DEPENDENCE OF THE RADIATION INTENSITY OF A GAS LASER ON THE MAGNETIC FIELD

M. I. D'YAKONOV and V. I. PEREL'

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

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The dependence of the radiation intensity of a gas laser on longitudinal and transverse magnetic fields is considered on the basis of a simplified model. In the proposed model the area of the dips of the amplification curve is proportional to the intensity. It is shown that the radiation intensity has a minimum for zero magnetic field, and also when the Zeeman splitting of the levels is equal to the resonator detuning. In addition, an intensity minimum may occur in a transverse magnetic field when the Zeeman splitting is equal to twice the resonator detuning. The intensity minima are interpreted as the result of merging of the dips in the amplification curve. When the dips merge, a nonmonotonic dependence of the generation frequency on the magnetic field may also occur.

THE amplification of radiation intensity of a gas laser when a weak longitudinal magnetic field is included has been examined in a series of experiments.[1-3] The present paper proposes a simplified model which permits an intuitive interpretation of this phenomenon by considering the dips in the amplification curve. Moreover, this model shows that the radiation intensity should have a second minimum when the Zeeman splitting Ω coincides with the resonator detuning δ. When $\Omega \sim \delta$ a nonmonotonic dependence of the shift of generation frequency on the magnetic field may manifest itself, similar to that which occurs for a weak magnetic field.[1] The effect of a weak longitudinal magnetic field on the shift of generation frequency has been considered previously.[4-5]

We shall suppose here that the magnetic field is not large enough for it to exert any substantial influence of the discharge properties.

1. Let the upper and lower operating levels have total momenta j_1 and j_0 respectively. We shall number the magnetic sublevels of the upper state by the symbols m and m′, and the sublevels of the lower state by the symbols μ and μ'. We must then write three groups of equations for the density matrix elements of the system:

$$\frac{df_{mm'}}{dt} = -(i\Omega_{mm'}+\gamma_1)f_{mm'} + \frac{i}{\hbar}\sum_{\mu}[(\vec{\mathscr{E}}\mathbf{d}_{m\mu})f_{\mu m'} - f_{m\mu}(\vec{\mathscr{E}}\mathbf{d}_{\mu m'})] + \gamma_1 N_1 F(v)\delta_{mm'},$$

$$\frac{df_{\mu\mu'}}{dt} = -(i\Omega_{\mu\mu'}+\gamma_0)f_{\mu\mu'} + \frac{i}{\hbar}\sum_{m}[(\vec{\mathscr{E}}\mathbf{d}_{\mu m})f_{m\mu'} - f_{\mu m}(\vec{\mathscr{E}}\mathbf{d}_{m\mu'})] + \gamma_0 N_0 F(v)\delta_{\mu\mu'},$$

$$\frac{df_{\mu m}}{dt} = (i\omega_0 - i\Omega_{\mu m} - \gamma_{10})f_{\mu m} + \frac{i}{\hbar}\Big[\sum_{m_1}(\vec{\mathscr{E}}\mathbf{d}_{\mu m_1})f_{m_1 m} - \sum_{\mu_1} f_{\mu\mu_1}(\vec{\mathscr{E}}\mathbf{d}_{\mu_1 m})\Big]. \qquad (1)$$

Here s is the coordinate along the resonator axis, ω_0 is the transition frequency for zero magnetic field,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial s}, \qquad \Omega_{mm'} = (m-m')\Omega_1,$$

$$\Omega_{\mu\mu'} = (\mu-\mu')\Omega_0, \qquad \Omega_{\mu m} = \mu\Omega_0 - m\Omega_1,$$

Ω_1 and Ω_0 are the frequency intervals between the Zeeman sublevels of the upper and lower states, $\vec{\mathscr{E}}$ is the electric field strength in the resonator, $\mathbf{d}$ is the atomic dipole moment operator, γ_1 and γ_0 are the natural widths of the upper and lower states, and $\gamma_{10} = (\gamma_1 + \gamma_0)/2$. Pumping is assumed to be isotropic and homogeneous so that atoms are produced on the first and ground levels with a Maxwellian velocity distribution. F(v) is the Maxwell distribution normalized to unity; N_1 has the meaning of the population of an individual upper state sub-level which would be created by pumping if there were no electromagnetic wave field in the resonator; N_0 is a similar quantity for the lower state.

