MEASUREMENT OF THE λ-TRANSITION TEMPERATURE AND DENSITY MAXIMUM OF LIQUID He^4

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Submitted to JETP editor July 5, 1965

J. Exptl. Theoret, Phys. (U.S.S.R.) **50**, 844–852 (April, 1966)

The vapor pressure of He^4 at the λ point and the temperature difference between the λ point (T_λ) and the temperature of the maximum fluid He^4 density ($T_{max\,\rho}$) are measured with high accuracy. The position of the λ point was determined on the basis of the specific heat curve and the sharp change in the heat transfer. The position of the density maximum was determined on the basis of the change in the nature of the convection. The vapor pressure at the λ point is found to be $P_\lambda = 37.80 \pm 0.03$ mm Hg (0° C, g = 980.665 cm/sec²), $T_\lambda^{(58)} = 2.172_0 \pm 0.0003$° K. The temperature difference is $T_{max\,\rho} - T_\lambda = 0.0065 \pm 0.0005$°.

At the present time, there is no clear physical picture explaining the nature of the λ transition of liquid helium and its properties in the vicinity of the λ point. The λ transition is used as an excellent reference point; however, data on the measurements of temperature of the λ transition differ more widely than the experimental errors given by the authors.

Thus, for example, Long and Meyer,[1] taking for T_λ the maximum temperature at which there is still no difference between the readings of manometers measuring the pressure in an external tank and some in thermal contact with the internal tank, give $P_\lambda = 38.10 \pm 0.02$ mm Hg. (The pressure here and below is given for 0° C, and g = 980.665 cm/sec², while the temperature is on the scale T_{58}.[2]) Using a similar method, Hoare and Zimmerman[3] obtained $P_\lambda = 37.87 \pm 0.02$ mm Hg. Roberts and Sydoriak,[4] observing the maximum sound attenuation in liquid helium with account of corrections as given by Landau and Khalatnikov[5] and Chase,[6] obtained $P_\lambda = 37.80 \pm 0.01$ mm Hg, Buckingham and Fairbank,[7] from measurements of the specific heat of liquid helium, found $T_\lambda = 2.172 \pm 0.002$° K ($P_\lambda = 37.8 \pm 0.2$ mm Hg).

At several millidegrees above the λ point there is a maximum in the density of liquid He^4. From the estimates of Edwards[8] on the basis of measurements of the index of refraction of liquid helium, the temperature of the maximum density $T_{max\,\rho}$ is larger than T_λ by 0.001°. From measurements of the dielectric constant, Chase, Maxwell and Millett[9] found that $T_{max\,\rho} - T_\lambda = 0.005$°, while Kerr and Taylor,[10] measuring the density of liquid helium with a pycnometer, obtained $T_{max\,\rho} - T_\lambda = 0.006$°. The temperature of the λ transition in these measurements was taken to be the inflection point on the curve of the dependence of density on temperature.

The large scatter in the experimental data relative to the λ point and the density maximum of liquid helium, and also the desire to know how uniquely T_λ and $T_{max\,\rho}$ are determined by the different methods, served as the reason for setting up the present experiment.

For the measurements, an apparatus was prepared which made it possible simultaneously to determine accurately the vapor pressure of liquid helium and to measure the temperature at three points differing in height, at the same time it was sufficiently adiabatic to serve as a calorimeter. The apparatus consisted of three concentric dewars. The first two dewars were made of glass, the third of metal: its outer part was made of copper and its inner of stainless steel. Liquid nitrogen was placed in the outer dewar, liquid He^4 in the metal, and 10.6 ± 0.08 g of He^4 was condensed in the external dewar.

