CONTRIBUTION TO A PHENOMENOLOGICAL THEORY OF SOUND ABSORPTION NEAR SECOND-ORDER PHASE TRANSITION POINTS

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The interaction between a sound wave and the thermal fluctuations of a certain intrinsic parameter of the system under consideration is studied. This interaction leads to a renormalization of the velocity and also to an additional sound absorption. The purpose of the research was to elucidate the features of sound propagation near second-order phase transition points. Therefore, the characteristic transition parameter plays the role of the intrinsic parameter. It is shown that increase in the fluctuations of the latter near the transition point leads to a decrease in the sound velocity and an increase in the absorption coefficient in both the asymmetric and symmetric phases. The frequency dependence of the velocity and absorption coefficient is investigated.

As was discovered in a whole series of experiments (see for example [1-4]), the sound absorption increases in the vicinity of second-order phase transition points. Several researches [5,6] have been devoted to the theoretical explanation of this "anomaly." These are based on the results of Mandel'shtam and Leontovich as applied to the problem under discussion.[8] These authors calculated the sound absorption associated with the relaxation of an internal parameter. In order for the relaxation mechanism for sound absorption of Mandel'shtam-Leontovich to come into play, it is necessary that the equilibrium value of the internal parameter be dependent on the density of the substance, or, more exactly, that it change during passage of the sound wave.

When studying the anomaly of absorption near a second-order phase transition point, we encounter still another case. For a number of materials, the symmetry of the more symmetric phase is such that there is no linear coupling in them between the acoustic deformations and the characteristic transition parameter, which plays the role of the internal parameter mentioned above.[5] It is not possible to explain the experimentally observed anomaly of the sound absorption in the symmetric phase of such materials within the framework of the Mandel'shtam-Leontovich theory.

We shall consider below, the sound absorption brought about by the interaction of the sound wave with the thermal fluctuations of the internal parameter. We shall be interested only in the absorption anomaly near second-order phase transition points, although the calculations given below are also of interest in the study of other problems, for example, the propagation of sound in a liquid. As experimental results show,[7] the absorption coefficient of acoustic waves at hypersonic frequencies is proportional to the square root of the frequency in a number of liquids. Such a frequency dependence will be observed below for the sound absorption coefficient brought about by fluctuations of the internal parameter, in the region of frequencies which is much higher than the reciprocal of the relaxation time. We recall that the sound absorption coefficient computed by Mandel'shtam and Leontovich [8] in this region of frequencies is independent of the frequency.

The interaction of the sound wave with fluctuations of an internal parameter has already been investigated by Pippard,[9] who studied the effect on propagation of a sound wave of the fluctuations arising from the onset of the superfluid phase in normal helium at temperatures close to the transition temperature. Here, however, it was necessary to make a number of assumptions of a model character, the degree of approximation of which is not clear.[1)]

While the calculation given below has a general character, the nature of the fluctuating internal parameter contained in it affects only the form of the

[1)]Ginzburg has repeatedly pointed out the possibility of a different type of "pre-transition" effect in the symmetric phase, associated with the fluctuations of the characteristic parameter (for example, see[10]).

equations of motion for this quantity. The equations of motion used below refer to a characteristic transition parameter of the type of the $\alpha \rightleftarrows \beta$ transition in quartz. In the case of transitions to the superconducting and superfluid states, it would possibly be necessary to take into account the quantum-mechanical nature of the characteristic parameter, but this is not done in the present work.

1. The characteristic transition parameter (we shall denote it by η) is an internal parameter of the system, which has the meaning of a certain internal deformation. For phase transitions in ideal crystals, for example, the transition parameter is associated with the relative displacements of the sublattices; it corresponds to the Born optical deformation. Our first goal is to obtain the equations of motion for the acoustic and "internal" deformations, associated with a change in η. Here it is most convenient to begin with the expression for the density of the common potential energy of the internal and acoustic deformations. We shall use the free energy of the material for the latter.

In the spirit of the Landau theory, we shall assume that, near the transition point, the potential (free) energy of the system per unit volume can be represented in the form of the series

$$u(\eta, v) = \alpha(v, T)\eta^2 + \tfrac{1}{2}\beta\eta^4 + \tfrac{1}{2}\lambda v^2. \tag{1}$$

Here v is a small dimensionless parameter characterizing the volume of the system, $v = | V - V_{\Theta} | V_{\Theta}$, V_{Θ} being the volume occupied by the system at the transition temperature $T = \Theta$. In the symmetric phase, $\alpha > 0$, while in the asymmetric phase, $\alpha < 0$. Therefore $\alpha(0, \Theta) = 0$. It follows from the form of Eq. (1) that we are considering only phase transitions of the second kind, far from the critical Curie point.