The field $\vec{\mathscr{E}}$ in the resonator has the form

$$\vec{\mathscr{E}} = [\mathbf{e}(t) + \mathbf{e}^*(t)] \sin ks, \quad \mathbf{e}(t) = \mathbf{E}e^{i\omega t},$$

$$k = \omega_n / c = \pi n / L, \tag{2}$$

where ω_n is the natural resonator frequency for the mode under consideration, ω is the frequency at which generation takes place, L is the resonator length.

In order to find the radiation intensity and shift of generation frequency relative to ω_n, it is necessary to determine from (1) the quantity $f_{\mu m}(s, v, t)$ and to calculate the dipole moment (per unit volume) inducing the oscillations of the resonator mode under consideration:

$$\mathbf{P} = \frac{2}{L} \int_0^L ds \sin ks \int_{-\infty}^{\infty} dv \, \mathbf{D}(s, v, t),$$

$$\mathbf{D}(s, v, t) = \sum_{m\mu} f_{\mu m}(s, v, t) \mathbf{d}_{m\mu}. \tag{3}$$

The electric field strength in the resonator is then found from the equation[6]

$$\frac{d^2\mathbf{e}}{dt^2} + \frac{\omega_n}{Q}\frac{d\mathbf{e}}{dt} + \omega_n^2 \mathbf{e} = -4\pi \frac{d^2\mathbf{P}}{dt^2}. \tag{4}$$

2. To describe the basic effects which arise when a laser is placed in a magnetic field, it suffices to find the quantity **P** with an accuracy to terms of the third order in electric field strength **E**. In principle there is no difficulty in solving Eq. (1) to this degree of approximation[5] by following Lamb's procedure.[6] However the expressions obtained are exceedingly cumbersome, so there is reason to consider a simplified model which nevertheless validly reflects the basic characteristics of the phenomena under consideration and enables us to give them a simple physical interpretation. This model is based on two assumptions: 1) the g-factors of the upper and lower levels are equal ($\Omega_1 = \Omega_0 = \Omega$); this assumption is almost exact when applied to the basic lines of the He-Ne laser; 2) the density matrices of the upper and lower states may be written in the form $f_{mm'} = \delta_{mm'} f_1$, $f_{\mu\mu'} = \delta_{\mu\mu'} f_0$. This supposition may have some basis if depolarizing collisions with small change of velocity play a significant part. Generally speaking, however, the non-diagonal elements of $f_{mm'}$ and $f_{\mu\mu'}$ are not small, although the contribution from them does not alter the general pattern of the phenomena under consideration.

Under these suppositions one may replace the system of equations (1) with a simpler one:

$$\frac{df_1}{dt} = -\gamma_1 f_1 + \frac{i}{\hbar(2j_1+1)} \sum_q (-1)^q \mathscr{E}_q [D_{-q} - (D^*)_{-q}] + \gamma_1 N_1 F,$$

$$\frac{df_0}{dt} = -\gamma_0 f_0 - \frac{i}{\hbar(2j_0+1)} \times \sum_q (-1)^q \mathscr{E}_q [D_{-q} - (D^*)_{-q}] + \gamma_0 N_0 F,$$

$$\frac{dD_q}{dt} = (i\omega_0 + iq\Omega - \gamma_{10}) D_q + \frac{i}{\hbar} d^2 \mathscr{E}_q (f_1 - f_0),$$

$$d^2 = 1/3 \,|\, (j_1 \| d \| j_2) \,|^2. \tag{5}$$

Here $\mathscr{E}_q$ and D_q ($q = 0, \pm 1$) are the circular components of the vectors $\vec{\mathscr{E}}$ and **D** defined by formulas (2) and (3); the z axis is directed along the magnetic field; the quantity d^2 is connected with the probability W_{10} of spontaneous transition from an upper to a lower level:

$$W_{10} = 4\omega_0^3 d^2 / \hbar c^3 (2j_1 + 1).$$

Equation (5) can easily be solved with an accuracy to terms of third order in the field **E**[6] (the condition $E^2 d^2 \ll \hbar^2 \gamma_1 \gamma_0$ is the criterion for the validity of this approximation). Calculation of the dipole moment from formula (3) leads to the result

