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SOME FEATURES OF THE TWISTING OF QUANTUM AND CLASSICAL LIQUIDS

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The process of the turbulent twisting of quantum and classical liquids is investigated. It is shown that the twisting process differs significantly in the two cases. A qualitative explanation of the twisting of a quantum fluid is proposed on the basis of the Onsager-Feynman theory of quantum vortices.

It was shown in [1] that the dependence of $\log \Delta z = f(t)(\Delta z = z_\infty - z_i$, where z_∞ is the final meniscus depth and z_i is the meniscus depth at the time t_i, t is the time of twisting) for a liquid set into rotation is different in the case of helium II on the one hand and helium I and water on the other. In carrying out these experiments, the measurements of the depth of the meniscus were made visually (every 15 seconds) with the help of a KM-6 cathetometer.

In the present work, the results are given for experiments carried out by means of the technique of photographing the meniscus: the cine camera KS-50-B, controlled by a special electronic arrangement, photographed one frame every 3 seconds. The container with the helium was put into rotation after determination of the liquid level in the stationary state, while the final depth of the meniscus was established by the coincidence of the level in several successive frames of the film, viewed in enlarged form by means of a projector.

The problem is studied of the temperature at which the change in the character of the twisting takes place. Figure 1 shows the corresponding graphs: curve a is taken for helium I at the temperature $T = 2.186°K$ and curve b for helium II at $T = 2.164°K$. These curves have the character of the time variation corresponding to the normal and superfluid components of liquid helium respectively.[1] This makes it possible to confirm that the change in the character of the twisting takes place at the phase-transition point of liquid helium.

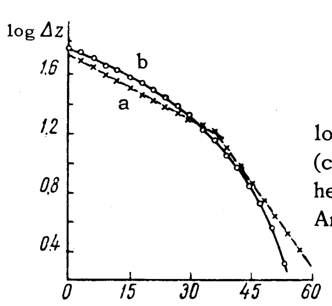

FIG. 1. Dependence of $\log \Delta z = f(t)$ for helium I (curve a, $T = 2.186°K$) and helium II (curve b, $T = 2.164°K$). Angular velocity $\omega_0 = 53.4\ \text{sec}^{-1}$.

As has already been pointed out above, water has the dependence $\log \Delta z = f(t)$, which is also characteristic of helium I. (see Fig. 2). The method of photographing the meniscus permitted us to study the time dependence of the depth of the meniscus in some detail. In particular, in the case of water, it was possible to establish the fact that the instant of kinking of the curve $\Delta z = f(t)$ for a given vessel shifts to the left with increase in the rotational velocity ω_0. The graph of this dependence (Fig. 3) represents a linearly decaying function.

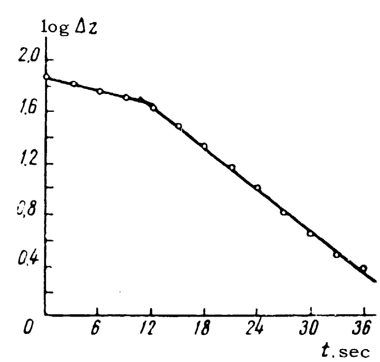

FIG. 2. Dependence $\log \Delta z = f(t)$ for water. Angular velocity $\omega_0 = 56.5\ \text{sec}^{-1}$.

In a successive series of experiments, we studied the time of complete twisting of helium I and helium II for different radii R of the container. The corresponding graphs (see Fig. 4) show that whereas in the case of helium I, the time of complete twisting increases linearly with increase in the radius of the vessel (curve a in Fig. 4), for

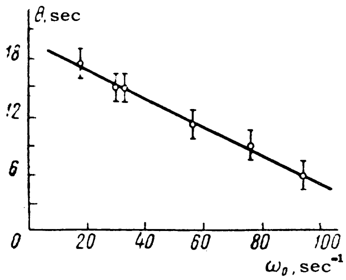

FIG. 3. Dependence of the instant of transition from one linear dependence to another (see Fig. 2) on the angular velocity for water.

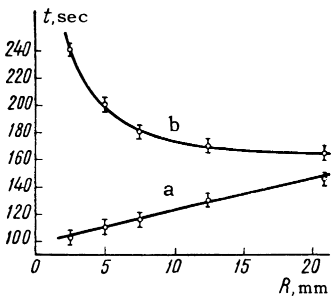

FIG. 4. Dependence of the time of complete twisting of helium I (curve a) and helium II (curve b) on the radius of the container; $\omega_0 = 30\ \text{sec}^{-1}$.

helium II, this time falls off according to a nonlinear law (see Fig. 4, curve b). Similar curves with smaller errors are obtained in the case in which the time of achieving a meniscus equal to $^3/_4$ of its equilibrium depth was determined for the same velocity of rotation (Fig. 5). The dependences of the time for the meniscus to reach $^3/_4$ of its equilibrium depth as a function of the angular velocity are plotted in Fig. 6 (for helium I, curve a, and for helium II, curve b); similar dependences exist also in the case in which the time of complete growth of the meniscus is measured.

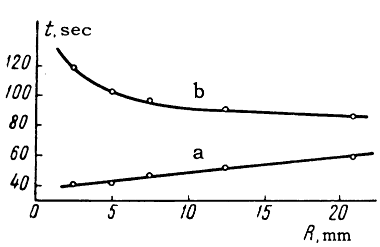

FIG. 5. Dependence of the time of the penetration of the meniscus to 3/4 of its equilibrium depth for helium I (curve a) and helium II (curve b) on the radius of the container: $\omega_0 = 30\ \text{sec}^{-1}$.

