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A STUDY OF SEMICONDUCTOR POLARIZATION BY TAKING INTO ACCOUNT SATURATION (TWO-BAND MODEL)

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We calculate the polarization vector of a semiconductor in a strong electromagnetic wave, using the two-band semiconductor model as a starting point. The description is based on the density matrix. Two relaxation times τ_1 and τ_2 are introduced phenomenologically into the density-matrix equation. The first of these times can be related to recombination and the second to polarization relaxation of the system. Polarization connected with interband transitions only is calculated; intraband transitions are not taken into account in the present work. Analytic expressions are obtained for the absorption and dispersion coefficients by taking into account the strong electromagnetic field for two models: in one model the electron energies are uniformly distributed over the bands, and in the other the parabolic shape of the band is taken into account. Nonuniform energy distribution of the electrons within the band results in an asymmetry of the absorption and dispersion curves. The calculation is performed in the approximation of the first harmonic with respect to field strength E and in the zeroth approximation with respect to D (D is the population difference of the bands), and also by taking into account the second harmonic in D. Allowance for the latter leads to deformation of the bands, corresponding to an increase of the width of the forbidden band. Only resonance terms are retained in the formulas presented.

1. INTRODUCTION. INITIAL EQUATIONS

THE use of electromagnetic radiation from lasers makes it possible to investigate the dependence of the coefficients of absorption and refraction in semiconductors on the strength of the monochromatic field. For a theoretical description of these phenomena it is necessary to determine the dependence of the complex dielectric constant of the semiconductors on the magnitude of the field, or, in other words, to calculate the polarization vector with account of saturation.

The question of allowance for saturation in semiconductors was considered by Krokhin[1-3]. He calculated the relaxation of the population difference in the bands in the Fermi quasilevel approximation. The present paper is devoted essentially to polarization in a strong field. We investigated, in particular, the frequency dependence of the absorption coefficient and of the dispersion, taking the polarization dissipation into account. This enables us to describe more accurately the frequency dependence near the edge of the absorption band.

We start out with a two-band model of the semiconductor. The index 1 denotes quantities pertaining to the valence band and index 2 the conduction band. The state of electrons of the system is characterized by four density matrices:

$$\rho_{11}(\mathbf{p}, \mathbf{p}', t); \quad \rho_{22}(\mathbf{p}, \mathbf{p}', t),$$
$$\rho_{12}(\mathbf{p}, \mathbf{p}', t), \quad \rho_{21}(\mathbf{p}, \mathbf{p}', t). \qquad (1.1)$$

The first two describe the states in the valence and conduction bands, respectively, while the last two describe the transitions between bands.

The polarization vector is determined by the expression

$$\mathbf{P}(t) = en \sum_{a,b} \int \mathbf{r}^{ba}_{\mathbf{p}',\mathbf{p}} \rho^{ab}_{\mathbf{p},\mathbf{p}'} d\mathbf{p}\, d\mathbf{p}'; \quad a, b = 1, 2. \qquad (1.2)$$

Here $\mathbf{r}^{ab}_{\mathbf{p},\mathbf{p}'}$ are the corresponding matrix elements, e the electron charge, and n the average concentration.

Let us formulate the principal limitations.

A. We consider processes due only to band transitions. Under this condition we have

$$\overset{(11)}{\mathbf{r}_{\mathbf{p},\mathbf{p}'}} = \overset{(22)}{\mathbf{r}_{\mathbf{p},\mathbf{p}'}} = 0. \qquad (1.3)$$

B. We take into account only direct transitions between bands. Under this condition, taking into account the momentum conservation law, the matrix element of the transitions can be written in the form

$$\mathbf{r}^{(12)}_{\mathbf{p},\mathbf{p}'} = \mathbf{r}^{(21)}_{\mathbf{p},\mathbf{p}'} = C\frac{\mathbf{p}}{|\mathbf{p}|}\delta(\mathbf{p}-\mathbf{p}') \equiv \mathbf{r}_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{p}'), \tag{1.4}$$

where C is a constant quantity. This quantity can also be complex, and then $C^{(12)} = C^{(21)*}$ and $\mathbf{r}_{\mathbf{p}}^{(12)} = \mathbf{r}_{\mathbf{p}}^{(21)*}$.