The saturated vapor pressure in the inner dewar was measured by a manometer of special construction. The manometer (Fig. 1) consisted of a vertical cylindrical tube made of glass and separated by partitions into two sections. The upper section was joined with the lower by a tube of inside diameter 6 mm. The manometer was filled with mercury. The lower section of the manometer was connected with the space of the inner dewar by means of a metallic hose of diameter 10 mm. The reading of the manometer was by

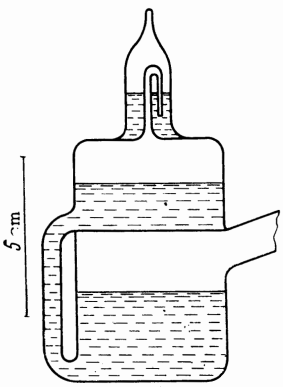

FIG. 1. Manometer for the measurement of the saturated vapor pressure of liquid He⁴ near the critical point.

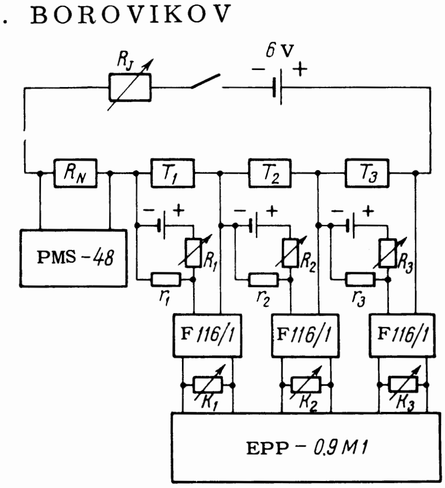

FIG. 3. Electrical measuring circuit for recording thermometer readings: T_1, T_2, T_3—resistance thermometers, R_J, R_i, K_i, r_i—resistors, R_N standard resistor, PMS-48—low resistance potentiometer, F116/1—constant current microvoltmeter, EPP-0.9M1—electronic automatic potentiometer.

means of a KM-5 cathetometer. The random scatter in the pressure measurement amounted to ±0.005 mm Hg, while the systematic error brought about by the inaccuracy of determination of the mercury level and by the lack of constancy in the room temperature did not exceed ±0.03 mm Hg.

The general form of the inner dewar is shown in Fig. 2. A flat constantan heater 1 was placed on the outside of the bottom of the inner container with He^4; at helium temperature, this heater had a resistance of 50.4Ω. Thermometers 2 were placed in the container at distances of 25, 80, and 135 mm from the bottom; the upper thermometer was located no further than 1 mm from the free surface of the He^4.

To guarantee the adiabaticity of the liquid He^4 in the internal dewar a bottleneck was made in its upper part; this was accomplished by a tube of stainless steel of diameter 10 mm, length 40 mm and wall thickness 0.1 mm. In order to prevent the incidence of thermal radiation on the inner container with the liquid He^4, a copper screen 3 making thermal contact with the He^4 of the middle dewar was located above the bottleneck. Heat conduction to the inner vessel in the range $T < T_\lambda$ at a temperature of He^4 in the middle dewar of about 4° K amounted to 0.1–0.2 milliwatt and was brought about chiefly by the layer of liquid He^4. In the temperature range $T > T_\lambda$, the total heat conduction to the inner vessel was principally due to the heat transfer above by the gas and by the thermal conductivity of the walls of the container, and was equal to about 0.06–0.07 milliwatt. This guaranteed passage through the temperature range $T_{\max\,\rho} - T_\lambda$ in time $t \geq 1$ hour.

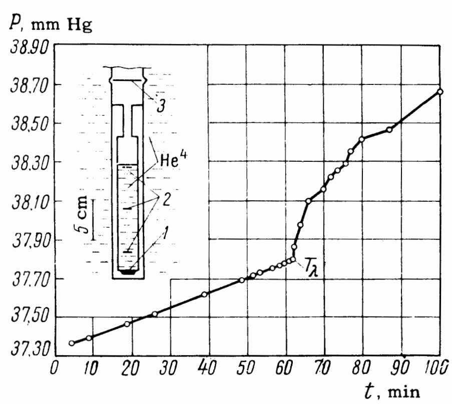

FIG. 2. Structure of the inner dewar: 1—heater, 2—thermometers, 3—screen. The curve shows the variation of pressure in the dewar in warming from above by means of the gas.

