CYCLOTRON-PHONON RESONANCE IN SEMICONDUCTORS

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We consider the absorption of an electromagnetic field by the electrons of a semiconductor in a quantized magnetic field. We show that scattering of electrons by optical phonons of frequency ω_0 leads to resonance absorption at frequencies $\omega = |m\omega_H \pm \omega_0|$, where ω_H is the cyclotron frequency and m is an integer. The shape and intensity of the absorption peak are calculated. The possibility of experimental realization of the effect is considered.

INTRODUCTION

FOR electrons of a semiconductor situated in a magnetic field, there are two types of resonances connected with the cyclotron frequency ω_H. These are cyclotron resonance, when the frequency ω of the high-frequency electric field coincides with ω_H, and the magnetophonon resonance of Gurevich and Firsov [1,2], when the frequency ω_0 of the optical phonons coincides with ω_H. In either case, satisfaction of the resonance condition ensures intense transfer of electrons between the neighboring Landau levels. It is natural to visualize a process in which the transfer is the result of simultaneous action of two factors, i.e., when the absorption of a quantum of the high frequency field is accompanied by simultaneous emission or absorption of a phonon. We shall call resonance of this type cyclotron-phonon resonance.

The first to point out the possibility of such an effect was apparently Klinger [3], who calculated the high-frequency conductivity of a semiconductor in a quantizing magnetic field[1)]. He started in his calculations from the Kubo formula using the Van Hove method. However, the effect was not investigated in detail in his paper and the possibilities of its experimental observation were not discussed.

We find it of interest, in addition, to check the possibility of calculating the coefficient of absorption by usual perturbation-theory methods [5]. All the possible processes of importance to us are shown in Figs. 1—3. Figure 1 pertains to the case $\omega_H > \omega_0$. In this case, as can be readily verified, the absorption of a quantum can be connected only with a transition to a higher Landau level. The resonance frequencies will be $\omega = \omega_H \mp \omega_0$. (The upper and lower signs will pertain henceforth to phonon emission and absorption, respectively). The case $\omega_H < \omega_0$ is shown in Fig. 2. In this case only transitions with phonon emission are possible, the upward transitions (Fig. 2a) causing resonance at $\omega = \omega_0 + \omega_H$, and the downward transitions (Fig. 2b) at $\omega = \omega_0 - \omega_H$. Finally, Fig. 3 shows

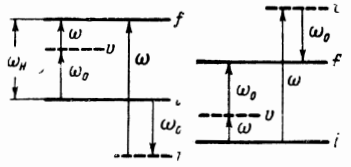

FIG. 1. Transitions that lead to resonant absorption if the cyclotron frequency ω_H is larger than the frequency of the optical phonons ω_0.

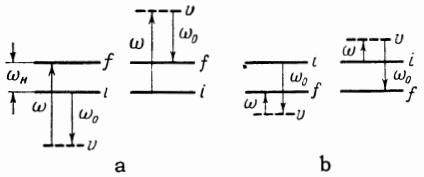

FIG. 2. Transitions which lead to resonant absorption if the cyclotron frequency ω_H is smaller than the frequency of the optical phonons ω_0: a — transitions of an electron from a lower Landau level to a higher one, b — transitions of an electron from a higher Landau level to a lower one.

1)When the manuscript was being readied for press, the authors learned of the pending publication of a paper by Uritskii and Shuster[4], in which the impossibility of the effect in question is also pointed out. The authors are grateful to Z. I. Uritskii and G. V. Shuster for the opportunity to become acquainted with this paper prior to publication.

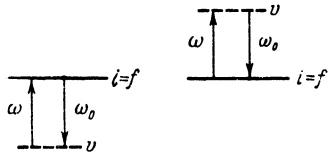

FIG. 3. Transitions which lead to resonant absorption when the electron remains on the same Landau level.

transitions in which the electron remains on the same Landau level.

It is obvious that, just as in the case of magnetophonon resonance, umklapp is possible not only between neighboring Landau levels. Therefore ω_H can be replaced by its multiple $m\omega_H$ both in the diagrams and under resonance conditions. When $\omega_H \approx \omega_0$ the cyclotron-phonon resonance will occur at frequencies $\omega \approx m'\omega_H$. The resonance $m' = 0$ will by its nature be magnetophonon, since it corresponds to a "static" field. The resonance at $m' = 1$ occurs at cyclotron frequency, and the processes in question yield simply a correction to the ordinary cyclotron resonance, which is obtained without phonon participation. The resonances at $m' = 2, 3, \ldots$ correspond to multiple cyclotron frequency; they can also be obtained without phonon participation, if account is taken of the spatial inhomogeneity of the high-frequency field.