Expanding $u(\eta, v)$ in a series in $\eta' = \eta - \eta_0$ and $v' = v - v_0$ (η_0 and v_0 are the equilibrium values of η and v), we get

$$u(\eta', v') = \alpha(0, T)\eta'^2 + \alpha_v' v'\eta'^2 + \delta\,(\mathrm{grad}\,\eta')^2 + \tfrac{1}{2}\lambda v^2 \tag{2a}$$

for $T > \Theta$ and

$$u(\eta', v') = -2\alpha(v_0, T)\eta'^2 + 2\alpha_v'\eta_0 v'\eta' + \alpha_v' v'\eta'^2 + \delta\,(\mathrm{grad}\,\eta')^2 + \tfrac{1}{2}\lambda v^2 \tag{2b}$$

for $T < \Theta$. Here $\eta_0^2 = -\alpha/\beta$, $v_0 = -\alpha_V'\eta_0^2/\lambda$. In obtaining Eqs. (2) we assumed that $\alpha_V'^2/\beta\lambda \ll 1$; usually, this ratio is of the order of 0.1. In the expression for the potential energy density, a term $\delta\,(\mathrm{grad}\,\eta')^2$ is added. This takes into account in first approximation the correlation between the values of η' at the different points.[12] In what follows, we shall omit the primes on η and v. In general, the terms $\alpha_{VV}''\eta^2 v^2$ and $2\alpha_{VV}''\eta_0\eta v^2$ in Eqs. (2a) and (2b) should be taken into account. However, as additional investigations show, account of these terms leads only to insignificant corrections to the final results.

We emphasize that the transition from Eqs. (1), which pertain to the spatially homogeneous case, to Eqs. (2a) and (2b), in which $\eta = \eta(r, t)$ and $v = v(r, t)$, is by no means obvious. The latter can apply in the best case only to long wave oscillations in η and v. Therefore, in the expression

$$\eta(\mathbf{r}, t) = \int \eta(\omega, \mathbf{k})\, e^{i\mathbf{kr}+i\omega t}\, d\mathbf{k}\, d\omega \tag{3}$$

we shall carry out the integration only over the region $k < k_m$ (the quantity k_m is a parameter of the theory). Obviously the same applies to the function $v(r, t)$. For fixed k_m the procedure described above for the transition to the spatially inhomogeneous case becomes incorrect in the immediate vicinity of the transition point.[13] Just this circumstance establishes the inapplicability of the theory given here in the immediate vicinity of the transition point, even when the expansion (1) is valid in this region.

We first assume that the kinetic energy density and the dissipation function are quadratic in the corresponding velocities. The set of equations obtained under these assumptions has the form

$$\chi\dot{\eta} + \alpha\eta - \delta\nabla^2\eta + \alpha_v'\eta v = f(\mathbf{r}, t), \tag{4a}$$

$$\rho\ddot{v} - \lambda\nabla^2 v - \alpha_v'\nabla^2\eta^2 = 0 \tag{4b}$$

for $T > \Theta$, and

$$\chi\dot{\eta} - 2\alpha\eta - \delta\nabla^2\eta + \alpha_v'\eta_0 v + \alpha_v'\eta v = f(\mathbf{r}, t), \tag{5a}$$

$$\rho\ddot{v} - \lambda\nabla^2 v - 2\alpha_v'\eta_0\nabla^2\eta - \alpha_v'\nabla^2\eta^2 = 0 \tag{5b}$$

for $T < \Theta$. Everywhere in these equations, terms are omitted that are nonlinear in v. We shall proceed in similar fashion in what follows. In Eqs. (4a) and (5b), we have also omitted the inertial terms, thus assuming that the approach of the quantity η to its equilibrium value has a relaxational character. Account in these equations of terms proportional to η presents no difficulty but together with this it does not lead to essentially new results for the acoustic frequencies smaller than $(\alpha/\mu)^{1/2}$ and $\chi/4\mu$; (the latter two quantities represent respectively the frequency and the attenuation coefficient of the optical ("internal") vibrations).