For reliable and reproducible temperature measurement, thermometers were made of lead brass[1)] and was a wire of 0.05 cm in diameter and 5 cm in length, stretched on a flat picture frame. The resistance of each thermometer at helium temperature was about 1Ω while the temperature sensitivity was 0.8Ω/deg (for currents of 0.5–5 mA). The thermometers had a linear dependence of resistance on temperature in the region from 2.1 to 2.7° K. In the temperature range from 2.154 to 2.172° K, the linearity was observed with accuracy better than 1%. The thermometers were calibrated against the vapor pressure of liquid He^4, and their readings were recorded by an automatic recorder by means of the circuit shown in Fig. 3. This circuit, with the help of the resistors K_1, R_i, and r_i, made it possible to determine the individual temperature sensitivity in each thermometric channel. The maximum temperature sensitivity of

[1)]The conductor of lead brass was made in the department of Materials Technology of the Academy of Sciences Institute for Physical Problems, under the direction of N. N. Mikhailov.

the circuit was 4×10^{-5} °K for a single division of the graph of the recorder.

MEASUREMENT OF T_λ

The first method of determination of the λ point consisted in the determination of the kink in the curve of relative pressure vs time in the inner dewar for a small heat influx through the free surface of the liquid helium.

The experiments were carried out in the following way: the temperature of liquid He^4 of the middle dewar, in which the inner dewar was placed, was lowered to $T < T_\lambda$. The liquid He^4 in the inner dewar was also cooled to $T < T_\lambda$ because of the condensation of its vapor on the copper screen (Fig. 2) and the copper walls of the upper part of the inner dewar. Then the liquid He^4 in the middle dewar was heated and its temperature was maintained at 4° K. Under these conditions the temperature, and also the vapor pressure of the liquid He^4 in the inner dewar, began to increase smoothly.

As is seen from Fig. 2, the shape of the curve changes sharply after reaching a pressure equal to 37.80 mm Hg. We took this point of discontinuity $P_{\lambda a}$ to be the λ point. Within the limits of accuracy of the experiment, $P_{\lambda a}$ did not depend upon the heat influx to the inner vessel (from 0.1 to 0.2 milliwatt) and was equal to

$$P_{\lambda a} = 37.80 \pm 0.03 \text{ mm Hg},$$

which corresponds to

$$T_{\lambda a} = 2.172_0 \pm 0.0003° \text{ K}.$$

It should be noted that after reaching the temperature $T_{\lambda a}$, the appearance of a noticeable difference in temperature between the upper and lower layers of liquid He^4 in the dewar was observed.

The second method of determination of T_λ was based on the fact that the heat transfer from the solid to the superfluid helium was more effective than to the non-superfluid helium; therefore, the resistance of the thermometers changes abruptly on going through the λ point.

Measurements were carried out in the following way: the temperature of the liquid He^4 of the inner dewar was established at $T < T_\lambda$, and that in the outer dewar at $T \approx 4$° K. A constant current was maintained in the thermometers in the limits from 0.5 to 5 mA, which corresponded to a heat flux density through the surface of the thermometers from 3.2 to 320 microwatt/cm^2. The thermometers were calibrated against the vapor pressure of He^4 in the range below the λ point; the accuracy of the calibration was ±0.00005° relative, and ±0.0003° absolute. The temperature of the liquid He^4 in the inner dewar was slowly raised and a jump was observed in the readings of the thermometers on going through the λ point. It appeared that the size of the temperature jump ΔT of the thermometers is a linear function of the heat flux density through the surface of the thermometers.

