kT the temperature in energy units.

If we relate in the obvious manner the influx of point defects to the variation of the area of the dislocation loop, then we can usually obtain on the basis of (2) and (3) the rates of growth of the dislocation loops:[5, 6]

$$V^v = a^2 D^* A(R)[\Delta^* - \varkappa\cos^2\varphi - R^{-1}B(R)],$$

$$V^i = a^2 D^* A(R)[-\Delta^* + \varkappa\cos^2\varphi - R^{-1}B(R)], \quad (4)$$

where

$$D^* = c_0^v D^v + c_0^i D^i, \quad \varkappa = \frac{\sigma\omega}{kT},$$

$$\Delta^* = \frac{c_0^v \Delta^v D^v - \Delta^i D^i c_0^i}{D^*},$$

$\varkappa > 0$ corresponds to tension and $\varkappa < 0$ to compression. It will be shown later that in the case of interest to us Δ^* has the same sign as $\varkappa$, and therefore $\Delta^*/\varkappa > 0$.

For those orientations for which $\cos^2\varphi < \Delta^*/\varkappa$, all the interstitial loops must decrease ($V^i < 0$), and ultimately they are completely "dissolved" in the crystal. As to the vacancy loop, some of them are also dissolved, and those having radii larger than a certain critical value R_{cr}^i increase in size. The critical radius is determined by the obvious relation

$$B(R_{cr}^v)/R_{cr}^v = \Delta^* - \varkappa\cos^2\varphi \text{ for } \Delta^* > \varkappa\cos^2\varphi. \quad (5)$$

Analogously, in the angle interval $\cos^2\varphi > \Delta^*/\varkappa$ all the vacancy loops are dissolved, and the fate of the interstitial ones depends on the ratio of their radii to the corresponding critical radius R_{cr}^i, defined by the equation

$$B(R_{cr}^i)/R_{cr}^i = -\Delta^* + \varkappa\cos^2\varphi \text{ for } \Delta^* < \varkappa\cos^2\varphi. \quad (6)$$

For tension ($\varkappa > 0$) and for compression ($\varkappa < 0$) the regions of the angles φ in which either vacancy or interstitial loops grow are interchanged.

Let us assume that all the spatial orientations of the dislocation loops have approximately the same probability. Then the characteristic critical radius of the dislocation loops R_{cr}, as follows from (5) and (6), is connected only with the quantity Δ^*. Since in order of magnitude $B(R) \sim a(G\omega/kT)$, we have for R_{cr} the estimate

$$R_{cr} \sim \frac{a}{\Delta^*}\left(\frac{\omega G}{kT}\right). \quad (7)$$

If the dimensions of the crystal grain greatly exceed the characteristic critical radius of the dislocations ($R_0 \gg R_c$), then at sufficiently late stages of the flow process the grain contains a large number of dislocation loops, whose radii greatly exceed the average critical dimension R_{cr}. These are precisely the dislocations which determine the course of the developed flow of the material. Indeed, the contribution of the different dislocation loops to the rate of plastic deformation is proportional to their radii (their lengths), and therefore the principal role in the creation of flow of materials should belong to the dislocations whose radii greatly exceed the corresponding critical values, since their over-all perimeter determines the total flow of the point defects.

We note that dislocation loops with radii smaller than the critical values can also be disregarded because they are "dissolved" very rapidly. Since furthermore intense "pumping over" of the material takes place between vacancy and interstitial loops of large dimensions, the complete dissolution of the small dislocations does not influence

the over-all balance of the matter carried by the diffusion fluxes. We shall henceforth disregard such dislocations, confining ourselves in all formulas to integration with respect to R, starting with $R_{cr}(\varphi)$.

Let us use this circumstance and simplify expressions (4) for the growth rates of the dislocation loops, leaving out the last term in the brackets, which is connected with the linear tension force of the dislocation:

$$V^{v} = a^2 D^* A(R)[\Delta^* - \varkappa \cos^2\varphi], \qquad R > R_{cr}{}^{v}(\varphi);$$

$$V^{i} = a^2 D^* A(R)[-\Delta^* + \varkappa \cos^2\varphi], \quad R > R_{cr}{}^{i}(\varphi). \quad (8)$$

In addition, we note that the dependence of the function A(R) on the dislocation radius is very weak, and therefore in differentiation with respect to R in (1) it can be regarded as constant:

$$\partial f / \partial t + V \partial f / \partial R = Q. \quad (9)$$

The velocity V in (9) is now given by formulas (8), and therefore Eq. (9) determines the functions f(R, φ) only if $R > R_{cr}(\varphi)$, where $R^{v}_{cr}(\varphi)$ and $R^{i}_{cr}(\varphi)$ are obtained in turn from (5) and (6).