From (1.2)—(1.4) we obtain the following expression for the polarization vector:

$$\mathbf{P}(t) = en\int \mathbf{r}_{\mathbf{p}}(\rho^{(12)}_{\mathbf{p},\mathbf{p}} + \rho^{(21)}_{\mathbf{p},\mathbf{p}})\,d\mathbf{p}. \tag{1.5}$$

Thus, the polarization vector is determined by the functions $\rho^{(12)}_{\mathbf{p},\mathbf{p}}$ and $\rho^{(21)}_{\mathbf{p},\mathbf{p}'}$ with the momenta p and p′ coinciding. Instead of using identical indices p, p and a, a, we shall write only one index each, p and a (a = 1, 2).

Under the foregoing limitations, the equations for the functions $\rho_{\mathbf{p}}^{(1)}$ and $\rho_{\mathbf{p}}^{(12)}$ are

$$\frac{\partial\rho_{\mathbf{p}}^{(12)}(t)}{\partial t} = -\frac{i}{\hbar}[\mathscr{E}_1(\mathbf{p}) - \mathscr{E}_2(\mathbf{p})]\rho_{\mathbf{p}}^{(12)}(t) - \frac{1}{\tau_{\mathbf{p}}^{(12)}}\rho_{\mathbf{p}}^{(12)}(t)$$

$$+\frac{ie}{\hbar}\mathbf{E}(t)\mathbf{r}_{\mathbf{p}}^{(12)}(\rho_{\mathbf{p}}^{(2)}(t) - \rho_{\mathbf{p}}^{(1)}(t)), \tag{1.6}$$

$$\frac{\partial\rho_{\mathbf{p}}^{(1)}(t)}{\partial t} = -\frac{1}{\tau_{\mathbf{p}}^{(1)}}[\rho_{\mathbf{p}}^{(1)}(t) - \rho_{\mathbf{p}}^{(1)0}(t)]$$

$$+\frac{ie}{\hbar}\mathbf{E}(t)(\mathbf{r}_{\mathbf{p}}^{(12)}\rho_{\mathbf{p}}^{(21)} - \rho_{\mathbf{p}}^{(12)}\mathbf{r}_{\mathbf{p}}^{(21)}). \tag{1.7}$$

Here $\rho_{\mathbf{p}}^{(1)0}$ is the specified distribution over the momenta in the valence band. In the absence of external sources, say electron injection, it coincides with the equilibrium distribution. $\mathscr{E}_{1,2}(\mathbf{p})$ is the energy of the electrons in band 1 or 2; $\tau_{\mathbf{p}}^{(1)}$ and $\tau_{\mathbf{p}}^{(12)}$ are respectively the relaxation times for the distribution functions in band 1 and for the transition between bands. The equations for the functions $\rho_{\mathbf{p}}^{(2)}$ and $\rho_{\mathbf{p}}^{(21)}$ follow from (1.6) and (1.7).

We shall find it convenient to introduce the function

$$D(\mathbf{p}, t) = \rho_{\mathbf{p}}^{(2)}(t) - \rho_{\mathbf{p}}^{(1)}(t), \tag{1.8}$$

which defines the difference in population of the particles with momentum p in the bands.

C. We assume that

$$\tau_{\mathbf{p}}^{(1)} = \tau_{\mathbf{p}}^{(2)} = \tau_1; \qquad \tau_{\mathbf{p}}^{(12)} = \tau_{\mathbf{p}}^{(21)} = \tau_2. \tag{1.9}$$

This means that the relaxation times $\tau_{\mathbf{p}}^{(1)}$, $\tau_{\mathbf{p}}^{(2)}$, and $\tau_{\mathbf{p}}^{(12)}$, which in general are functions of the momenta, are replaced by averaged relaxation times which no longer depend on p.