By inserting the random forces $\beta(r, t)$ in the right hand sides of (4a) and (5a) we account for the fluctuations of η. As is seen from (4a), for $T > \Theta$, the action of the sound wave on the internal deformation, corresponding to the quantity η, reduces to a change in the elastic modulus of this internal deformation. It is natural that if the fluctuations are absent, then such changes in the elastic modulus cannot denote any interaction between the acoustic and the internal deformations. We note that in our set of equations, we have considered only the friction forces arising in internal deformation, and have neglected the viscous forces which arise in purely acoustic deformation. For an account of these latter, we would need to add to Eqs. (4a) and (5b) terms proportional to $\partial\nabla^2 v/\partial t$. Calculations show that the coefficient of sound attenuation brought about by the viscosity (for a purely acoustic deformation) depends more strongly on the temperature than the coefficient of attenuation associated with the internal deformations.

2. We now proceed to the solution of the set of equations of motion. We first consider the case $T > \Theta$. We express the solution of Eq. (4a) in the form of a series, each successive term of which contains the parameter v in a higher degree than the previous: $\eta = \eta^0 + \eta' + \ldots$. The function $\eta^0 = \eta^0(\mathbf{r}, t)$ describes the spatially inhomogeneous fluctuations of η, not disturbed by the sound wave. By assuming, as usual, that $f(\mathbf{r}, t)$ corresponds to jolts uncorrelated in time and space,

$$\langle f(\mathbf{r}, t) f(\mathbf{r}', t')\rangle = \chi k_B T \delta(t-t')\delta(\mathbf{r}-\mathbf{r}'), \tag{6}$$

we find

$$\langle \eta^0(\omega', \mathbf{k}')\eta^0(\omega'', \mathbf{k}'')\rangle = \frac{1}{(2\pi)^4}\frac{\chi k_B T \delta(\omega'+\omega'')\delta(\mathbf{k}'+\mathbf{k}'')}{(\alpha+\delta k'^2)^2+\chi^2\omega'^2}. \tag{7}$$

For $\eta'(\mathbf{r}, t)$, we have

$$\eta^1(\mathbf{r}, t) = -\alpha_v' \int g_1(t-t', \mathbf{r}-\mathbf{r}') \times \eta^0(t', \mathbf{r}') v(t', \mathbf{r}') dt' d\mathbf{r}'. \tag{8}$$

Here $g_1(t-t', \mathbf{r}-\mathbf{r}')$ is the Green's function of Eq. (4a) without the terms which contain v. By substituting $\eta^0 + \eta'$ in place of η in Eq. (4b), we get

$$\rho\ddot{v} - \nabla^2\Big(\lambda v - 2\alpha_v'^2 \int g_1(t-t', \mathbf{r}-\mathbf{r}') \times \eta^0(t, \mathbf{r})\eta^0(t', \mathbf{r}') v(t', \mathbf{r}') dt' d\mathbf{r}'\Big) = 0. \tag{9}$$

It is seen from (9) that the problem has been reduced to the study of sound propagation in a medium with random inhomogeneities, the statistical properties of which are known.

A similar procedure can also lead to a set of equations relative to the region $T < \Theta$. First of all, we note that the fourth term on the left hand side of (5a) should be put on the right hand side and the solution of Eq. (5a) can then be represented in the form of a sum $\eta = \eta_1 + \eta_2$, of the solution of two inhomogeneous equations, on the right sides of which are the variables $-\alpha_V'\eta_0 v$ and $f(\mathbf{r}, t)$, respectively. Here the function $\eta_1(\mathbf{r}, t)$, reflects the process of relaxation of η to the new equilibrium value, which changes with frequency, while $\eta_2(\mathbf{r}, t)$ corresponds to the thermal fluctuations of η. In the theory of Mandel'shtam and Leontovich, applied to the case of second-order phase transitions by Landau and Khalatnikov, the action of the sound wave on that degree of freedom, which corresponds to the characteristic (internal) parameter, is described only by the function $\eta_1(\mathbf{r}, t)$. We only just now saw that such an effect does not always take place, but only in the presence of a linear relation between the deformations (in our case, a change in volume) and transition parameter. In the symmetric phase, this linear relation is often excluded by symmetry considerations. Along with this, the quadratic dependence of the volume deformation on the characteristic parameter is always permitted by the symmetry. Account of this quadratic dependence is a fundamental part of the present research. Here, we have essentially considered terms of higher order than in the previous researches,[5,6] but in a number of cases, precisely these terms describe the effects of interest in the first nonvanishing approximation. The action of the sound wave on the fluctuations of η is the concrete mechanism by which the quadratic relation mentioned above comes into play in the problem considered here.