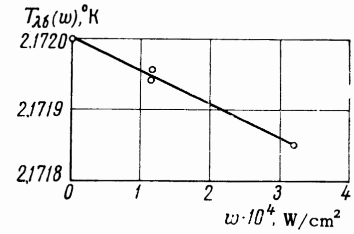

FIG. 4. Temperatures $T_{\lambda b}(w)$ for which jumps occur in the thermometer readings, as a function of the heat flux density w through its surface.

$$\Delta T = k_1 w, \quad k_1 = 2 \cdot 10 \text{cm}^2 \cdot °\text{K/W},$$
$$[w] = [\text{W/cm}^2].$$

The values of the temperatures $T_{\lambda b}(w)$ at which the jump is observed in the readings of the thermometers are shown in Fig. 4, and are described by the relations

$$T_{\lambda b}(w) = 2.172_0 - k_2 w, \quad k_2 = 0.5 \text{cm}^2 \cdot °\text{K/W},$$
$$[w] = [\text{W/cm}^2],$$

which, as $w \to 0$, gives

$$T_{\lambda b} = 2.172_0 \pm 0.0003° \text{ K}.$$

The third method of measurement of the λ point was to determine the specific heat near the λ transition and from it T_λ under the assumption of the validity of the temperature dependence of the specific heat obtained by Buckingham and Fairbank.[7] According to their data, near the λ transition,

$$C[\text{ J/g} \cdot °\text{K}] = 4.55 - 3 \log (T_\lambda - T) \quad \text{for } T < T_\lambda. \quad (1)$$

Measurements of the specific heat were carried out with simultaneous measurement of the pressure of the saturated vapors and the automatic recording of the readings of the thermometers. The specific heat of the liquid He^4 throughout the whole interval of the measured temperatures was determined according to the formula

$$C = q / m\Delta T,$$

where q is the quantity of heat obtained in the heat pulse, m the mass of liquid He^4, ΔT the change in temperature of the liquid He^4 brought about only

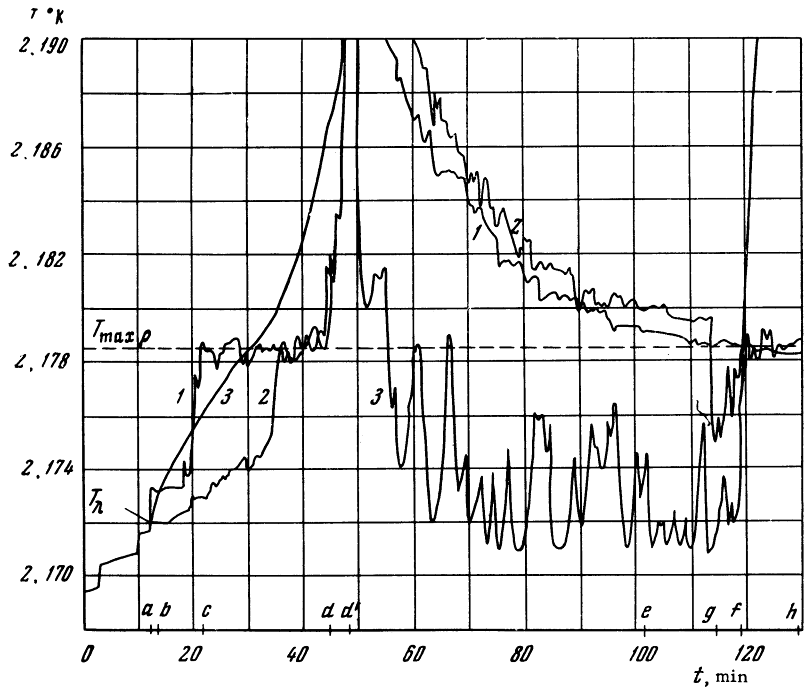

FIG. 5. Readings of the lower (1), middle (2) and upper (3) thermometers (explanation in text).

by the heat pulse. (The heat capacity of the empty vessel amounted to 0.03% of the heat capacity of the helium.)

The value of ΔT was determined from the calibrated recording of the reading of the thermometers on the tape of the automatic recorder in terms of the distance between the heating curves before and after the temperature pulse. The curves were straight parallel lines within ±0.0001°. The character of the curve is seen in Fig. 5 (Section 0 - a). It was impossible to obtain the value of the heat capacity of the liquid He^4 in the temperature range $T_\lambda < T < T_{max\,\rho}$ with out apparatus because of the inhomogeneity of the heating of the liquid He^4.

