THE BEHAVIOR OF A SEMICONDUCTOR IN A STRONG RESONANT RADIATION FIELD

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The susceptibility $\kappa(\omega)$ of the valence electrons is determined for an intrinsic semiconductor which interacts with a free radiation field (steady state), and the carrier distribution function is found. The saturation which follows from the expression for κ stabilizes the natural frequencies of the field.

BASOV, Bogdankevich, and Devyatkov [1] and also Vavilov et al.[2] analyzed the possibility of creating a semiconductor laser excited by fast electrons. As in p-n junction injection lasers, the stabilization of the short-wave limit of the radiation spectrum is observed at a certain excitation level of the crystal. It will be shown below that this effect results from the saturation of the valence-electron susceptibility κ. To determine κ, let us consider the interaction of a semiconductor with a strong resonant ($\omega \approx \Delta$) radiation field. The term "strong" means that the behavior of the system cannot be described by perturbation theory.[3] We assume that the photon density is much smaller than the carrier concentration and that the radiative transitions have a clearly expressed collective character. Their times (in first-order perturbation theory) are very small; thus, in the case of a continuous spectrum

$$\tau_{rad} \sim 10^{-11}/N \text{ [sec]},$$

where N is the number of photons in the volume V; in the presence of a single field oscillator

$$\tau_{rad} \sim 10^{-13}/N \text{ [sec]}.$$

On the other hand, the characteristic relaxation times (also in first-order perturbation theory) due to scattering by acoustic phonons at a temperature $T \approx 20\,^\circ$K, are

$$\tau_{ph} \gtrsim 10^{-12} \text{ sec}.$$

Restricting ourselves to temperatures of $\sim 10\,^\circ$K ($T \ll \theta_D$), we may neglect the influence of optical phonons. Thus, for a sufficiently high number of photons N, the dynamics of the system has to be described in the first approximation by a Hamiltonian which allows only for the carrier interaction with the radiation field.[3] Because τ_{rad} decreases with increasing transition energy, a population growth on moving upwards in the band is possible.

Consider a steady state with specified carrier densities and field intensity in the volume V occupied by the crystal and by the radiation field. The field is represented by a single oscillator with frequency ω. We define κ by the expression

$$\mathbf{j} = \frac{\hbar\omega^2 V}{c_0}\varkappa(\omega)\mathbf{A}_\omega, \tag{1}$$

where κ is the susceptibility operator. The electron current $\mathbf{j}$ and the vector potential $\mathbf{A}_\omega$ (divided by $\hbar$) have the form

$$\mathbf{j} = \sum_p (\alpha a_p^+ b_{-p}^+ + \alpha^* b_{-p} a_p), \qquad \mathbf{A}_\omega = \beta c_0 (c + c^+),$$

$$\alpha = (e/m)\xi, \qquad \beta = (2\pi/\hbar\omega V)^{1/2}\mathbf{n}. \tag{2}$$

Here a, b, and c are the electron and photon operators, ξ is the momentum matrix element; e and m are the electron charge and mass; $\mathbf{n}$ is the polarization vector, and c_0 is the light velocity in vacuo. The summation in (2) is with respect to the wave vectors $\mathbf{p}$ and the spins σ. Subsequently we will deal with linear polarization only, and the quantization direction is given by the vector $\mathbf{n}$, therefore we will omit the vector designations.

Let us write the equations of motion by means of a full Hamiltonian

$$\frac{dj}{dt} = i\sum_p (\varepsilon_c(p) + \varepsilon_v(p))(\alpha a_p^+ b_{-p}^+ - \alpha^* b_{-p} a_p)$$

$$+ i|\alpha|^2\beta\gamma(\omega)(c - c^+),$$

$$\frac{d}{dt}\Big(\sum_p \alpha a_p^+ b_{-p}^+ - \alpha^* b_{-p} a_p\Big) = i\sum_p (\varepsilon_c(p) + \varepsilon_v(p))$$

$$\times(\alpha a_p^+ b_{-p}^+ + \alpha^* b_{-p} a_p) - i|\alpha|^2\beta\gamma(\omega)(c + c^+), \tag{3}$$

$$\frac{d}{dt}(c - c^+) = -i\omega(c + c^+) - i\beta\sum_p (\alpha a_p^+ b_{-p}^+ + \alpha^* b_{-p} a_p),$$

