PULSED STIMULATED EMISSION IN A HYDROGEN-ATOM BEAM LASER

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Pulsed stimulated emission in a laser using a hydrogen atom beam is considered for the case of two relaxation times. It is shown that if a number of conditions are met, the polarization excited in the generator by the pump pulse depends only on the number of active particles in the resonator, and that decay of the stimulated signal follows an exponential law with an exponent $1/T_2$.

THE most interesting experiments that can be carried out with hydrogen-beam lasers are those aimed at determining the relaxation times of hydrogen atoms colliding with atoms or molecules of a different gas or with the walls of the storage bulb.

In experiments of this type the flux of the hydrogen atoms in the excited states into the resonator is relatively small, so that the condition for self-excitation of the generator is not satisfied, that is, the field in the resonator is equal to zero. An illumination flash, which produces polarization in the resonator, is applied to such an underexcited generator from the outside; after the flash is turned off, the attenuation of the signal is observed and the relaxation time of the hydrogen atoms is determined from the attenuation time.

This is the experimental setup used to determine the relaxation times of hydrogen atoms on the wall of a storage bulb,[1] under the assumption that there is only one relaxation time ($T_1 = T_2$). The most interesting physical situation, however, which makes it possible to obtain considerable information concerning the interatomic interactions, occurs when the hydrogen atoms relax on atoms or molecules of a different gas.[2] In this case $T_1 \neq T_2$.[1)]

We consider below the effect of an illumination flash on an underexcited laser operating with a beam of hydrogen atoms in the storage bulb with which are mixed in atoms or molecules of a different gas (that is, in the case of two relaxation times T_1 and T_2), and determine the conditions under which observation of pulsed stimulated emission makes it possible to determine reliably the relaxation times of the hydrogen atoms on the atoms or molecules of the admixture gas.

If the flux of active particles is assumed to be stationary, then the variation of E (field amplitude), P (polarization amplitude), and the particle number N can be described by the following system of equations:[3, 4]

$$\dot{E} = -\gamma_3 E + 2\pi\omega_0 P, \quad \dot{P} = -\gamma_2 P + d^2 NE/\hbar,$$
$$\dot{N} = \gamma_1(N_0 - N) - EP/\hbar. \tag{1}$$

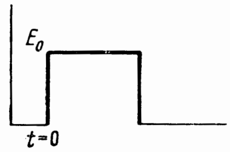
Here $\gamma_1 = 1/T_1$ and $\gamma_2 = 1/T_2$, where T_1 and T_2 are the transverse and longitudinal relaxation times; $\gamma_3 = \omega_0/2Q$ describes the losses in the resonator; ω_0 is the transition frequency (we assume that it coincides with the generator frequency); Q is the quality factor of the resonator; N_0 is the initial number of active particles, d is the matrix element of the dipole moment.

We shall assume that the illumination pulse has a rectangular form[2)] (see the figure). Up to the point $t = 0$ the field is $E = 0$. This means that $P = 0$ and $N = N_0$. To the right of the point $t = 0$ the field is $E = E_0$ and it is necessary to solve a system consisting of the last two equations in (1), putting $E = E_0 = \text{const}$. From this system we obtain for the polarization the following equation:

$$\ddot{P} + (\gamma_1 + \gamma_2)\dot{P} + \left\{\gamma_1\gamma_2 + \left(\frac{dE_0}{\hbar}\right)^2\right\}P - \frac{d^2E_0}{\hbar}\gamma_1 N_0 = 0. \tag{2}$$

[1)]In a solid the times T_1 and T_2 are essentially different: for ruby, for example, $T_1 \sim 10^{-3}$ sec and $T_2 \sim 10^{-11}$ sec; in a gas medium, T_1 and T_2 differ by only a factor of several times.

[2)]Generally speaking, the illumination flash differs from rectangular, but if it is sufficiently long, then the rise and fall of the pulse can be neglected. It is assumed, of course, that the rise and fall times of the pulse are considerably smaller than T_1 and T_2, that is, considerably smaller than 1 sec as applied to a gaseous medium.



Shape of illumination flash

Its solution is

$$P = \exp\,[-\gamma_1 + \gamma_2)t/2]\,(A\cos\omega t + B\sin\omega t) - A, \quad (3)$$

where

$$A = d^2E_0\gamma_1N_0/\hbar[\gamma_1\gamma_2 + (dE_0/\hbar)^2],$$
$$B = dN_0[1 + \gamma_1(\gamma_1+\gamma_2)/2(dE_0/\hbar)^2],$$
$$\omega^2 = \gamma_1\gamma_2 + (dE_0/\hbar)^2 - [(\gamma_1+\gamma_2)/2]^2. \quad (4)$$

If we assume that the field E_0 is sufficiently strong, that is,

$$dE_0/\hbar \gg \max\{(\gamma_1+\gamma_2)/2, \quad (\gamma_1\gamma_2)^{1/2}, \quad \gamma_1\},$$

then

$$P = dN_0\left\{\exp\left[-\frac{(\gamma_1+\gamma_2)t}{2}\right]\left[\frac{\gamma_1}{\omega^*}\cos\omega^*t + \sin\omega^*t\right] - \frac{\gamma_1}{\omega^*}\right\},$$

$$\omega^* = dE_0/\hbar. \quad (5)$$

The illumination must be turned off at an instant when P is maximal, that is, $\dot{P} = 0$. In the zeroth approximation we find that the polarization is maximal at the instant $t = \pi/2\omega^*$ and is equal to

$$P_{max} = dN_0, \quad (6a)$$

while the number of active particles reaches at this instant a value

$$N^* = N_0\gamma_2/\omega^*. \quad (6b)$$

If we take into account terms of the first order of smallness, then at the instant when $\omega t = \pi/2$ the polarization reaches a value

$$P = dN_0\{1 - (\gamma_1+\gamma_2)/2\omega^* - \gamma_1/\omega^*\}. \quad (7)$$

From this we see immediately that in order for the induced polarization to be independent of the field E_0, accurate to, say, 1%, it is necessary to have $\gamma_2/\omega^* < 0.01$, that is, if $Q = 5 \times 10^5$, $d = 10^{-20}$, and $V = 10^3$ cm^3, then the illumination pulse power should be not larger than 10^{-10} W (at a pulse duration 10^{-2} sec).