In the measurement of the heat capacity of the liquid He^4 in the range $T > T_{max\,\rho}$, pulses of duration 20 sec were used which gave off 0.4 Joule of heat energy in the heater 1 (Fig. 2). This brought about an intense convection in the liquid He^4 and guaranteed a more homogeneous heating of the helium in the dewar; however the accuracy of determination of the heat capacity in this case was about 50%, and the measurements were carried out only in order to check that there was no major error in the method of measurement.

For measurement of the heat capacity of the liquid He^4 in the region $T > T_{max\,\rho}$, pulses of 20 sec duration were used, which contained 0.101, 0.145 and 0.227 J. This made possible the determination of C with an accuracy of about 10%. The results of the measurements of the heat capacity and to curves corresponding to Eq. (1) with T_λ equal to 2.1716 and 2.1720° K are shown in Fig. 6. As is seen, these curves are extreme but still lie within the limits of error of measurement in the experimental data for all values of the specific heat capacity. Thus, taking the absolute error in the determination of T into account, we obtain

$$T_{\lambda c} = 2.1718 \pm 0.0004° \text{ K}.$$

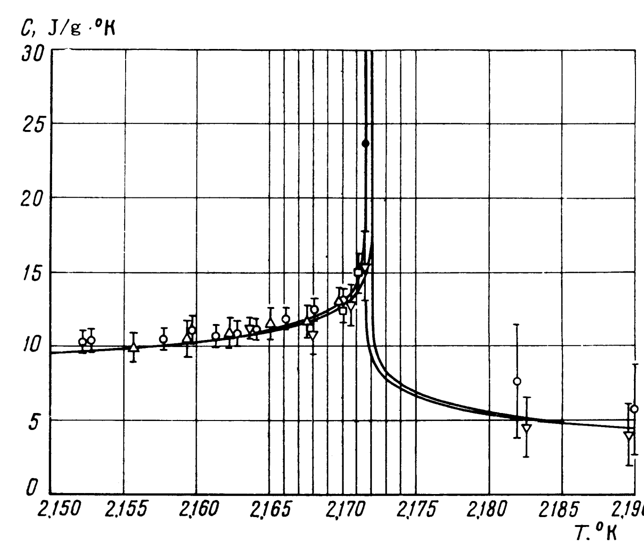

FIG. 6. Specific heat of liquid He^4 near the λ point. The points are our measurements; the different symbols correspond to different experiments. The solid curves correspond to the variation of the specific heat of liquid helium heat T_λ obtained from the data of Buckingham and Fairband[7] for T_λ taken by us to be equal to 2.1716 and 2.1720°K.

MEASUREMENT OF $T_{max\,\rho}$

The measurement of $T_{max\,\rho}$ was based on a determination of the temperature of the lower layers of helium in the dewar, for which convection in the dewar began to take place or, conversely, ceased. It is well known[11] that for a liquid (in the given case, for helium I) located in a vertical tube (dewar), along which there is a constant gradient $\partial T/\partial h$, the boundary of convective instability is the value

$$\partial T / \partial h = -67.4\ \varkappa\nu / r^4 g\beta, \tag{2}$$

where κ is the temperature conductivity, ν the kinematic viscosity, β the coefficient of thermal expansion of liquid He^4, r the internal radius of the dewar, h the height of the layers of liquid He^4 in the dewar, and g the acceleration due to gravity. If use is made of Eq. (2) and the results of the research of Chase, Maxwell and Millett,[9] from which it follows that $T_{max\,\rho} - T_\lambda = 0.005°$ and

$$\beta = 4.9\cdot 10^{-2} + 2.1\cdot 10^{-2} \log (T - T_\lambda)$$

$$\text{for } (T - T_\lambda) < 0.03°,$$

then one can estimate the maximum temperature difference $|T' - T_{max\,\rho}|$ between the upper layers of helium located at temperature $T_{max\,\rho}$, and the lower layers located at the temperature T', for which convection in the dewar has still not appeared.