The rate of creation of dislocation loops Q(R, φ), which enters in (9), is connected with the character of the dislocation source. In the model assumed by us for the dislocation sources, it can be written with sufficient accuracy in the form

$$Q(R,\varphi) = \frac{\rho(R,\varphi)}{\tau(R,\varphi)}\theta[R - R_{cr}(\varphi)],$$

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases} \quad (10)$$

where $\rho(R, \varphi)$ is the volume density of the dislocation segments of length R (and of corresponding orientation), capable of forming the dislocations, and $\tau(R, \varphi)$ is the time of creation of a single dislocation with parameters R and φ, i.e., the time necessary to increase, by diffusion, the length of the dislocation segment in question by an amount of the order of R.

Since the curvature of the dislocation segment increases during the course of dislocation formation, the radius of the dislocation must at all time be larger than the critical value. But in this case, as already noted above, the velocity V is practically independent of the radius, and we can put

$$\tau = R / V. \quad (11)$$

Formula (11) is only an estimate when it comes to the accurate numerical value of τ, but in our opinion it is perfectly valid for an explanation of the main relation.

We substitute (10) and (11) in (9):

$$\frac{\partial f}{\partial t} + V(t)\frac{\partial f}{\partial R} = V(t)\frac{\rho(R,\varphi)}{R}, \quad R > R_{cr}(\varphi). \quad (12)$$

In Eq. (12) the very weak dependence of the velocity V on the radius R has very little effect, but the time variation of the velocity, i.e., the relation $\Delta^* = \Delta^*(t)$, is very important. On this basis, when solving (12), we can assume the rate of growth of the dislocation loop to be a function of the time only: V = V(t). To find this function we must write out a balance equation for the matter carried by the diffusion fluxes.

3. STATIONARY SUPERSATURATION OF POINT DEFECTS

Let us analyze first the evolution of the solution of Eq. (12) in order to ascertain the influence of the initial distribution of the dislocation loops in the form of the function f(R, φ) for large times. An examination of the role of the dislocations of different sizes, carried out in Sec. 2, has led us to the conclusion that in the most important region of the dimensions the velocity V can be independent of the dislocation radius. Under such a condition, the solution of (12) of interest to us can be obtained quite simply. If the distribution of the dislocation loops by dimensions is described at the initial instant of time (t = 0) by the function $f_0(R, \varphi)$, then the distribution function of the loops at any instant of time t is given by the following solution of Eq. (12):

$$f(R,t) = f_0[R - L(t)]\theta[R - L(t)] + \int_{R-L(t)}^{R} \rho(z)\theta(z - R_{cr})\frac{dz}{z}, \quad (13)$$

where

$$L(t) = \int_0^t V(t')\,dt',$$

and all the functions depend on the angle φ as a parameter. We have purposely introduced the function $\theta(z)$ as a multiplier in the first term of the right side of (13), in order to take into account the physical meaning of the argument of the function $f_0(R)$ (there are no dislocations with negative radii).

Naturally, the function (13) describes the distribution of the dislocation loops inside the grain, i.e., only for $R < R_0$. It follows from this that the first term in (13), which depends on the initial distribution of the dislocations, makes a contribution only during the initial state of development of the flow of the material, so long as $L(t) < R_0$. For suf-

ficiently long times ($t > t_{cr}$, where t_{cr} is determined by the equation $L(t_{cr}) = R_0$), the function (13) ceases to depend on the initial distribution of the dislocation loops. It is easy to see that the characteristic time t_{cr}, beyond which the "memory" of the initial distribution vanishes, has an order-of-magnitude estimate

$$t_{cr} \sim \frac{R_0}{V} \sim \frac{aR_0}{D^*} \frac{\ln(R_0/r_0)}{\Delta^*}. \tag{14}$$

Inasmuch as for $t > t_{cr}$ we have by definition $I(t) > R_0$, the lower limit of integration of the second term of (13) for such times is actually equal to $R_{cr}(\varphi)$. Consequently, for long times $t > t_{cr}$ the distribution function is of the form

$$f(R, \varphi, t) = \int_{R_{cr}(\varphi)}^{R} \rho(z, \varphi) \frac{dz}{z} \quad \text{for } R > R_{cr}(\varphi),$$
$$f(R, \varphi, t) = 0 \quad \text{for } R < R_{cr}(\varphi). \tag{15}$$

Thus, the question of the time variation of the distribution function at large times is uniquely related with the question of the change of the corresponding critical radius of the dislocation loops, which is determined in turn by the form of the function $\Delta^* = \Delta^*(t)$.