$$P_q = \chi_q e_q, \tag{6}$$

where the nonlinear polarizability χ_q is given by the expression

$$\chi_q = \frac{d^2}{\hbar} \int_{-\infty}^{\infty} \frac{(f_1 - f_0)\, dv}{\delta - q\Omega + kv - i\gamma_{10}}, \tag{7}$$

$$f_1 - f_0 = (N_1 - N_0) F(v) \left[1 - \sum_q I_q r_q(v) \right]. \tag{8}$$

We have introduced here the following symbols: $\delta = \omega - \omega_0$,

$$I_q = \frac{d^2 |E_q|^2}{2\hbar^2 \gamma_{10}} \left[\frac{1}{(2j_1+1)\gamma_1} + \frac{1}{(2j_0+1)\gamma_0} \right]$$

is the dimensionless radiation intensity with polarization q, and

$$r_q(v) = \frac{\gamma_{10}^2}{(\delta - q\Omega + kv)^2 + \gamma_{10}^2} + \frac{\gamma_{10}^2}{(\delta - q\Omega - kv)^2 + \gamma_{10}^2}. \tag{9}$$

The integral in formula (7) can be calculated if we assume that $\gamma_{10} \ll ku$ (u is the most probable atomic velocity). We then obtain

$$\chi_q = a_q - \sum_{q_1} b_{qq_1} I_{q_1}, \tag{10}$$

$$a_q = \beta e^{-\xi_q^2} \left(i + \frac{2}{\sqrt{\pi}} \int_0^{\xi_q} e^{t^2} dt \right), \tag{11}$$

$$b_{qq_1} = \beta e^{-\xi_q^2}\left[\frac{\gamma_{10}}{(q_1-q)\Omega - 2i\gamma_{10}} + \frac{\gamma_{10}}{2\delta - (q_1+q)\Omega - 2i\gamma_{10}}\right];$$
$$\beta = \frac{\pi^{1/2}d^2}{\hbar ku}(N_1 - N_0), \quad \xi_q = \frac{\delta - q\Omega}{ku}. \qquad (12)$$

Formulas (10)–(12) give the final expression for the polarizability for the model we have chosen.

3. Before using formulas (10)–(12) for further calculations we note an important consequence of the simplified model adopted here which allows us to obtain basic qualitative results without computations. Following Bennet,[7] we introduce the auxiliary quantity $\chi_q(\omega_c)$ side by side with the nonlinear polarizability χ_q. This quantity denotes the polarizability for a weak signal of frequency ω_c propagating in a medium in which the velocity distribution of excited atoms is distorted by a strong field of frequency ω. It is clear that

$$\chi_q(\omega_c) = \frac{d^2}{\hbar}\int_{-\infty}^{\infty}\frac{(f_1 - f_0)\,dv}{\omega_c - \omega_0 - q\Omega + kv - i\gamma_{10}}, \qquad (13)$$

where $f_1 - f_0$ is determined by formula (8) just as in expression (7). Comparison of formulas (7) and (13) shows that

$$\chi_q(\omega_c)\,|_{\omega_c=\omega} = \chi_q. \qquad (14)$$

$\chi_q^{(0)}(\omega_c)$ denotes the polarizability of the medium for a weak signal when the polarizability is not distorted by a strong field. This quantity is given by expression (13), in which $(N_1 - N_0)F(v)$ must be inserted instead of $f_1 - f_0$. We set $\chi_q = \chi_q' + i\chi_q''$. Then with the help of relationships (7), (8), and (13), and neglecting terms of order I_q^2, an expression may be obtained for the quantity

$$S = 4\pi\int_{-\infty}^{\infty}[\chi_q^{(0)''}(\omega_c) - \chi_q''(\omega_c)]\,d\omega_c = 8\pi^2\gamma_{10}\sum_q \chi_q'' I_q. \qquad (15)$$

The quantity S has a simple meaning. The presence of a strong wave field leads to the formation of dips in the amplification coefficient for a weak signal.[7] The widths of the dips are determined by the quantity γ_{10}, and S is a quantity proportional to the total area of these dips. We note that this does not depend on q in the model which has been adopted, i.e., it is identical for all polarizations.