From (1.6) and (1.7) we obtain, subject to conditions (1.9), the following closed system of equations:

$$\frac{\partial D(\mathbf{p}, t)}{\partial t} = -\frac{1}{\tau_1}(D(\mathbf{p}, t) - D^0(\mathbf{p}))$$

$$-\frac{2ie}{\hbar}(\mathbf{r}_{\mathbf{p}}^{(12)}\rho_{\mathbf{p}}^{(21)} - \rho_{\mathbf{p}}^{(12)}\mathbf{r}_{\mathbf{p}}^{(21)})\mathbf{E}(t), \tag{1.10}$$

$$\frac{\partial\rho_{\mathbf{p}}^{(12)}(t)}{\partial t} = -\frac{i}{\hbar}[\mathscr{E}_1(\mathbf{p}) - \mathscr{E}_2(\mathbf{p})]\rho_{\mathbf{p}}^{(12)}(t) - \frac{1}{\tau_2}\rho_{\mathbf{p}}^{(12)}(t)$$

$$+\frac{ie}{\hbar}\mathbf{r}_{\mathbf{p}}^{(12)}D(\mathbf{p}, t)\mathbf{E}(t). \tag{1.11}$$

2. EXPRESSIONS FOR THE POPULATION DIFFERENCE D(p, t) AND FOR THE POLARIZATION VECTOR P(t) WITH ALLOWANCE FOR SATURATION

Equations (1.10) and (1.11) coincide in form with the corresponding equations for the system with two discrete levels[4]. The only difference is that the quantities in (1.11) depend upon p. This dependence, however, enters parametrically, and therefore comes into play only when calculating characteristics that are averaged over p, for example, the polarization vector.

The electric field intensity is specified in the form

$$\mathbf{E}(t) = \mathbf{E}\cos\omega t. \tag{2.1}$$

In the stationary case, in the approximation of the zeroth harmonic for D and the first harmonic for E, we obtain the following expression for the functions $\rho_{\mathbf{p}}^{(12)}(t)$ and D(p):

$$\rho_{\mathbf{p}}^{(12)}(t) = -\frac{e}{2\hbar}\mathbf{r}_{\mathbf{p}}^{(12)}D(\mathbf{p})\frac{\mathbf{E}\,e^{i\omega t}}{\omega_{21} - \omega - i/\tau_2}, \tag{2.2}$$

$$D(\mathbf{p}) = D^0(\mathbf{p})\frac{(\omega_{21} - \omega)^2 + \tau_2^{-2}}{(\omega_{21} - \omega)^2 + \tau_2^{-2} + R}. \tag{2.3}$$

We put here

$$R = \frac{\tau_1}{\tau_2}\frac{e^2}{\hbar^2}|\mathbf{r}_{\mathbf{p}}^{(12)}\mathbf{E}|^2.$$

We retain only the resonance terms in expressions (2.2) and (2.3).

From (1.5), (2.2), and (2.3) follows an expression for the polarization vector with account of saturation

$$\mathbf{P}_i(t) = -\frac{e^2n}{2\hbar}\int |\mathbf{r}^{(12)}|_{ij}^2\,\mathbf{E}_jD^0(\mathbf{p})$$

$$\times\frac{(\omega_{21} - \omega)\cos\omega t + \tau_2^{-1}\sin\omega t}{(\omega_{21} - \omega)^2 + \tau_2^{-2} + R}\,d\mathbf{p}. \tag{2.4}$$

Here $D^0(\mathbf{p}) = \rho_{\mathbf{p}}^{(2)0} - \rho_{\mathbf{p}}^{(1)0}$ is the specified difference of the distributions over the momenta in the bands.