When the concentration of the point defects is small, the change in their supersaturation occurs in our model as a result of the settling of the defects on the dislocations (or evaporation from dislocation loops). It is easily understood that the rate of increase of the supersaturation of the vacancies (i.e., actually the rate of increase of their concentration in the volume of the material) is given by the relation

$$c_0^{v} \frac{d\Delta^{v}}{dt} = -\omega \int_0^{\pi} \sin\varphi \, d\varphi \int \left[\left(\frac{\delta N^{v}}{\delta t} \right)_{v} f^{v}(R, \varphi) + \left(\frac{\delta N^{v}}{\delta t} \right)_{i} f^{i}(R, \varphi) \right] 2\pi R \, dR. \tag{16}$$

A similar formula can be written out in perfectly obvious fashion also for the rate $d\Delta^i/dt$.

Strictly speaking, a contribution to $d\Delta^v/dt$ is made also by the total flux of vacancies going into the formation of new dislocation loops (to ensure the operation of the dislocation sources). We shall assume, however, that the average radius of the produced dislocations l is much smaller than the dimension of the grain ($l \ll R_0$). Under this condition, as already explained in Sec. 2, the role of the diffusion fluxes going to formation of new loops is insignificant in the overall balance of the transported matter and they can be disregarded.

Let us substitute in (16) the corresponding expressions for $\delta N/\delta t$ from (2) and (3), and let us effect simplifications connected with the importance of growing dislocations only:

$$\frac{d\Delta^{v}}{dt} = -\omega D^{v} A^* \int_0^{\pi} \left[\Delta^{v} - \frac{\sigma\omega}{kT} \cos^2\varphi \right] \times [F_v(\varphi) + F_i(\varphi)] \sin\varphi \, d\varphi. \tag{17}$$

The constant $A^* \sim A(R_0)$, and the function $F(\varphi)$ determine the total perimeter of all the dislocation loops of a given type participating in the process, with orientation characterized by the angle φ:

$$F(\varphi) = 2\pi \int_{R_{cr}(\varphi)}^{R_0} f(R, \varphi) R \, dR \quad \text{for} \quad R_{cr}(\varphi) > 0,$$
$$F(\varphi) = 0 \text{ for } R_{cr}(\varphi) < 0, \tag{18}$$

where for sufficiently long times the function $f(R, \varphi)$ is given by expression (15):

$$F(\varphi) = 2\pi \int_{R_{cr}(\varphi)}^{R_0} R \, dR \int_{R_{cr}(\varphi)}^{R} \rho(z, \varphi) \frac{dz}{z} = \pi \int_{R_{cr}(\varphi)}^{R_0} \rho(z, \varphi) (R_0^2 - z^2) \frac{dz}{z}. \tag{19}$$

Since by assumption the average radius of the produced dislocations l is much smaller than the dimension of the grain R_0, the ratio (19) can be simplified ($R_{cr}(\varphi) > 0$):

$$F(\varphi) = \pi R_0^2 \int_{R_{cr}(\varphi)}^{\infty} \rho(z, \varphi) \frac{dz}{z}. \tag{20}$$

We see that the total perimeter of all the dislocations of specified orientation, which determines the supersaturation of the point defects, is proportional to the crystal grain cross section area.

In analogy with (17), we have for the rate of growth of supersaturation of interstitial atoms

$$\frac{d\Delta^{i}}{dt} = -\omega D^{i} A^* \int_0^{\pi} \left[\Delta^{i} + \frac{\sigma\omega}{kT} \cos^2\varphi \right] \times [F_v(\varphi) + F_i(\varphi)] \sin\varphi \, d\varphi. \tag{21}$$

The most important feature of (17) and (21) is the presence in them of stationary solutions, i.e., solutions which do not depend on the time: $d\Delta^v/dt = d\Delta^i/dt = 0$.