Let us return to the set of equations (5a)-(5b). In a way completely analogous to what was done above, we find that the solution of Eq. (5a) (with accuracy to terms of higher order in v) has the form

$$\eta(\mathbf{r}, t) = \eta^0(\mathbf{r}, t) - a_v' \times \int (\eta_0 + \eta^0(\mathbf{r}', t')) g_2(\mathbf{r}-\mathbf{r}', t-t') v(t', \mathbf{r}') dt' d\mathbf{r}'. \tag{10}$$

Here $g_2(\mathbf{r}, t)$ is the Green's function of Eq. (5a) without the terms containing v. The expression for $\langle \eta^0(\omega', \mathbf{k}')\eta^0(\omega'', \mathbf{k}'')\rangle$ for $T < \Theta$ is obtained from (7) by replacing α by -2α. Substituting (10) in (5b) and keeping only the terms that are linear in v, we find

$$\rho\ddot{v} - \nabla^2\Big(\lambda v - 2\alpha_v'^2 \eta_0^2 \int g_2(\mathbf{r}-\mathbf{r}', t-t') v(t', \mathbf{r}') dt' d\mathbf{r}' - 2\alpha_v'^2 \eta_0 \int [\eta^0(\mathbf{r}, t) + \eta^0(\mathbf{r}', t')] \times g_2(\mathbf{r}-\mathbf{r}', t-t') v(t', \mathbf{r}') dt' d\mathbf{r}'$$

$$- 2\alpha_v'^2 \int g_2(t-t', \mathbf{r}-\mathbf{r}') \times \eta^0(t, \mathbf{r})\eta^0(t', \mathbf{r}')v(t', \mathbf{r}')\,dt'\,d\mathbf{r}' \Big) = 0. \quad (11)$$

Equation (11), as also Eq. (9), describes the propagation of the sound wave in a medium with random inhomogeneities, but in the absence of (9) there is in addition a term not connected with the fluctuations (third term on the left side). Retaining only this term, we obtain the results of Landau and Khalatnikov.[5]

3. We now turn to the investigation of Eq. (9). The first two components on the left side of this equation describe the propagation of the initial, unrenormalized wave, the third component causes in particular, the renormalization of the characteristics of the mean wave field. A regular procedure can be developed for such a renormalization (see, for example, [14]). However, if we consider that the renormalizations change the wave vector of the initial wave only slightly, then the equation for the mean wave field can be obtained by averaging the coefficients of Eq. (9). This corresponds to keeping only the terms in the renormalized series that are quadratic in η. It is obvious that here we neglect scattering from the fluctuations of η: in the region $T > \Theta$, only second-order scattering takes place, as can be seen from (9); this gives a contribution $\sim\eta^4$.

For a plane monochromatic wave $v = ve^{-i\omega t + i\mathbf{q}\cdot\mathbf{r}}$, the averaged Eq. (9) is rewritten in the form

$$-\omega^2 + q^2c_0^2 - q^2K^1(\omega, \mathbf{q}) = 0, \quad (12)$$

$$K^1(\omega, \mathbf{q}) = \frac{2\alpha_v'^2}{\rho} \iiint_0^\infty g_1(\tau, \mathbf{R}) \langle \eta(t, \mathbf{r}) \times \eta(t-\tau, \mathbf{r}-\mathbf{R}) \rangle e^{i\omega\tau + i\mathbf{q}\mathbf{R}}\,d\tau\,d\mathbf{R}, \quad (13)$$