The time $t = \pi/2\omega^*$ is that time interval during which the illumination should be turned on in order for the polarization to reach a maximum value. If the illumination flash is chosen longer or shorter, then the polarization will be smaller than P_{max}. We see that this time is determined by the amplitude of the field and decreases with increasing field. There exists, however, a low limit for the quantity t, determined by the time of establishment of the field in the resonator $2Q/\omega_0$. In the calculation we neglect this process. Therefore, if we again specify an accuracy of, say, 1% for the experiment, then $2Q/\omega t$ should be smaller than 1%. When $Q = 5 \times 10^4$ and $\omega_0 = 10^{10}$ we obtain $t > 10^{-3}$ sec.

Assume that at the instant $t = \pi/2\omega^*$ the illumination flash is suddenly turned off. Then the field in the resonator will reach, after a short time $\sim 2Q/\omega_0$ (time of establishment of the field in the resonator) a value determined by the values of P_{max} and N^* from expressions (6), while the values of P and N themselves remain practically unchanged during this time. Then this field will slowly attenuate within a time that is determined by the relaxation times of the hydrogen atoms. The behavior of the field is determined by the complete system (1) with the initial conditions $P_0 = P_{max}$ and $N_0 = N^*$. It is easily reduced to the form

$$\frac{1}{\gamma_3}\left(\dddot{E} - \frac{\ddot{E}\dot{E}}{E}\right) + \ddot{E} - \frac{\dot{E}^2}{E} = \frac{2\pi\omega_0 d^2\gamma_1}{\hbar\Upsilon_3}E\left\{N^* - \frac{\hbar\Upsilon_2\Upsilon_3}{2\pi\omega_0 d^2}\right\}$$
$$- \gamma_1\dot{E} - \left(\frac{dE}{\hbar}\right)^2\frac{\dot{E}}{\Upsilon_3} + \frac{d^2E^3}{\hbar^2}, \quad (8)$$

or, recognizing that the rate of attenuation of the field is considerably smaller than γ_3, and assuming this velocity to be much larger than $dE/\hbar$, we obtain

$$\ddot{E} - \frac{\dot{E}^2}{E} + \gamma_1\dot{E} + \gamma_1\Upsilon_2\left\{1 - \frac{N^*\cdot 2\pi\omega_0 d^2}{\hbar\Upsilon_2\Upsilon_3}\right\}E = 0. \quad (9)$$

Solution of (9) with account of the boundary conditions is of the form

$$E = E_{in}\exp\left\{-\Upsilon_2\left[1 - \frac{N^*\cdot 2\pi\omega_0 d^2}{\hbar\Upsilon_2\Upsilon_3}\right]t\right\}. \quad (10)$$

For practically any N_0, even near the threshold, the term

$$2\pi\omega_0 d^2N^*/\hbar\gamma_2\gamma_3 \ll 1,$$

if we take a sufficiently large illumination of the field E_0, that is, the form of the attenuating signal is determined entirely by the quantity γ_2.

In order for equation (9), and consequently also the solution (10), to be correct, it is also necessary that $(dE_{in}/\hbar\gamma_2)^2$ be sufficiently small. This condition can be reduced to the form

$$(4\pi QN_0d^2/\hbar\gamma_2)^2 \ll 1.$$

The threshold condition of the generator takes the form

$$4\pi QN_0^{\,thr}d^2/\hbar\gamma_2 = 1.$$

From this we see that if we do not wish the error to exceed 1%, then N_0 must be one tenth as small as N_0^{thr}.

Thus, for a clean-cut formulation of the experiment aimed at determining the relaxation times with accuracy α, the following three conditions must be satisfied:

$$\gamma_2/\omega^* < \alpha, \quad 2\omega^*/\pi\gamma_3 < \alpha, \quad N_0 < N_0^{thr}\sqrt{\alpha}.$$

It is also necessary that the illumination pulse duration be $t = \pi/2\omega^*$. When $Q = 5 \times 10^4$, $v = 10^3$ cm^3, $\gamma_2 \sim 1$ sec^{-1}, and $\alpha = 0.01$ we find that the power of the illumination signal should lie in the interval $10^{-8} - 10^{-10}$ W at a flash duration of 10^{-3}–10^{-2} sec, respectively. The flux of the active particles should be smaller than 0.1 of the threshold flux.

[1] H. M. Goldenberg, Preprint, Harvard Univ., 1960.

[2] H. C. Berg, Phys. Rev. **137**, A1621 (1965).

[3] Kh. Yu. Khaldre and R. V. Khokhlov, Izv. VUZov, Radiofizika **1**, 60 (1958).

[4] A. N. Oraevskiĭ and A. V. Uspenskiĭ, Trudy, Phys. Inst. Acad. Sci. **31**, 96 (1965).

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