The stationary values of Δ^v and Δ^i are determined by two equations obtained by setting the left sides of (17) and (21) equal to zero. However, Δ^v and Δ^i taken separately do not enter themselves in the expressions of interest to us, since the dis-

location-width growth process depends on the quantity Δ^*, i.e., only their linear combination. It is therefore sufficient for us to consider only one equation for Δ^*. This equation is obtained by subtracting (21) from (17) and reduces to the differential form of the material balance. In the stationary mode $[d(c_0^v \Delta^v - c_0^i \Delta^i)/dt = 0]$ the equation for Δ^* is

$$\int\limits_{V_v(\varphi)>0} V_v(\varphi) F_v(\varphi) \sin\varphi d\varphi = \int\limits_{V_i(\varphi)>0} V_i(\varphi) F_i(\varphi) \sin\varphi d\varphi, \quad (22)$$

where the dependence of the velocities $V(\varphi)$ on the angle φ is determined by the relation (4).

Equation (22) has the simplest form in the case when the distribution of the sources of the dislocation loops does not depend on their orientation. The functions $F(\varphi)$ are positive by definition, and therefore it follows from (8) that (22) has a solution when

$$\Delta^* = u_0^2 \varkappa, \quad 0 < u_0^2 < 1, \quad (23)$$

and the positive property $u_0^2 = u_0^2(\sigma/G)$ for $\rho = \rho(z)$ is a root of the following transcendental equation:

$$\int\limits_0^{u_0} (u_0^2 - u^2)\Psi(u_0^2 - u^2) du = \int\limits_{u_0}^1 (u^2 - u_0^2)\Psi(u^2 - u_0^2) du, \quad (24)$$

where

$$\Psi(u_0^2 - u^2) \equiv \Phi[\sigma G^{-1}(u_0^2 - u^2)] = F_v(\varphi), \quad u = \cos\varphi. \quad (25)$$

Since the right and left sides of (24) have a similar structure, the parameter u_0^2 of interest to us should have a value of the order of unity: $-u_0^2 \sim 1 - u_0^2 \sim 1$. It follows from (22) and (24) that u_0 does not depend on the same dimension.

Thus, an external load produces in the crystal a stationary supersaturation of point defects, characterized by a quantity $\Delta^* = u_0^2 \varkappa$. The exact value of the parameter u_0 depends on the form of the function $\rho(z)$ and on the value of the load σ.

Equation (24) can be readily transformed into ($\varkappa > 0$)

$$\int\limits_{R_{cr}^v(\pi/2)}^{\infty} \rho(z)\frac{dz}{z} \int\limits_0^{u_1(z)} (u_0^2 - u^2) du = \int\limits_{R_{cr}^i(0)}^{\infty} \rho(z)\frac{dz}{z} \int\limits_{u_2(z)}^1 (u^2 - u_0^2) du, \quad (26)$$

where

$$u_1(z) = \cos\varphi_1, \quad R_{cr}^v(\varphi_1) = z; \quad u_2(z) = \cos\varphi_2, \quad R_{cr}^i(\varphi_2) = z.$$

For compression ($\varkappa < 0$) the symbols "v" and "i" in (26) are interchanged.

Let us illustrate the solution (26), using a very simple example and assuming that $a/l \ll \sigma/G$. Since, as follows from (5) and (6),

$$R_{cr}^v\left(\frac{\pi}{2}\right) \sim R_{cr}^i(0) \sim \frac{a}{4\pi}\left(\frac{G}{\sigma}\right)\ln\left(\frac{R_{cr}}{r_0}\right),$$

the integration limits in both integrals of (26) are much smaller, under the assumptions made, than the average dimensions of the produced dislocations, and can therefore be replaced by zero. In addition, we can simultaneously assume here that $u_1(z) = u_2(z) = u_0$ and (26) is replaced by the equation

$$\int\limits_0^1 (u^2 - u_0^2) du = 0,$$

from which it follows that $u_0^2 = 1/3$.

In the general case, when the requirement $a/l \ll \sigma/G$ is not satisfied, the connection between the parameter u_0 and the form of the function $\rho(z)$ and the magnitude of the load is much more complicated. In our model, however, we can pretend only to yield the correct order of magnitude of the quantities characterizing the flow of the crystal, the concrete value of this parameter does not have great significance.

The formulated result, which consists in the presence of stationary supersaturation of the point defects, shows that an ensemble of dislocation loops of different type, situated in an external stress field, can ensure the realization of a definite stationary regime that differs qualitatively from the coalescence process.[5] Whereas in the coalescence process the supersaturation of the point defect decreases and tends to zero with time time,[5] a certain forced constant supersaturation of the point defect, caused by the action of the elastic field on the dislocations occurs in the stationary flow of material in question. It turns out that after this flow is established neither the distribution of the dislocation loops by dimensions (15) nor rates of their growth (8) change with time, and plastic deformation at a constant rate takes place in the crystal.