It is obvious that cyclotron-phonon resonance can be observed only provided all the levels are sufficiently narrow and the frequencies are sufficiently monochromatic. The Landau levels are smeared by the longitudinal thermal motion by an amount T and by the collisions with the lattice by an amount τ^{-1}, which is determined by the electron relaxation time τ. (Here and throughout $\hbar = 1$ and $k = 1$, i.e., the temperature and the frequencies are measured in energy units.) Therefore, disregarding the "static" case, we shall assume that the frequency is high ($\omega\tau \gg 1$), the magnetic field is strong ($\omega_H\tau \gg 1$) and quantizing ($\omega_H \gg T$). The phonon frequency will be monochromatic if interaction takes place with the long-wave optical phonons. The characteristic lengths that determine the variation of the wave function of the electron in the magnetic field H will be: the thermal length $\lambda_T = (2mT)^{-1/2}$ for the direction along the field, and the magnetic length $\lambda_H = (2m\omega_H)^{-1/2}$ for the direction transverse to the field. For all real fields and temperatures, these lengths are large compared with the lattice constant, and therefore the significant phonons will be long-wave. We note that under the conditions assumed $\lambda_H \ll \lambda_T$.

1. GENERAL FORMULA FOR THE ABSORPTION COEFFICIENT

The absorption coefficient $K(\omega)$ connected with the described transitions can be calculated by the same method as was proposed by Frohlich[5] for the calculation of infrared absorption by free carriers. The entire difference lies only in the fact that in our problem the states of the carriers are quantized in the magnetic field. In this method the interaction between the electrons and the high frequency field H_R or the lattice H_L are regarded as perturbations, which are turned on simultaneously and cause transitions of the required type in the second order of perturbation theory in $H' = H_R + H_L$.

Assume that a certain initial state

$$|i\rangle = |\alpha, \ldots N(\mathbf{q}) \ldots, \ldots N(\mathbf{k}) \ldots\rangle \equiv |\alpha, 0, 0\rangle \quad (1.1)$$

is described by the state of the electron α and by the occupation numbers of the phonons $N(q)$ and of the photons $N(k)$. Here q and k are the phonon and photon momenta; the phonons are assumed longitudinal, and it is understood that the photon polarization is defined. We are interested in a transition to the final state due to the absorption of a phonon

$$|f\rangle = |\alpha', \ldots N(\mathbf{q}) \pm 1, \ldots, \ldots N(\mathbf{k}) - 1 \ldots\rangle \equiv |\alpha', \pm\mathbf{q}, -\mathbf{k}\rangle, \quad (1.2)$$

at which the state of the electron changes and emission or absorption of a phonon takes place.

The Hamiltonian of such a transition will be

$$\tilde{H} = H'(H^0 - E)^{-1}H' = H_L(H^0 - E)^{-1}H_R + H_R(H^0 - E)^{-1}H_L, \quad (1.3)$$

where H^0 is the energy of the interacting electrons, phonons, and photons. All terms containing either H_L or H_R twice have been discarded, since they do not yield transitions of interest to us. The matrix elements of the transition will be

$$\langle i|\tilde{H}|f\rangle = \sum_v \frac{\langle i|H_L|v\rangle\langle v|H_R|f\rangle}{E_i - E_v} + \sum_v \frac{\langle i|H_R|v\rangle\langle v|H_L|f\rangle}{E_i - E_v}. \quad (1.4)$$

In the first term of (1.4) the transitions go through virtual states $|v\rangle = |\alpha'', \pm\mathbf{q}, 0\rangle$, i.e., at first the number of phonons changes, and then the number of photons. In the second term $|v\rangle = |\alpha'', 0, -\mathbf{k}\rangle$, i.e., at first the number of photon changes, and then the number of phonons. Figures 1, 2, and 3 show these two types of transitions on the left and on the right, respectively.