$c_0^2 = \lambda/\rho$. The dependence of the quantity $K^1(\omega, \mathbf{q})$ on $\mathbf{q}$ can be neglected if the acoustic wavelength is much greater than the correlation radius of the fluctuations of η, i.e., if $\alpha \gg \delta q^2$. Since $q \sim 10^3$, while $\delta \sim \alpha'_\Theta d^2$, where $d \sim 10^{-8}$ cm, the inequality under discussion is satisfied for $(T - \Theta)/\Theta \gg q^2d^2 \sim 10^{-10}$, that is, it is satisfied with ample margin in the region of interest to us. Therefore,

$$K^1(\omega, \mathbf{q}) \approx K^1(\omega, 0) = K(\omega).$$

Further, writing $K(\omega) = K_1(\omega) + iK_2(\omega)$ and $q = q_1 + iq_2$, we find

$$q_1 = \frac{\omega}{c_0}\left(1 + \frac{1}{2}\,\frac{K_1(\omega)}{c_0^2}\right), \quad (14)$$

$$q_2 = \frac{1}{2}\frac{\omega}{c_0}\frac{K_2(\omega)}{c_0^2}. \quad (15)$$

Here we have assumed that $|K(\omega)| \ll c_0^2$.

We now find the expression for $K(\omega)$. From (4a), we get

$$g_1(\omega, \mathbf{k}) = \frac{1}{(2\pi)^4}\frac{1}{\alpha + \delta k^2 + i\omega\chi}. \quad (16)$$

Also, employing (7), we get the relation

$$K(\omega) = \frac{2\alpha_v'^2}{\rho}\frac{\chi k_B T}{(2\pi)^4} \times \int \frac{d\omega''\,d\omega'\,d\mathbf{k}\delta_+(\omega' + \omega'' - \omega)}{(\alpha + \delta k^2 + i\omega'\chi)[(\alpha + \delta k^2)^2 + \chi^2\omega''^2]}. \quad (17)$$

Integrating (17) over the frequency and the angles of the vector $\mathbf{k}$, we find:

$$K_1(\omega) = \frac{\alpha_v'^2}{\pi^2\rho} k_B T \int_0^{k_m} \frac{k^2dk}{4(\alpha + \delta k^2)^2 + \chi^2\omega^2}, \quad (19)$$

$$K_2(\omega) = \frac{\alpha_v'^2}{2\pi^2\rho} k_B T\omega\chi \int_0^{k_m} \frac{k^2\,dk}{(\alpha + \delta k^2)[4(\alpha + \delta k^2)^2 + \chi^2\omega^2]}. \quad (20)$$

The integrals appearing in (19) and (20) have been computed only for those regions in which the relation $\delta k_m^2 \ll \alpha$ or $\delta k_m^2 \gg (\alpha^2 + \chi^2\omega^2/4)^{1/2}$ is satisfied. In the first case, we can set $\delta = 0$ in the integrand and multiply it by $4\pi k_m^3/3$. In this case, we get a relaxational dependence on the frequency for the absorption coefficient and the sound velocity. There is great interest in the second case. We can assume here that the upper limit of integration is infinite. As a result, we get

$$K_1(\omega) = \frac{\alpha_v'^2\sqrt{2}\,k_B T}{8\pi\rho\delta\chi\omega}\left[\left(\frac{\alpha^2}{\delta^2} + \frac{\chi^2\omega^2}{4\delta^2}\right)^{1/2} - \frac{\alpha}{\delta}\right]^{1/2}, \quad (21)$$

$$K_2(\omega) = \frac{\alpha_v'^2 k_B T}{4\pi\rho\delta\chi\omega}\left\{\frac{1}{\sqrt{2}}\left[\left(\frac{\alpha^2}{\delta^2} + \frac{\chi^2\omega^2}{4\delta^2}\right)^{1/2} + \frac{\alpha}{\delta}\right]^{1/2} - \left(\frac{\alpha}{\delta}\right)^{1/2}\right\}. \quad (22)$$

For $\omega\tau \gg 1$ ($\tau = \chi/\alpha$ is the time for relaxation of the nonequilibrium value of η to its equilibrium value) $K_1(\omega) \approx K_2(\omega)$. Taking (14) and (15) into account, we can establish the fact that the sound velocity in this region increases with frequency according to the law

$$c = c_0 - b / 2c_0\sqrt{\omega}, \quad (23)$$

i.e., it approaches the velocity of sound for a fixed value of η, as of course it should. The absorption coefficient in this region is