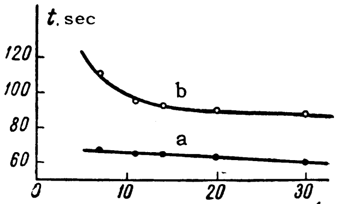

FIG. 6. Dependence of the time of penetration of the meniscus to 3/4 of its equilibrium depth on the angular velocity for helium I at T = 2.20°K (curve a) and helium II at T = 1.38°K (curve b). Radius of the container R = 21 mm.

On the basis of graphs similar to those shown in Fig. 5, but taken for different angular velocities, and also graphs similar to those shown in Fig. 6, but for containers with different diameters, it was established that for helium II the dependence $t = f(v)$ (where v is the velocity on the periphery of the vessel) is a universal nonlinear function (for different ω_0 and R; see Fig. 7). For helium I, such a dependence is not a universal one. In this case the dependence $t = f(\omega_0/R)$ is universal (see Fig. 8, curve a). The same dependence for helium II increases linearly (see Fig. 8, curve b) but is not a universal function.

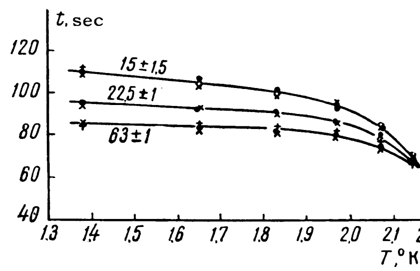

FIG. 7. Dependence of the time of penetration of the meniscus to 3/4 of the equilibrium depth for helium II on the temperature: ● – R = 21 mm, × – R = 12.5 mm, + – R = 7.5 mm, ○ – R = 5 mm. The number on the curves are the velocity on the periphery of the container; $v = \omega_0 R$ (in cm/sec).

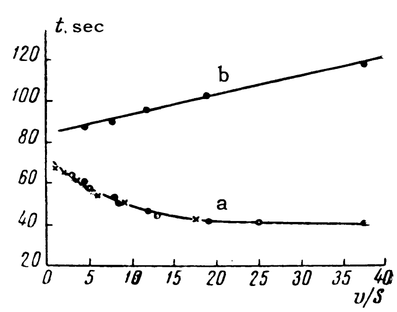

FIG. 8. Dependence of the time of penetration of the meniscus to 3/4 of its equilibrium depth on the ratio $v/S \equiv \omega_0/\pi R$ for helium I (curve a) and helium II (curve b). The different points on the curve correspond to the same diameters of the containers as in Fig. 6 (v – velocity on periphery of the container, S is the area of its cross section).

Proceeding to a discussion of the results, one must note in the first place that in all the experiments described the Reynolds numbers reach very high values: $Re \sim 5 \times 10^3$ for water and $Re \sim 5 \times 10^5 - 2 \times 10^7$ for helium I and the normal component of helium II. This evidently explains the fact that the observed times of twisting are much less than the value $\sim R^2/\nu \sim 10^6$ sec which one would have expected for laminar twisting (R is the radius of the vessel, ν the kinematic viscosity). It is obvious that the turbulent twisting, for which $\nu_{eff} \gg \nu$, must take place more rapidly than the laminar twisting. In connection with the turbulent character of the phenomena under consideration, there is no basis for explaining the data of Fig. 2 by means of the well-known formulas of laminar twisting of a classical liquid.[2]

So far as the twisting of helium II is concerned, in this case the turbulent character of the motion of the normal component (at the beginning of twisting) and the irregularity brought about by it in the formation of the Onsager-Feynman vortex system also make a detailed description of the process more difficult. However, several remarks can be made.

One can consider it a valid assumption that the formation of vortices in the superfluid component of helium II goes through an initial stage of formation of nuclei which subsequently unite and create a regularly ordered set of Onsager-Feynman vortices.[3-5] The turbulent motion of the normal component accelerates the motion of the "vortex shreds" towards each other; in the case of laminar flow the motion would be diffuse and would be completed much later.

On the other hand, Andronikashvili and Kaverkin[6] have shown that the meniscus of rotating helium II begins to fall away from the surface of the vessel, gradually moving toward the central region which, at the initial stage of twisting, remains flat (while the meniscus of a classical liquid falls away almost instantaneously along the entire diameter of the vessel and only subsequently becomes deeper). Obviously, this circumstance illustrates the reverse action of the vortex formation process in the superfluid component on the twisting of the normal component. By being accelerated by the turbulent motion of the latter, the vortex formation process retards the twisting of the normal part of helium II, in accord with the assumption made above by Andronikashvili on the suppression of classical turbulence by quantum turbulence.

In connection with the dependence of the time for establishment of the meniscus of helium II on ω_0 and R (Fig. 5 or Fig. 6, curve b), one can make the following rough estimates. The total number of vortices formed at the end of the twisting process is proportional to the cross-sectional area of the container and the angular velocity: $N \sim R^2\omega_0$; the number of vortices n formed per unit time in a unit length of the periphery of the vessel depends on the difference of the mean velocities of the normal and superfluid liquids, which changes from $\omega_0 R$ at the beginning of twisting to zero at the instant of its completion. It is natural to assume, therefore, that

$$t \sim \frac{N}{Rn} \sim \frac{\omega_0 R}{(\omega_0 R)^{\alpha}} = (\omega_0 R)^{1-\alpha} = \frac{1}{v^{\alpha-1}}.$$

Thus, the universal dependence on the velocity, which is also given by experiment, is explained. The experimental data (decrease of t with increase of v) show that $\alpha > 1$. The numerical estimate, according to the data of Fig. 4, gives $\alpha \sim 1.3$.

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