Figure 1a shows the dependence of the quantity $4\pi\chi_q''(\omega_c)$, proportional to the amplification coefficient, on the frequency of a weak signal ω_c in the absence of a magnetic field. There are two dips in this case: one at $\omega_c = \omega \approx \omega_n$ (ω_n is the natural resonator frequency), the other symmetrical to the first relative to the center of the curve ω_0. The generation condition

$$4\pi\chi_q'' = 1/Q \qquad (16)$$

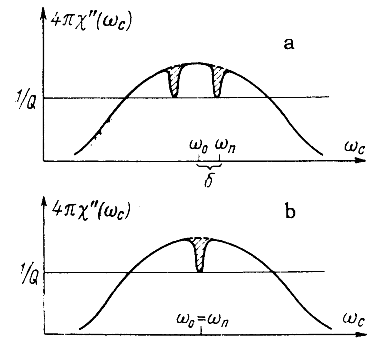

FIG. 1

demands that the bottom of each dip should touch the horizontal line $4\pi\chi_q''(\omega_c) = 1/Q$. The quantity S represents the area of the shaded regions in Fig. 1a. It follows from formulas (15) and (16) that

$$\sum_q I_q = \frac{Q}{2\pi\gamma_{10}}S. \qquad (17)$$

Thus the radiation intensity is proportional to the total area of the dips in the amplification coefficient for a weak signal. This statement is always valid for a two-level system generally adopted in the analysis of a gas laser in the absence of a magnetic field. Generally speaking it is incorrect when Zeeman splitting in a magnetic field is taken into account. However in the model adopted here it is valid in this case also, as will be shown below. This circumstance serves as the basis of further qualitative consideration. In particular, the well-known effect of the lessening of radiation intensity upon exact tuning of the resonator to the center of the curve[6, 8] follows from formula (17). This situation is represented in Fig. 1b. On comparison with Fig. 1a, it is clear that the area of the dips and consequently the radiation intensity also has decreased.

4. We shall consider the influence of a longitudinal magnetic field on the radiation intensity of a gas laser with no defined direction of polarization. The z axis in this case is directed along the laser axis and $q = \pm 1$, which corresponds to left or right handed circular polarization of the radiation. If both polarizations are present in the radiation then the condition

$$4\pi\chi_1'' = 4\pi\chi_{-1}'' = 1/Q \qquad (18)$$

must be fulfilled.

It is clear from formulas (15) and (18) that in this case the total intensity and the area of the dips in the amplification coefficient for each polarization are proportional to each other. Figure 2 shows

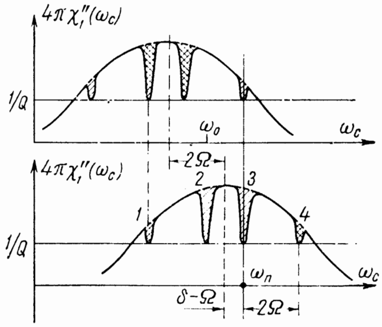

FIG. 2

the amplification coefficient for left-hand (upper diagram) and right-hand (lower diagram) polarization in the presence of a magnetic field. The maxima of these curves are now separated by 2Ω (we recall that Ω is the separation between Zeeman sublevels which is considered to be the same for upper and lower states).

Let us examine the plot of the quantity $4\pi\chi_1''(\omega_c)$. There are four dips on this curve altogether. The singly shaded dips are associated with the distortion of the atom-velocity distribution under the influence of the strong field E_1 ("own" dips). The doubly shaded dips are associated with the effect of the strong field E_{-1} ("foreign" dips). The strong field E_{-1} which is left-hand circularly polarized diminishes the population difference for atoms whose velocity satisfies the condition $\omega_0 - \Omega \pm kv = \omega$. These atoms participate in the amplification of a weak signal which is right-hand circularly polarized with frequency $\omega_c = \omega_0 + \Omega + kv$. Thus "foreign" dips arise in the $4\pi\chi_1''(\omega_c)$ curve for $\omega_c = \omega_0 + \delta + 2\Omega$ and $\omega_c = \omega_0 - \delta$, which are indicated by double shading in Fig. 2. The dips in the $4\pi\chi_{-1}''(\omega_c)$ curve come about in a similar manner.