4. RATE OF PLASTIC DEFORMATION

The rate of plastic deformation due to the change in the areas of the closed dislocation loops is determined by the formula[7]

$$\dot{\varepsilon}_{ik}^{pl} = \frac{d}{dt}\sum\frac{1}{2}(s_i b_k + \varepsilon_k b_i), \qquad \mathbf{s} = s\mathbf{n}, \tag{27}$$

where **b** are the Burgers vectors of the dislocations, s the areas of the dislocation loops, and **n** the vectors normal to the dislocation loops; the summation takes into account all the dislocation loops per unit volume. The directions of the vectors **b** and **n** are related in the usual manner with the direction of circuiting around the dislocation loop.[7] In a case of prismatic dislocation loops **b** = b**n** for interstitial dislocation loops and **b** = −b**n** for the vacancy ones.

If we choose the z axis on the direction of the effective axis of the external load, then formula (27) for the nonvanishing elements $\dot{\varepsilon}_{ik}$ assumes in our case the form (b = a)

$$\begin{aligned}\dot{\varepsilon}_{zz} &= 2a\Big\{-\int_{\varphi_0}^{\pi/2} V_v(\varphi)F_v(\varphi)\cos^2\varphi\sin\varphi d\varphi \\ &+ \int_0^{\varphi_0} V_i(\varphi)F_i(\varphi)\cos^2\varphi\sin\varphi d\varphi\Big\} \\ &= \frac{4\pi}{\ln(8R_0/a)}(c_0^v D^v + c_0^i D^i)\frac{\sigma\omega}{kT} \\ &\times\Big\{\int_0^{u_0} u^2(u^2-u_0^2)\Psi(u_0^2-u^2)du \\ &+ \int_{u_0}^{1} u^2(u^2-u_0^2)\Psi(u^2-u_0^2)du\Big\}.\end{aligned} \tag{28}$$

Analogously

$$\begin{aligned}\dot{\varepsilon}_{xx} = \dot{\varepsilon}_{yy} &= \frac{2\pi}{\ln(8R_0/a)}(c_0^v D^v + c_0^i D^i)\frac{\sigma\omega}{kT}\cdot \\ &\times\Big\{\int_0^{u_0}(1-u^2)(u^2-u_0^2)\Psi(u_0^2-u^2)du \\ &+ \int_{u_0}^{1}(1-u^2)(u^2-u_0^2)\Psi(u^2-u_0^2)du\Big\}.\end{aligned} \tag{29}$$

Using (24), we can represent (29) in a different form:

$$\dot{\varepsilon}_{xx} = \dot{\varepsilon}_{yy} = -{}^1/_2\,\dot{\varepsilon}_{zz}. \tag{30}$$

As expected, $\dot{\varepsilon}_{ii} = 0$.

The first striking factor is that the rate of plastic deformation is proportional to the square of the linear dimensions of the grain. Indeed, if we substitute (25) and (20) in (28) and (30), then it becomes clear that all the elements of the tensor $\dot{\varepsilon}_{ik}$ are equal to the products of the cross section area by certain functions of the external load. In addition, a natural linear connection between the rate of flow of the material and the density of the sources of the dislocation loops follows from (29) and (30).

Let us subject (28) to the transformations that have previously led us to Eq. (26):

$$\begin{aligned}\dot{\varepsilon}_{zz} &= \frac{4\pi^2}{\ln(8R_0/a)}(c_0^v D^v + c_0^i D^i)\frac{\sigma\omega}{kT}R_0^2 \\ &\times\Big\{\int_{R_{cr}^{v}(\pi/2)}^{\infty}\rho(z)\frac{dz}{z}\int_0^{u_1(z)}u^2(u^2-u_0^2)du \\ &+ \int_{R_{cr}^{i}(0)}^{\infty}\rho(z)\frac{dz}{z}\int_{u_2(z)}^{1}u^2(u^2-u_0^2)du\Big\}.\end{aligned} \tag{31}$$

Assuming $a/l \ll \sigma/G$, formula (31) takes the form

$$\dot{\varepsilon}_{zz} = \frac{M}{\ln(8R_0/a)}(c_0^v D^v + c_0^i D^i)\frac{\sigma\omega}{kT}\frac{R_0^2}{l_0}n, \tag{32}$$

where $M = 4\pi/45$,

$$\frac{n}{l_0} = 4\pi\int_0^{\infty}\rho(R)\frac{dR}{R}, \qquad n = 4\pi\int_0^{\infty}\rho(R)dR,$$

n is the total number of sources of dislocations of all dimensions per unit volume of the crystal.