We note that when account is taken of the second harmonic of the population difference D, it is necessary to replace ω_{21} in all the expressions by

$$\omega_{21}+\frac{e^2\mathbf{E}^2r^2\omega}{\hbar^2(4\omega^2+\tau_1^{-2})}.$$

Thus, the forbidden band broadens. This nonlinear effect is small in magnitude compared with the saturation effect. In formulas (2.2)—(2.4) we have retained only the resonance terms. In the more general case, it is necessary to write, for example, in lieu of (2.4),

$$\begin{aligned}\mathbf{P}_i(t)=&-\frac{e^2n}{2\hbar}\int|\mathbf{r}^{(12)}|_{ij}^2\,\mathbf{E}_jD^0(\mathbf{p})\\&\times\left\{\frac{(\omega_{21}-\omega)\cos\omega t+\tau_2^{-1}\sin\omega t}{(\omega_{21}-\omega)^2+\tau_2^{-2}+R}\right.\\&\left.+\frac{(\omega_{21}+\omega)\cos\omega t-\tau_2^{-1}\sin\omega t}{(\omega_{21}+\omega)^2+\tau_2^{-2}+R}\right\}d\mathbf{p}.\end{aligned}$$

3. ABSORPTION COEFFICIENT OF ELECTROMAGNETIC RADIATION

From (2.4) we obtain an expression for the absorption coefficient $\alpha(\omega)$, which is the imaginary part of the complex dielectric constant, due to transition between bands:

$$\alpha(\omega)=-\frac{e^2n}{2\hbar}\int(\mathbf{r}_\mathbf{p}\mathbf{e})^2D^0(\mathbf{p})\frac{\tau_2^{-1}\,d\mathbf{p}}{(\omega_{21}-\omega)^2+\tau_2^{-2}+R}. \tag{3.1}$$

Here **e** is a unit vector along the field **E**.

In order to display the main characteristic features of the frequency dependence of the absorption, let us consider first a rather crude approximation, namely, we assume that the system is isotropic, that is,

$$|\mathbf{r}^{(12)}|_{ij}^2={}^1\!/_3r^2\delta_{ij}.$$

We assume that $\rho_\mathbf{p}^{(1)(0)}$ and $\rho_\mathbf{p}^{(2)(0)}$ are equilibrium distributions, that is, there are no external sources to cause redistribution of electrons over the bands. The distributions of the energies over these bands are replaced by a uniform distribution. Then

$$\rho_1(\mathscr{E})=1/\Delta_1,\quad \rho_2(\mathscr{E})\ll\rho_1.$$

Here Δ_1 is the width of the valence band. We denote by Δ the width of the forbidden band. Under these conditions the expression for the absorption coefficient is

$$\alpha(\omega)=\frac{e^2nr^2}{6\Delta_1A}\left\{\operatorname{arctg}\frac{\Delta_1+\Delta-\hbar\omega}{\hbar A}\tau_2-\operatorname{arctg}\frac{\Delta-\hbar\omega}{\hbar A}\tau_2\right\}, \tag*{(3.2)*}$$

$$A=(1+\tau_1\tau_2e^2r^2\mathbf{E}^2/3\hbar^2)^{1/2}. \tag{3.3}$$

A plot of $\alpha(\omega)$ is shown in Fig. 1.

At frequencies $|\Delta-\omega\hbar|\ll\Delta_1$ and $\Delta_1\gg\hbar/\tau_2$ we get from (3.2)

$$\alpha(\omega)=\frac{e^2nr^2}{6\Delta_1A}\left\{\frac{\pi}{2}+\operatorname{arctg}\frac{\hbar\omega-\Delta}{\hbar A}\tau_2\right\}. \tag{3.4}$$

In a weak field A = 1 in (3.2) and (3.4). In a strong field when A ≫ 1, expression (3.4) becomes

$$\alpha(\omega)=\frac{ern\hbar}{2\Delta_1E\sqrt{3\tau_1\tau_2}}\left\{\frac{\pi}{2}+\operatorname{arctg}\frac{\hbar\omega-\Delta}{erE}\left(\frac{3\tau_1}{\tau_2}\right)^{1/2}\right\}.$$

It follows from (3.2)—(3.4) that in a strong field the transition region at the edge of the absorption band broadens, and the absorption coefficient decreases at all frequencies.