The effect of a field with one polarization on the amplification coefficient for another with a different polarization is clear from formula (10). Generally speaking it occurs when there are no depolarizing collisions which lead to the equalization of the populations in the Zeeman sublevels.[5] In the model adopted here however the dips caused by the field E_{-1} in the $4\pi\chi_1''(\omega_c)$ curve should be exactly the same as in the $4\pi\chi_{-1}''(\omega_c)$ curve. It suffices in what follows to consider one of the curves in Fig. 2, for example $4\pi\chi_1''(\omega_c)$. As is clear from Fig. 2, a merging of dips in pairs 1,2 and 3,4 occurs when the magnetic field tends to zero ($\Omega \to 0$). Thus the radiation intensity decreases. Another anomaly occurs when the Zeeman splitting Ω becomes equal to the resonator detuning δ. "Own" dips 2 and 3 merge for $\Omega = \delta$ and the radiation intensity also decreases.

It is not difficult to calculate the radiation intensity for polarizability χ_q from Eqs. (18) and formulas (10)–(12). Expressions for intensity turn out to be simplest in the two cases indicated below:

1) $\delta \gg \gamma_{10}$, $|\Omega - \delta| \sim \gamma_{10}$;

$$I_1 = 2\varphi(\delta - \Omega)\left[1 + \frac{\gamma_{10}^2}{(\delta - \Omega)^2 + \gamma_{10}^2}\right]^{-1}; \qquad (19a)$$

2) $\delta = 0$,

$$I_1 + I_{-1} = 2\varphi(\Omega)\left[1 + \frac{\gamma_{10}^2}{\Omega^2 + \gamma_{10}^2}\right]^{-1}. \qquad (19b)$$

Here we have introduced the symbol

$$\varphi(x) = 1 - (4\pi\beta Q)^{-1} \exp (x / ku)^2. \qquad (20)$$

Of course, formulas (19) can be used only if $\varphi > 0$.

5. The shift of the generation frequency ω_1 (for right-hand circularly polarized oscillations) from the natural resonator frequency ω_n is determined by the real part of the polarizability

$$2(\omega_1 - \omega_n) / \omega_n = -4\pi\chi_1'. \qquad (21)$$

The presence of a dip in the amplification curve $4\pi\chi_1''(\omega_c)$ at a certain frequency corresponds to nonmonotonic behavior of the dispersion $4\pi\chi_1'(\omega_c)$ close to this frequency (see Fig. 3). The length of

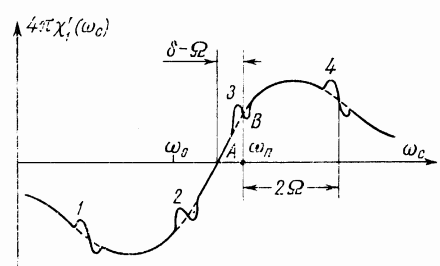

FIG. 3

the section AB in Fig. 3 is proportional to the shift of generation frequency. When dips 3 and 4 in Fig. 2 merge, the bend 4 of the dispersion curve in Fig. 3 "travels" to point B, which leads to a nonmonotonic dependence on the magnetic field of the shift of generation frequency. Such non-monotonicity will occur for $\Omega = 0$ and for $\Omega = \delta$, i.e., in the same places where there are radiation intensity minima. We note that in fact this non-monotonicity will appear only for sufficiently large pumping. The smaller the ratio γ_{10}/ku, the lower the pumping at which the non-monotonic frequency shift will appear.

We give formulas describing the dependence of the beat frequency $\Delta = \omega_1 - \omega_{-1}$ on the magnetic field close to the singularities:

1) $\delta \gg \gamma_{10}$, $|\Omega - \delta| \sim \gamma_{10}$;

$$\frac{\Delta}{\omega_n} = 4\pi\beta\left[\frac{1}{2}F\left(\frac{2\delta}{ku}\right) - \frac{1}{\sqrt{\pi}}\frac{\delta - \Omega}{ku} - \varphi(\delta - \Omega)\frac{\gamma_{10}(\delta - \Omega)}{(\delta - \Omega)^2 + 2\gamma_{10}^2}\right]; \qquad (22)$$

2) $\delta = 0$;

$$\frac{\Delta}{\omega_n} = 4\pi\beta\left[F\left(\frac{\Omega}{ku}\right) - \varphi(\Omega)\frac{\gamma_{10}\Omega}{\Omega^2 + 2\gamma_{10}^2}\right].$$

Here F(x) denotes the function

$$F(x) = \frac{2}{\sqrt{\pi}}e^{-x^2}\int_0^x e^{t^2}\,dt.$$

We note that the parameter which enters into formulas (10), (19), and (22) may be expressed in the form

$$\beta = \frac{1}{4\pi Q}\frac{N_1 - N_0}{(N_1 - N_0)_0},$$

where $(N_1 - N_0)_0$ is the threshold overpopulation in the absence of a magnetic field and for exact resonator tuning ($\Omega = 0$, $\delta = 0$).