$$q_2 = b\sqrt{\omega} / 2c_0^3. \quad (24)$$

Here and in Eq. (23),

$$b = \alpha_v'^2 k_B T / 8\pi\rho\delta^{3/2}\sqrt{\chi}.$$

We recall that the absorption coefficient and the sound velocity calculated according to the equations of Mandel'shtam and Leontovich are independent of frequency in this range of frequencies with accuracy up to terms of order $1/\omega^2\tau^2$. At low frequencies, the absorption coefficient is proportional to the square of the frequency and depends strongly on the temperature $q_2 = 1/(T - \Theta)^{3/2}$. The quantity $\lambda - K_1(0)$ represents the renormalized compressional modulus, while $K_1(0)$ corresponds to the anomaly of this modulus. For $\alpha \ll \delta k_m^2$, the quantity $K_1(0) \sim (T - \Theta)^{-1/2}$. As was noted above, in the region of application of the theory, the anomaly in the compressional modulus is less than its normal part. In this region, the expression for the anomalous part of the elastic modulus, obtained in the present work, is identical with the corresponding expression from the previous research,[13] where the anomalies of the thermodynamic quantities were computed by a different method.

4. We now investigate Eq. (11). One must transform from this equation to the equation for the mean field. Here, we make use of the results of [13]. Writing (11) symbolically in the form of the relation $Lv = 0$, and expanding the operator L in a sum of operators of zero, first and second orders in η, $L = L_0 + L_1 + L_2$, we get for the mean field $\langle v \rangle$

$$(L_0 + \langle L_2 \rangle - \langle L_1 M_0 L_1 \rangle)\langle v \rangle = 0. \tag{25}$$

Here $M_0 = L_0^{-1}$, i.e.,

$$M_0 \psi(\mathbf{r}, t) = \int G_0(\mathbf{r} - \mathbf{r}', t - t')\psi(\mathbf{r}', t')\,d\mathbf{r}'\,dt',$$

where G_0 is the Green's function of the equation $L_0 v = 0$. In (25), terms not higher than second order in η are considered. We write Eq. (25) in the explicit form

$$\begin{aligned}\langle \ddot{v} \rangle &+ \nabla^2 \Big(\frac{\lambda}{\rho}\langle v \rangle - \frac{2\alpha_v'^2}{\rho}\eta_0^2 \\ &\times \int g_2(\mathbf{r} - \mathbf{r}', t - t')\langle v(t', \mathbf{r})\rangle dt'\,d\mathbf{r}' \\ &- \frac{2\alpha_v'^2}{\rho}\int g_2(t - t', \mathbf{r} - \mathbf{r}')\,\eta(t, \mathbf{r})\,\eta(t', \mathbf{r}')\langle v(t', \mathbf{r}')\rangle\,dt'\,d\mathbf{r}' \\ &- \frac{4\alpha_v'^4\eta_0^2}{\rho^2}\int g_2(\mathbf{r} - \mathbf{r}', t - t')\,G_0(\mathbf{r}' - \mathbf{r}'', t' - t'') \\ &\times \nabla_{\mathbf{r}''}^2 g_2(\mathbf{r}'' - \mathbf{r}''', t'' - t''')\langle[\eta(\mathbf{r}, t) + \eta(\mathbf{r}', t')]\,[\eta(\mathbf{r}'', t'') \\ &+ \eta(\mathbf{r}''', t''')]\rangle\langle v(t''', \mathbf{r}''')\rangle\,dt'\,dt''\,dt'''\,d\mathbf{r}'\,d\mathbf{r}''\,d\mathbf{r}''' \Big) = 0,\end{aligned} \tag{26}$$

where the integration over t''' is carried out from $-\infty$ to t'', over t'' from $-\infty$ to t', and so on.

As calculations show, account of the latter component on the left side of (26) leads to corrections of order $\Delta\lambda/\lambda$ where $\Delta\lambda = \alpha_V'^2/\beta$ is the jump in the elastic modulus in the phase transition. Inasmuch as we are interested in the case of second-order phase transitions, far from the critical Curie point, these corrections can be neglected. Throwing away the last term in Eq. (26) and substituting in it $\langle v \rangle = \langle v \rangle e^{-i\omega t + i\mathbf{q}\cdot\mathbf{r}}$, we rewrite it in a form analogous to (12), with the one difference that in place of the quantity $K^1(\omega, \mathbf{q})$, we have the quantity $Q^1(\omega, \mathbf{q})$, where

$$Q^1(\omega, \mathbf{q}) = \frac{2\alpha_v'^2\eta_0^2}{\rho}\frac{1}{-2\alpha + \delta q^2 - i\omega\chi} + \frac{2\alpha_v'^2}{\rho}\iint_0^\infty g_2(\tau, \mathbf{R})\,\eta(t, \mathbf{r})\,\eta(t - \tau, \mathbf{r} - \mathbf{R})\,e^{i\omega\tau - i\mathbf{q}\mathbf{R}}\,d\tau\,d\mathbf{R}. \tag{27}$$