When E = 0 the width of the edge of the band δ is of the order $1/\tau_2$, and in the presence of a field

$$\delta\sim A/\tau_2. \tag{3.5}$$

The width of the entire absorption band is determined by the width of the valence band Δ_1. This follows from (3.2). It must be noted once more, however, that within the framework of the present paper we take no account of the interband transitions that lead to an additional limitation on the width of the absorption line.

Let us consider now a more accurate expression for the absorption coefficient. We start with the case of a weak field.

When E = 0 expression (3.1), after integration over the angles, becomes

$$\alpha(\omega)=\frac{e^2nr^2}{6\hbar\tau_2}\int_0^{\Delta_1}\frac{\rho_1(\mathscr{E})\,d\mathscr{E}}{[(\mathscr{E}+\Delta)/\hbar-\omega]^2+\tau_2^{-2}}, \tag{3.6}$$

where

$$\rho_1(\mathscr{E})=\frac{3}{2}\frac{\mathscr{E}^{1/2}}{\mathscr{E}_0^{3/2}}=\frac{4\pi(2\mu)^{3/2}}{(2\pi\hbar)^3}\frac{V}{N}\mathscr{E}^{1/2}, \tag{3.7}$$

$\mathscr{E}_0$ is the maximum energy of electrons at T = 0^0,

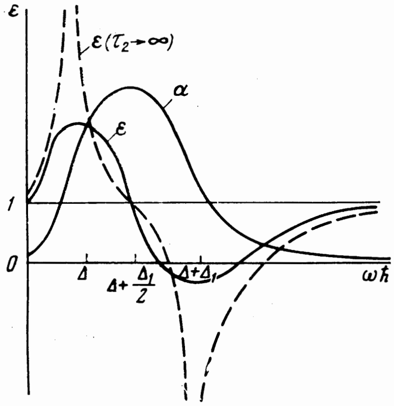

FIG. 1

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*arctg ≡ $\tan^{-1}$.

$\mu = m_1 m_2/(m_1 + m_2)$ is the reduced effective mass of electrons. In (3.6), as above, it is assumed that the valence band is completely filled and the conduction band is empty.

When $1/\tau_2 \to 0$ there follows from (3.6) the well-known formula for the absorption coefficient[5,6]:

$$\alpha(\omega) = \frac{\pi n e^2 r^2}{4\mathscr{E}_0^{3/2}} (\hbar\omega - \Delta)^{1/2}, \qquad \Delta < \hbar\omega < \Delta_1 + \Delta. \tag{3.8}$$

It follows from the foregoing that formula (3.8) does not take into account the finite nature of the relaxation time of the polarization of the medium (τ_2) and therefore does not describe with sufficient accuracy the frequency dependence of the absorption coefficient at the edge of the band.

Integrating in (3.6) with finite $1/\tau_2$ we obtain the following expression for the absorption coefficient

$$\alpha(\omega) = \frac{n e^2 r^2 \hbar}{8\mathscr{E}_0^{3/2} \tau_2} \left\{ \frac{1}{(q - p^2/4)^{1/2}} \left(\operatorname{arctg} \frac{\sqrt{\Delta_1} + p/2}{(q - p^2/4)^{1/2}} + \operatorname{arctg} \frac{\sqrt{\Delta_1} - p/2}{(q - p^2/4)^{1/2}} \right) - \frac{1}{p} \ln \frac{\Delta_1 + p\sqrt{\Delta_1} + q}{|\Delta_1 - p\sqrt{\Delta_1} + q|} \right\} \tag{3.8a}$$

$$p = \sqrt{2}\{[(\hbar\omega - \Delta)^2 + \hbar^2/\tau_2^2]^{1/2} - (\hbar\omega - \Delta)\}^{1/2},$$
$$q = [(\hbar\omega - \Delta)^2 + \hbar^2/\tau_2^2]^{1/2}. \tag{3.8b}$$

Three limiting cases are of importance:

1. $\Delta_1 \gg \hbar\omega, \Delta, \hbar/\tau_2$—independent band. Under these conditions we get from (3.8a)

$$\alpha(\omega) = \frac{1}{8} \frac{\pi n e^2 r^2}{\mathscr{E}_0^{3/2}} \frac{\hbar}{\tau_2 p}. \tag{3.9}$$

2. $|\hbar\omega - \Delta| \gg \Delta_1$. From (3.8a) it follows that when $\hbar\omega \to \infty$ but when Δ_1 and Δ are finite we get $\alpha(\omega) \to 0$. This means that the absorption band has a finite width.