The absorption coefficient is now calculated in the following manner[6,7]:

$$K(\omega)=\frac{\sqrt{\varepsilon(\omega)}}{c}n\frac{V}{N(\omega)}(1-e^{-\omega/T})$$
$$\times \mathrm{Av}_i\sum_f 2\pi|\langle i|\bar{H}|f\rangle|^2\delta(E_i-E_f). \quad (1.5)$$

Here $\epsilon(\omega)$ is the real part of the dielectric constant, for which, by assumption, there is no dispersion in the considered region of frequencies. c is the velocity of light in vacuum, n the electron concentration, V the normalization volume, and $N(\omega)$ the number of photons in the initial state with frequency $\omega = ck/\sqrt{\epsilon(\omega)}$. The summation over the final states f denotes summation over α and $\pm\mathbf{q}$. Averaging over the initial states i is formal averaging over α and $N(\mathbf{q})$. Since we are considering only the absorption of photons, we introduce the factor in the round brackets, which takes into account the stimulated emission of the photons [8].

Written down in greater detail, the matrix element of the transition (1.4) takes the form

$$\langle\alpha,0,0|\bar{H}|\alpha',\pm\mathbf{q},-\mathbf{k}\rangle$$
$$=\sum_{\alpha''}\left[\frac{\langle\alpha,0|H_R|\alpha'',-\mathbf{k}\rangle\langle\alpha'',0|H_L|\alpha',\pm\mathbf{q}\rangle}{\varepsilon_\alpha-\varepsilon_{\alpha''}+\omega}\right.$$
$$\left.+\frac{\langle\alpha,0|H_L|\alpha'',\pm\mathbf{q}\rangle\langle\alpha'',0|H_R|\alpha',-\mathbf{k}\rangle}{\varepsilon_\alpha-\varepsilon_{\alpha''}\mp\omega(\mathbf{q})}\right]. \quad (1.6)$$

The difference of the energies in the δ-function is

$$E_i-E_f=\varepsilon_\alpha-\varepsilon_{\alpha'}\mp\omega(\mathbf{q})+\omega. \quad (1.7)$$

Therefore, from the energy denominator in the second term we can eliminate the frequency of the phonon, replacing it by $\epsilon_\alpha - \epsilon_{\alpha''} - \omega$.

The Hamiltonians of the perturbations H_L and H_R are taken in the usual form. The interaction with the lattice is

$$H_L=\sum_{\mathbf{q}}ic_{\mathbf{q}}e^{i\mathbf{qr}}b_{\mathbf{q}}+\mathrm{c.c.} \quad (1.8)$$

Here b_q is the phonon vanishing operator. In place of c_q it will be convenient to use the quantity

$$B(\mathbf{q})=V(2\pi)^{-3}|c_{\mathbf{q}}|^2, \quad (1.9)$$

which does not depend on the normalization volume. As usual, we shall assume that $B(q)$ and $\omega(q)$ are isotropic. The interaction with the high frequency field is

$$H_R=\frac{e}{mc}\mathbf{PA}', \quad (1.10)$$

where

$$\mathbf{P}=-i\nabla+\frac{e}{c}\mathbf{A}^0 \quad (1.11)$$

is the generalized momentum in a constant magnetic field with vector potential $\mathbf{A}^0$, and $\mathbf{A}'$ is the vector potential of the high frequency field. In the quantized form

$$\mathbf{A}'=\sum_{\mathbf{k}}\left(\frac{2\pi c^2}{\omega\varepsilon(\omega)V}\right)^{1/2}\mathbf{e}_{\mathbf{k}}e^{i\mathbf{kr}}a_{\mathbf{k}}+\mathrm{c.c.}, \quad (1.12)$$

where $\mathbf{e}_k$ is a unit polarization vector and a_k is the photon vanishing operator.

To calculate the matrix elements we choose H along z and $A^0 = -Hy$ along x. Then $\alpha \equiv (lp_xp_z)$. The characteristic quantity for p_z is the thermal momentum $p_T = \lambda_T^{-1}$, and for p_x it is the magnetic momentum $p_H = \lambda_H^{-1}$. The matrix elements of the interaction with the lattice are expressed in the following form:

$$\langle lp_xp_z,0|H_L|l'p_x'p_z',\pm\mathbf{q}\rangle$$
$$=\pm ic_{\mathbf{q}}(N(q)+1/2\pm1/2)\delta(p_x,p_x'\pm q_x)\delta(p_z,p_z'\pm q_z)$$
$$\times\exp\{\pm2p_xq_y/p_H^2\}M_{ll'}(\pm\mathbf{q}_\perp), \quad (1.13)$$