Thus, for sufficiently intense loads ($\sigma \gg G(a/l)$), the rate of flow of the material is directly proportional to the first power of σ:

$$\dot{\varepsilon}_{zz} \sim D^* n\left(\frac{R_0^2}{l_0}\right)\frac{\omega\sigma}{kT}. \tag{33}$$

Using (33) as an example, we see also the characteristic dependence of the rate of flow on the density of the dislocation grid or the dimensions of the mosaic blocks in the crystalline grain (on the quantity l_0). Naturally, for a fixed grain dimension the rate of plastic deformation increases with increasing density of the dislocation grid (with decreasing l_0). On the other hand, for a fixed block structure of the grain (fixed l_0), the rate of flow increases with increasing grain dimension, the dependence on the grain dimension being stronger than the dependence on l_0.

For smaller loads (and fixed l_0), a nonlinear dependence of ε on σ should appear. In the general case this dependence is very complicated and is determined essentially by the form of the function ρ(R).

In order to illustrate the nonlinear dependence of the rate of flow on the external load, let us consider below an example of the case when the function ρ(R) describes a Gaussian distribution:

$$\rho(z) = \rho_0\exp\left\{-\frac{(z-l)^2}{2\lambda^2}\right\}, \qquad \rho_0 = \frac{n}{4\pi\sqrt{2\pi}\,\lambda}. \tag{34}$$

We substitute (34) in (24) in (31). Then the integrals entering in (24) and (31) take the form

$$J(R)=\int_R^\infty \rho(z)P(z)\,dz=\rho_0\int_R^\infty \exp\left\{-\frac{(z-l)^2}{2\lambda^2}\right\}P(z)\,dz, \tag{35}$$

where P(z) is a certain continuous function, which vanishes at the lower limit of integration; P(z) $= P_0(z-R)^\alpha$ as $z \to R$ (P_0 = const, $\alpha > 0$).

Let us assume that $R^v_{cr}(\pi/2) - l \gg \lambda$ and $R^i_{cr}(0) - l \gg \lambda$. Then the condition $R - l \gg \lambda$ is satisfied in (35), making it easy to obtain asymptotic estimates for these integrals

$$J(R)\cong \rho_0 P_0 \exp\left\{-\frac{(R-l)^2}{2\lambda^2}\right\}\int_0^\infty \exp\left\{-\frac{(R-l)x}{\lambda^2}\right\}x^\alpha dx$$

$$=\rho_0 P_0\Gamma(\alpha+1)\left(\frac{\lambda^2}{R-l}\right)^{\alpha+1}\exp\left\{-\frac{(R-l)^2}{2\lambda^2}\right\}, \tag{36}$$

where $\Gamma(\alpha)$ is the gamma function.

The main dependence on R in (36) is given by the exponential function, and therefore if $R_{cr} - l \gg \lambda$ Eq. (26) can have a solution only if $R^v_{cr}(\pi/2) \approx R^i_{cr}(0)$, i.e., when $u_0^2 \approx 1/2$.

We use now the asymptotic expression (36) to estimate the flow rate by means of formula (31). When $R_{cr} - l \gg \lambda$, the main load dependence of the rate of plastic deformation is given by a function of the type $\exp[-(R-l)^2/2\lambda^2]$, where $R_{cr} \sim a(G/\sigma)$. Thus, when the load is decreased, the rate of flow decreases exponentially. In other words, when the critical radii exceed the dimensions of all dislocations "generated" in the grain, the rate of flow becomes vanishingly small. In such a formulation, the last deduction is perfectly natural and, of course, does not depend on the concrete form of the distribution function (34).

We must bear in mind, however, that at low loads, when the discussed mechanism of crystal flow does not work in practice, the diffusion mechanism of transfer of matter from certain surfaces of the crystal grain to others comes into being and becomes predominant.[2-4] Consequently, at a fixed the dislocation-grid density (fixed l_0) the diffusion-dislocation mechanism which we have considered can determine the flow of crystals under relatively large loads (at which the gliding of dislocations still does not begin).

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