3. $1/\tau_2 \to 0$. In this limiting case formula (3.8a) coincides with formula (3.8).

We note once more that formula (3.8) does not describe quite accurately the dependence of the absorption coefficient on the frequency even near the edge of the band. Indeed, for example, when $\hbar\omega = \Delta$, it follows from (3.8) that $\alpha(\omega) = 0$, and from (3.8a) we get

$$\alpha(\omega)\big|_{\omega=\Delta/\hbar} = \frac{\sqrt{2}}{8} \frac{\pi n e^2 r^2}{\mathscr{E}_0^{3/2}} \left(\frac{\hbar}{\tau_2}\right)^{1/2}. \tag{3.10}$$

We take account here of the fact that $\Delta_1 \gg \hbar/\tau_2$.

It follows from (3.9) that the absorption coefficient differs from zero also when $\hbar\omega < \Delta$.

Using formula (3.1) under the same assumptions, we can obtain an expression for the absorption coefficient with saturation taken into account. We present here somewhat simplified results. The simplification consists in replacing in the denominator of formula (2.4) the quantity $\cos^2(\mathbf{p}\cdot\mathbf{e})$ by its mean value, that is, by 1/3.

Under this condition, the coefficient $\alpha(\omega)$ is determined as before by formula (3.8a), provided we replace in it $1/\tau_2$ by A/τ_2, everywhere except in the coefficient preceding the curly bracket (A is defined by (3.3)).

When $\hbar\omega = \Delta$ we obtain, taking saturation into account, the following expression in lieu of (3.10):

$$\alpha\left(\omega = \frac{\Delta}{\hbar}\right) = \frac{\sqrt{2}}{8} \frac{n e^2 r^2}{\mathscr{E}_0^{3/2}} \left(\frac{\hbar}{\tau_2}\right)^{1/2} \left(1 + \frac{\tau_1 \tau_2 e^2 r^2 E^2}{3\hbar^2}\right)^{-1/4}. \tag{3.11}$$

From the foregoing formula it follows that the character of the frequency dependence of $\alpha(\omega)$ is different for the two analyzed approximations, although the general rules do coincide.

The use of the more approximate model, which was considered at the beginning of this section, is nonetheless expedient, since the energy distribution inside the band, employed in modern band theory, is to a considerable degree a consequence of model representations and not a consistent theory.

4. DISPERSION OF THE SYSTEM

We begin again with the cruder model considered at the beginning of Sec. 3. Within the framework of these approximations we obtain from (2.4) the following expression for the real part of the dielectric constant:

$$\varepsilon = 1 + \frac{\pi e^2 r^2 n}{\Delta_1} \ln \frac{(\Delta_1 + \Delta - \hbar\omega)^2 + \hbar^2/\tau_2^2 + e^2 E^2 r^2 \tau_1/3\tau_2}{(\Delta - \hbar\omega)^2 + \hbar^2/\tau_2^2 + e^2 E^2 r^2 \tau_1/3\tau_2}. \tag{4.1}$$

It follows from this that the frequency dependence $\epsilon(\omega)$ has a character that is close to the normal (see Fig. 1). The dashed curve shows the function $\epsilon(\omega)$ when $A/\tau_2 \to 0$, that is, without account of polarization dissipation.