$$\gamma(\omega) = \sum_{p=0}^{p(\omega)} (a_p^+ a_p + b_{-p}^+ b_{-p} - 1),$$

γ being constant in the steady state. Taking into account the inequality $\hbar p^2/2\mu \ll \Delta$ (μ is the reduced mass), and also the relationship

$$d^2j/dt^2 = -\omega^2 j,$$

we obtain the equation

$$(\Delta^2 - \gamma|\alpha|^2\beta^2 - \omega^2)j = \gamma \frac{|\alpha|^2(\Delta+\omega)}{c_0} A_\omega. \qquad (4)$$

Introducing (1) and averaging over the system wave function, we finally get

$$\varkappa(\omega) = \gamma(\omega) \frac{e^2|\xi|^2(\Delta+\omega)}{m^2\hbar\omega^2 V} \left\{\Delta^2 - \gamma(\omega)\frac{2\pi e^2|\xi|^2}{m^2\hbar\omega V} - \omega^2\right\}^{-1}$$

$$\gamma(\omega) = \sum_{p=0}^{p(\omega)} (\langle a_p^+ a_p\rangle + \langle b_{-p}^+ b_{-p}\rangle - 1). \qquad (5)$$

If $\omega^2 > \Delta^2$, the dispersion is negative in the case $\gamma(\omega) > 0$ and the system is excited. From (5) it then follows that

$$n_e(p) = n_h(p) > 1/2. \qquad (6)$$

The equality sign appears here because the operators $a_p^+ a_p$ and $b_{-p}^+ b_{-p}$ obey identical equations of motion. Since κ is finite when $\omega \to \Delta$ we assume that $\gamma(\omega)$ is described by the expression

$$\gamma(\omega) = \beta(\omega^2 - \Delta^2), \qquad (7)$$

where the factor β depends on the excitation level and on the photon density. From (7) we conclude that the population changes in the bands obey the law

$$n_e(\omega) = n_h(\omega) \sim \omega^{1/2}. \qquad (8)$$

The formula for γ actually defines the envelope of the intensity maxima of the observed spectral lines, if one identifies them with the various modes.

The second term in the curly brackets in (5), which can be omitted in the similar problem for a system of two-level molecules, now becomes important and determines the saturation. When this term is larger than $(\omega^2 - \Delta^2)$, κ tends to $-1/2\pi$, and $\varepsilon_e = 1 + 4\pi\kappa \to -1$, i.e., the electronic part of the permittivity becomes negative; the lattice component of the permittivity ε_{latt} is assumed to be specified and independent of ω. The saturation stabilizes the mode frequency $\omega \approx c_0/\varepsilon^{1/2}$. When the generator operates simultaneously in several modes, one of which (ω_{max}) is in saturation conditions, the frequencies of the others approach ω_{max} as the excitation level increases.

Taking for an estimate (see, e.g. [2]) $\omega_{max} = 2.541 \times 10^{15}$,

$$\omega_{max} = 2.541 \cdot 10^{15}\ \text{sec}^{-1},\quad \Delta = 2.512 \cdot 10^{15}\ ce\kappa^{-1},$$

$$|\xi|^2 \sim 10^{-38}\ \text{g}^2\text{cm}^2/\text{sec}^2,$$

we obtain for the saturation carrier density $\rho_s \approx 10^{19}$ cm^{-3}, and the corresponding limiting wave vector equals $\sim 8 \times 10^6$ cm^{-1}. To obtain agreement with the observed value $\omega_{max} - \Delta = 0.029 \times 10^{15}$ sec^{-1} one has to take for the reduced mass μ the value $\sim 10^{-27}$ g, which is quite possible because we are far from the bottom of the band.

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[1] N. G. Basov, O. V. Bogdankevich, and A. G. Devyatkov, DAN SSSR, **155**, 783 (1964), Soviet Phys. Doklady **9**, 288 (1964).

[2] V. S. Vavilov, E. L. Nolle, and V. D. Egorov, FTT **7**, 934 (1965); Soviet Phys. Solid State **7**, 749 (1965).

[3] V. F. Chel'tsov, JETP **48**, 531 (1965); Soviet Phys. JETP **21**, 353 (1965).

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