Figure 4 gives a schematic representation of the dependence of the radiation intensity (A) and the beat frequency (B) on the magnetic field for fairly large pumping. The increase of intensity in a weak longitudinal magnetic field has been examined in a series of papers.[1-3] Culshaw and Kannelaud[1] examined the non-monotonic behavior of the beat frequency in a weak magnetic field for $\delta = 0$. As far as we know they did not examine the singularities which occur for $\Omega = \delta$.

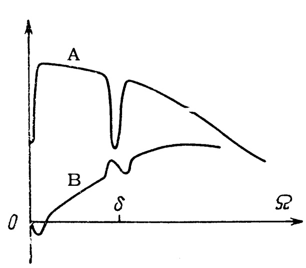

FIG. 4

No beats occur in a laser with Brewster windows, since the radiation polarization must remain linear. The dependence of intensity on magnetic field will have the same peculiarities as in a plane laser. The condition $\frac{1}{2}(4\pi\chi_1'' + 4\pi\chi_{-1}'') = 1/Q$ must be used instead of (18). From this with the help of formulas (10)–(12) in which $I_1 = I_{-1} = I/2$, we obtain the following expression for the radiation intensity I of a laser with Brewster windows:

$$I = 4[\psi(\Omega) + \psi(-\Omega)]^{-1}\left\{\varphi(\delta + \Omega)\exp\left[-\left(\frac{\delta + \Omega}{ku}\right)^2\right] + \varphi(\delta - \Omega)\exp\left[-\left(\frac{\delta - \Omega}{ku}\right)^2\right]\right\}, \qquad (23)$$

where

$$\psi(\Omega) = \exp\left[-\left(\frac{\delta - \Omega}{ku}\right)^2\right] \times \left[1 + \frac{\gamma_{10}^2}{\delta^2 + \gamma_{10}^2} + \frac{\gamma_{10}^2}{\Omega^2 + \gamma_{10}^2} + \frac{\gamma_{10}^2}{(\delta - \Omega)^2 + \gamma_{10}^2}\right],$$

the function $\varphi(x)$ is given by formula (20).

6. In the presence of a transverse magnetic field, we let the y axis lie along the resonator axis (the z axis is directed along the magnetic field). In this case the intensities I_x and I_z are determined from the equations

$$4\pi\chi_x'' = 1/Q, \quad 4\pi\chi_z'' = 1/Q, \qquad (24)$$

where $\chi_x = \frac{1}{2}(\chi_1 + \chi_{-1})$, $\chi_z = \chi_0$. Analysis of the dips in the $\chi_x''(\omega_c)$ and $\chi_z''(\omega_c)$ curves shows that in addition to the singularities at $\Omega = 0$ and $\Omega = \delta$ a radiation intensity minimum may occur for $\Omega = 2\delta$. This last singularity is associated with the merging of "own" and "foreign" dips and will thus appear only if light of both polarizations is present in the laser radiation. It is not difficult to obtain an expression for the intensity in this case using formulas (10)–(12) and (24).

7. The model employed in this paper allows us to give an intuitive interpretation to the dependence of laser radiation intensity and shift of generation frequency on the magnetic field. Calculations which we have carried out without assuming that the density matrices of the upper and lower states $f_{mm'}$ and $f_{\mu\mu'}$[9] are diagonal show that the qualitative results which we have obtained here are basically valid. However the influence of the non-diagonal elements is not small particularly for weak magnetic fields when the Hanle effect is imposed on the phenomena associated with the merging of dips which we have analyzed here.