We now consider the dispersion curve with allowance for the energy distribution in the valence band. As a result of integration under the same conditions as in Sec. 3, we obtain:

$$\varepsilon = 1 + \frac{e^2 n \pi r^2}{\mathscr{E}_0^{3/2}} B,$$

$$B = 2\sqrt{\Delta_1} - \frac{p}{4} \ln \frac{\Delta_1 + p\sqrt{\Delta_1} + q}{|\Delta_1 - p\sqrt{\Delta_1} + q|} - \sqrt{q - p^2/4} \times \left(\operatorname{arctg} \frac{\sqrt{\Delta_1} + p/2}{(q - p^2/4)^{1/2}} + \operatorname{arctg} \frac{\sqrt{\Delta_1} - p/2}{(q - p^2/4)^{1/2}} \right). \tag{4.2}$$

The quantities p and q are defined by (3.8b). Saturation is taken into account here in the same approximation as at the end of section 3.

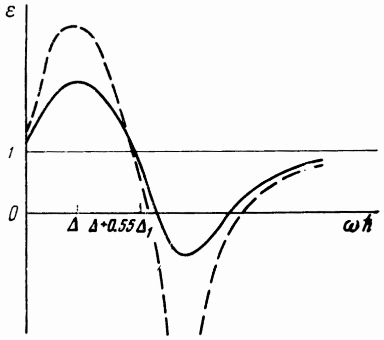

FIG. 2

In the limiting case $A/\tau_2 \rightarrow 0$ the expression for B in formula (4.2) simplifies greatly and takes the form

$$B = 2\sqrt{\Delta_1} - 2\sqrt{\Delta - \hbar\omega}\,\mathrm{arctg}\frac{\sqrt{\Delta_1}}{\sqrt{\Delta - \hbar\omega}}, \qquad \hbar\omega < \Delta;$$

$$B = 2\sqrt{\Delta_1} - \sqrt{\hbar\omega - \Delta}\ln\frac{\sqrt{\hbar\omega - \Delta} + \sqrt{\Delta_1}}{|\sqrt{\hbar\omega - \Delta} - \sqrt{\Delta_1}|}, \ \hbar\omega > \Delta. \quad (4.3)$$

This expression is valid for all ω, except regions with width of the order of A/τ_2 near the values $\hbar\omega = \Delta$ and $\hbar\omega = \Delta_1 + \Delta$.

The qualitative variation of the function $\epsilon(\omega)$ is shown in Fig. 2. The dashed curve shows the function $\epsilon(\omega)$ when $A/\tau_2 \rightarrow 0$.

From (4.2) we can obtain the value of $\epsilon(\omega)$ at the points $\hbar\omega = \Delta$ and $\hbar\omega = \sqrt{\Delta_1} + \Delta$, where this function reaches a maximum and a minimum. In the region of anomalous dispersion, $\epsilon = 1$ when $\hbar\omega \approx \Delta + 0.55\,\Delta_1$; when $\hbar\omega \rightarrow \infty$ we get $\epsilon(\omega) \rightarrow 1$.

All the singularities of the $\epsilon(\omega)$ curves coincide in the approximations outlined above. The only difference is that in the second case the value of ϵ at the point of the maximum and the minimum is not the same. This difference is especially noticeable when $A/\tau_2 \rightarrow 0$, when the maximum is finite and is equal to $2\sqrt{\Delta_1}$, and the minimum tends to infinity. This asymmetry is due to the fall-off of the distribution function at the upper edge of the valence band.

In conclusion we note that the expression (2.4) for the polarization vector is valid also for states with a negative temperature, when the population in some region of the conduction band is larger than the population in the corresponding region of the valence band. We do not present the calculations for this case, since they go beyond the scope of the present article and deserve a separate analysis. The latter pertains also to questions connected with account of the intraband transitions. Closely related to them is, finally, also the question of calculating the relaxation time τ_1 and τ_2. When account is taken of the interband transitions, one more relaxation time, τ_3, appears and determines the time of slowing down of electrons in the conduction band. τ_3 was calculated by Popov and Krokhin[7,8]. Apparently no corresponding calculations of the times τ_1 and τ_2 were made.

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