Proceeding as in the case $T > \Theta$, we find for the quantities Q_1 and Q_2 $[Q_1 + iQ_2 = Q(\omega) = Q^1(\omega, 0)]$:

$$Q_1(\omega) = \frac{4\alpha_v'^2}{\beta\rho}\frac{\alpha^2}{(-2\alpha)^2 + \chi^2\omega^2} + \frac{\alpha_v'^2 k_B T}{\rho\pi^2}\int_0^{k_m}\frac{k^2 dk}{4(-2\alpha + \delta k^2)^2 + \chi^2\omega^2}, \tag{28}$$

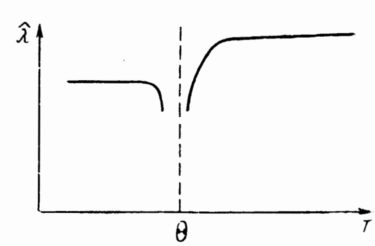
$$Q_2(\omega) = \frac{4\alpha_v'^2}{\beta\rho}\frac{\alpha\chi\omega}{(-2\alpha)^2 + \chi^2\omega^2} + \frac{\alpha_v'^2 k_B T\omega\chi}{2\rho\pi^2} \times \int_0^{k_m}\frac{k^2\,dk}{(-2\alpha + \delta k^2)[4(-2\alpha + \delta k^2)^2 + \chi^2\omega^2]}. \tag{29}$$

The first components on the right sides of (28) and (29) correspond to the results of Landau and Khalatnikov. The integrals therein, with accuracy up to the replacement of α by -2α are equivalent to the integrals entering into (19) and (20).

5. We now estimate a comparison of the results obtained here with experiment. The true ("renormalized") compression modulus is given by the expression

$$\hat{\lambda} = \begin{cases} \lambda - K_1(0) & \text{for } T > \Theta, \\ \lambda - Q_1(0) & \text{for } T < \Theta. \end{cases} \tag{30}$$

The dependence of $\hat{\lambda}$ on temperature is represented schematically in the drawing. Along with the jump in the compression modulus in the transi-



tion, there is also a decrease in the modulus in both phases near the transition point, which agrees qualitatively with experiment.

For a sound wave of low frequency ($\omega\tau \ll 1$), where $\tau = \chi/\alpha$ for $T > \Theta$ and $\tau = -\chi/2\alpha$ for $T < \Theta$, the expression for the absorption coefficient can be represented in the form

$$q_2 = \begin{cases} \dfrac{\omega^2\tau}{4}\dfrac{Dc^2}{c_0^3} & \text{for} \quad T > \Theta, \\ \dfrac{1}{2}\omega^2\tau\dfrac{c_0^2 - c_1^2}{c_0^3} + \dfrac{\omega^2\tau}{4}\dfrac{Dc^2}{c_0^3} & \text{for} \quad T < \Theta. \end{cases} \tag{31}$$

Here $c_0^2 - c_1^2 = \alpha_V'^2/\beta\rho$, where c_1 and c_0 have the meaning of the sound velocities in the symmetric and asymmetric phases respectively, sufficiently far from the transition point, in order that the fluctuations brought about by the anomaly not be important. At the same time, it must be sufficiently close that the inequality $|T - \Theta| \ll \Theta$ be satisfied (condition for the applicability of the phenomenological theory); Dc^2 is the anomaly in the sound velocity.

We see that when $c_0^2 - c_1^2 \sim Dc^2$ (and this usually takes place for $|T - \Theta|$ of the order of a fraction of a degree) the sound absorption coefficient in the symmetric phase is only several times smaller than the absorption coefficient in the nonsymmetric phase. Experiment [1,4] in general confirms this conclusion. Whether one can make a more accurate comparison of theory and experiment is doubtful, since reliable and unambiguously interpreted experimental data[2)] refer only to a single case—the He I – He II transition, and for this transition as has already been observed the calculations should be carried out again.

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