For example in the case of a longitudinal magnetic field the polarizability is given as before by formula (10) in which, however, the coefficients b_{qq_1} must be replaced by $\tilde{b}_{qq_1}$, where

$$\tilde{b}_{11} = C_1 b_{11},$$

$$\tilde{b}_{1,-1} = C_2 b_{1,-1} + C_3(\Omega)\frac{\beta}{2}\left(\frac{\gamma_{10}}{\delta - \Omega - i\gamma_{10}} - \frac{\gamma_{10}}{\Omega + i\gamma_{10}}\right)e^{-\xi_1^2},$$

$$\tilde{b}_{-q,-q_1}(\Omega) = \tilde{b}_{qq_1}(-\Omega),$$

where

$$C_1 = A_1(\gamma_1 + \gamma_0)\Gamma^{-1}, \qquad C_2 = (A_2\gamma_1 + A_3\gamma_0)\Gamma^{-1},$$

$$C_3 = \left(A_2\gamma_0\frac{\gamma_1}{\gamma_1 - 2i\Omega} + A_3\gamma_1\frac{\gamma_0}{\gamma_0 - 2i\Omega}\right)\Gamma^{-1},$$

$$\Gamma = {}^1/_9[(2j_0+1)\gamma_0+(2j_1+1)\gamma_1](2j_1+1)^{-1}(2j_0+1)^{-1}.$$

The quantities A_1, A_2 and A_3 are expressed in terms of 6j symbols

$$A_1 = \frac{1}{3}\begin{Bmatrix} 1 & 1 & 0 \\ j_0 & j_0 & j_1 \end{Bmatrix}^2 + \frac{1}{2}\begin{Bmatrix} 1 & 1 & 1 \\ j_0 & j_0 & j_1 \end{Bmatrix}^2 + \frac{1}{6}\begin{Bmatrix} 1 & 1 & 2 \\ j_0 & j_0 & j_1 \end{Bmatrix}^2,$$

$$A_2 = \begin{Bmatrix} 1 & 1 & 2 \\ j_1 & j_1 & j_0 \end{Bmatrix}^2, \qquad A_3 = \begin{Bmatrix} 1 & 1 & 2 \\ j_0 & j_0 & j_1 \end{Bmatrix}^2.$$

For the case $j_1 = 1$, $j_0 = 2$ (He-Ne laser) $A_1 = {}^{23}/_{450}$, $A_2 = {}^1/_{900}$, $A_3 = {}^7/_{300}$.

The deviation of the coefficients C_1 and C_2 from unity is caused by the fact that the populations of the various Zeeman sublevels are in fact not identical. The appearance of a second term in the expression for $\tilde{b}_{1,-1}$ is connected with the fact that the density matrices $f_{mm'}$ and $f_{\mu\mu'}$ are not diagonal. The dependence of C_3 on the magnetic field reflects the Hanle effect on the upper and lower levels. For $\Omega \gg \gamma_1, \gamma_0$ the coherence of the states is totally destroyed by the magnetic field and $C_3 \to 0$.

The assumption concerning the mixing of Zeeman sublevel populations, employed in this paper, leads for example to the E_{-1} field exerting an equal influence on polarizabilities χ_1 and χ_{-1}, as has been shown above. If such mixing is absent then the E_{-1} field influences the polarizability χ_1 less than the polarizability χ_{-1}. Thus for $\Omega = 0$ $b_{1,-1} = b_{11}$ always, but $\tilde{b}_{1,-1}/\tilde{b}_{11} = (A_2 + A_3)/A_1$. For $j_1 = 1$, $j_0 = 2$ this ratio equals ${}^{11}/_{23}$. It is interesting to note that for $j_1 = j_0 = {}^1/_2$ we have $\tilde{b}_{1,-1} = 0$, i.e., the polarizability χ_1 is determined exclusively by the field E_1. This is understandable since in the case given the transitions σ_+ and σ_- start and finish on different Zeeman sublevels.

Formula (19) of the present paper shows that for $\Omega = 0$ the radiation intensity has a minimum with width γ_{10}. The most notable effect associated with the presence of the non-diagonal elements of $f_{mm'}$ and $f_{\mu\mu'}$ is the fact that a second minimum may become superimposed (also for $\Omega = 0$) on this minimum, the width of the second minimum being determined by the lesser of the quantities γ_1 and γ_0.

The simplified model is insufficient for examining questions concerning the stability of different types of oscillations.[9] For example, it follows from this model that in the absence of a magnetic field, plane or circularly polarized states are states of neutral equilibrium. However exact calculations show that the circularly polarized state is unstable in this case.

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