If we take it into account in (2) that $\partial T/\partial h = |T' - T_{max\,\rho}|/h$, then

$$|T' - T_{max\,\rho}| \approx 6\cdot 10^{-5}\ °K.$$

This is the limiting accuracy with which one can determine experimentally the temperature of the maximum liquid density of He^4 in a given apparatus from measurements of the temperature of the appearance and disappearance of convection in liquid He^4 close to $T_{max\,\rho}$.

The determination of the difference $T_{max\,\rho} - T$ was carried out in three ways. The first method was based on the fact that if liquid He^4 initially at a temperature $T < T_{max\,\rho}$ is heated continuously from below, then the lower layer of helium will be heated up to $T > T_{max\,\rho}$, and the upper layer of helium, which has the temperature $T = T_{max\,\rho}$ will fall; thus the situation will be continued until all the liquid He^4 in the dewar is heated to $T \approx T_{max\,\rho}$. In other words, the readings of the lowest thermometer should remain close to some temperature which we shall take to be $T'_{max\,\rho}$. Then the temperature of the lower and middle layers will begin to rise as a consequence of the continuing convective heat transfer.

The experiment was carried out in the following fashion: liquid He^4 in the inner dewar was cooled to $T < T_\lambda$, the temperature of the outer dewar was kept constant close to 4° K. Then the temperature of the liquid He^4 was raised in the inner dewar to $T_\lambda < T_{max\,\rho}$ by heat pulses from the heater (Fig. 5, section 0 - a). In the next section (b - d′), a constant current of 2 mA passes through the heater; this current releases about 0.2 microwatt of heat energy in the heater. When the lower layers of liquid He^4 were heated to the temperature $T > T_{max\,\rho}$, subsequent liberation of heat in the heater brought about convection in the dewar (oscillatory character of temperature rise and an enormously large value of the heat transfer in helium I; section b - b′, curves 1 and 2). Here the temperature of the lower layers was kept constant with an accuracy of ±0.0005° close to $T'_{max\,\rho}$ (section c - d, curve 1). The difference $T'_{max\,\rho} - T_\lambda$ was determined from the calibration of the thermometers in the temperature region $T < T_\lambda$, which was extrapolated to the region of temperatures of helium I. The resultant value was

$$T'_{max\,\rho} - T_\lambda = 0.0063 \pm 0.0005°.$$

The next method of measurement of the quantity $T_{max\,\rho} - T_\lambda$ was based on the fact that if helium I at $T > T_{max\,\rho}$ is cooled from above, the convective cooling of the lower layers of helium in the dewar should cease close to the temperature maximum density. The experiment on the determination of this temperature is a continuation of the above and was carried out in the following way: when the entire liquid He^4 in the internal dewar was heated to $T > T_{max\,\rho}$, then the temperature of the liquid He^4 in the inner dewar was lowered by the increase in the boiling off of liquid He^4 from the outer dewar. The rate of pumping was maintained so that the upper surface of the liquid He^4 in the inner dewar was at a temperature below the assumed value of the temperature for maximum density (the section d′ - f, curve 3). The heat transfer in the dewar has in this case a convective character (section d′ - e in curves 1 and 2).

The viscous forces in liquid He^4 cause the motion of liquid He^4 in the lower part of the dewar to be damped out in about 20 minutes, while the lower layers of helium are cooled to the temperature for which its density will have the maximum value. When the temperature of the lower layers approaches a certain temperature $T''_{max\,\rho}$, then the heat transfer between the upper and lower layers almost ceases (section e - f, curve 1), indicating cessation of convection. The jumpwise nature of the decrease of temperature of the middle thermometer (point g, curve 2) takes place because of the short duration cooling of the central layers of the helium through the upper layer to the temperature $T < T_\lambda$. A nondamping mechanical motion of helium mixes the middle layer, which leads to an increase in the temperature of the helium close to the middle thermometer (section g - f, curve 2). As the temperature of the upper layers of liquid He^4 in the internal dewar increases to a temperature greater than the temperature of maximum density (section f - h, curve 3), the central layers of liquid He^4 are heated convectively to a temperature close to the temperature $T''_{\max\rho}$ of the lower layers, and there is no further heating of the upper layer (section f - h, curve 2). The lower layers of helium generally do not have any influence on the increase in the temperature of the upper layers (section f - h, curve 1). This indicates that the helium has a maximum density for a given temperature in the lower part of the dewar. The value obtained is

$$T''_{\max\rho} - T_\lambda = 0.0068 \pm 0.0005°.$$

The third method of determination of $T_{\max\rho} - T_\lambda$ is similar to the foregoing with this difference that the liquid He^4 is at the temperature $T < T_{\max\rho}$ and the convective character of the heat transfer from the upper layers of helium to the lower was observed in the heating of the upper layers. The temperature was brought to the temperature of maximum density $T'''_{\max\rho}$, at which the convective character of the heat transfer between the upper and lower layers disappeared.

The experiment went as follows: the liquid He^4 of the inner dewar was cooled to $T < T_\lambda$. The temperature of the outer helium dewar was maintained constant near 4°K. The liquid He^4 in the inner dewar was gradually heated because of the heat conduction from above through the saturated vapor and because of the heat conduction along the walls of the inner dewar. After passage through the λ point, the transfer of heat to liquid He^4 of the inner dewar from the upper layers to the lower has a clearly expressed convective character (Fig. 7, sections a - b, curves 1 and 2). The temperature of the upper layers increased continuously because of the heat intake through the saturated vapor of He^4 (curve 3). When the temperature of the lower layers of liquid He^4 rose to some value $T'''_{\max\rho}$ they ceased to feel the heating of the upper layers (curve 1, section b - d). The increase in the temperature of the upper layers (section c - d, curve 2) took place because of the intake of heat from above as a consequence of the temperature conductivity of the helium itself and the walls of the container. As a result the following value was obtained:

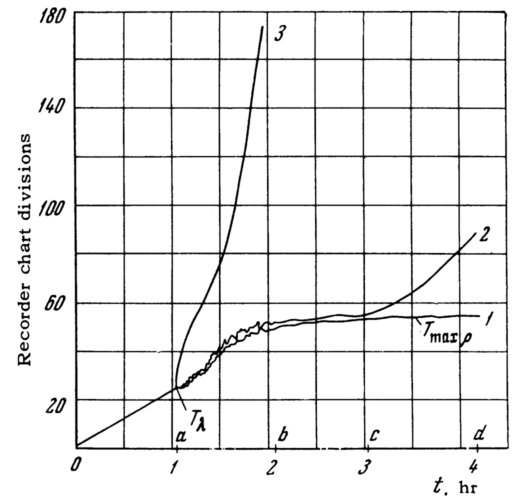

FIG. 7. Readings of the lower (1), middle (2) and upper (3) thermometers for warming liquid helium from above by means of the gas.

$$T'''_{\max\rho} - T_\lambda = 0.0064 \pm 0.0005°.$$

Thus one can assume that the different methods of determination of the λ-transition temperature give identical values within the limits of experimental accuracy. The following values of the pressure and the temperature of the λ transition are the most reliable:

$$P_\lambda = 37.80 \pm 0.03 \text{ mm Hg.} \qquad T_\lambda = 2.172_0 \pm 0.003° \text{ K.}$$

The temperature of the maximum density of liquid He^4 lies above the temperature of the λ point by

$$T_{\max\rho} - T_\lambda = 0.0065 \pm 0.0005°.$$

The results of measurement of T_λ agree with the data of Roberts and Sydoriak,[4] and Buckingham and Fairbank,[7] but are somewhat smaller than the values given by Long and Meyer[1] and by Hoare and Zimmerman.[3] This discrepancy apparently follows from the very nature of the method of measurement of P_λ which was used in [1,3]: in the passage through the λ transition the pressure increases not only in the outer tank but also in the inner one; therefore the observed difference in the pressure arises at a temperature somewhat exceeding T_λ.

The measured value $T_{\max\rho} - T_\lambda$ agrees within the limits of experimental accuracy with the experimentally obtained data for this value by Chase, Maxwell, and Millett,[9] and by Kerr and Taylor.[10]

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Translated by R. T. Beyer
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