where

$$M_{ll'}(\mathbf{q}_\perp)=\int_{-\infty}^{+\infty}dy\,\Phi_l^*(y)\,e^{iq_yy}\Phi_{l'}\left(y+\frac{cq_x}{eH}\right) \quad (1.14)$$

is the integral with oscillator functions Φ_l. If we introduce in the q_xq_y plane the polar coordinates $q_\perp$ and φ, then

$$M_{ll'}(\mathbf{q}_\perp)=\exp\{-iq_xq_y/p_H^2\}\exp^{i(l'-l)\varphi}Q_{ll'}(q_\perp/p_H). \quad (1.15)$$

We have introduced here the function Q usually employed in such problems, connected with the generalized Laguerre polynomial L by

$$Q_{ll'}(\varkappa)=(-1)^{l-l'}Q_{l'l}(\varkappa), \quad (1.16)$$

with

$$Q_{ll'}(\varkappa)=(-1)^{l-l'}(l'!/l!)^{1/2}\varkappa^{l-l'}L_{l'}^{l-l'}(\varkappa^2)e^{-\varkappa^2/2} \quad (1.17)$$

for $l > l'$. The Laguerre polynomials are defined in accordance with [9].

In the calculation of the matrix elements of the interaction H_R the field is assumed uniform, i.e., we stipulate in fact that $\lambda \gg \lambda_T$ and λ_H, thus imposing a lower bound on the temperature and an upper bound on the magnetic field. Under these assumptions, the transitions will be dipole and

$$\langle\alpha,0|H_R|\alpha',-\mathbf{k}\rangle=\frac{e}{mc}N(\omega)^{1/2}\left(\frac{2\pi c^2}{\omega\varepsilon(\omega)V}\right)^{1/2}\langle\alpha|P|\alpha'\rangle, \quad (1.18)$$

where P is the projection of (1.11) on the polarization direction $\mathbf{e}_k$.

In calculating the absorption for linear polarization, it is convenient to choose $\mathbf{e}_k$ in the y direction, and then

$$\langle\alpha|P|\alpha'\rangle\equiv\langle lp_xp_z|P|l'p_x'p_z'\rangle=\delta(p_x,p_x')\delta(p_z,p_z')p_{ll'}, \quad (1.19)$$

where $p_{ll'}$ are matrix elements of the momentum for the oscillator.

Substituting all the intermediate results in (1.6), we find from the square of the transition matrix element

$$|\langle i|\hat{H}|f\rangle|^2 = \left(\frac{e}{mc}\right)^2 N(\omega)\frac{2\pi c^2}{\omega\varepsilon(\omega)V}\frac{(2\pi)^3}{V}B(q) \times \left(N(q)+\frac{1}{2}\pm\frac{1}{2}\right)\delta(p_x, p_x'\pm q_x)\delta(p_z, p_{z'}'\pm q_z) \times D_{ll'}(\pm\mathbf{q}_\perp), \quad (1.20)$$

$$D_{ll'}(\mathbf{q}_\perp) = \left|\sum_{l''}\left[\frac{p_{ll''}M_{l''l'}}{\omega_H(l-l'')+\omega}+\frac{M_{ll''}p_{l''l'}}{\omega_H(l'-l'')-\omega}\right]\right|^2. \quad (1.21)$$

The dependence of $D_{ll'}$ on $\mathbf{q}_\perp$ is determined by the corresponding dependence in $M_{ll'}$. Substituting in the sum over l'' the explicit expressions for $p_{ll'}$ and $M_{ll'}$ in accordance with (1.15), and using the recurrence relations for the Laguerre polynomials, we can show that

$$D_{ll'}(\mathbf{q}_\perp) = \left|\frac{e^{i\varphi}}{\omega_H+\omega}+\frac{e^{-i\varphi}}{\omega_H-\omega}\right|^2\frac{1}{2}\left(\frac{q_\perp}{p_H}\right)^2\left|Q_{ll'}\left(\frac{q_\perp}{p_H}\right)\right|^2. \quad (1.22)$$

We now substitute (1.20) in expression (1.5) for K and go over to integration with respect to $\mathbf{q}$. The integration with respect to φ is easily carried out, since φ enters only in (1.22). The thermal averaging over the phonons leads to replacement of $N(q)$ by the Planck functions $N_T(q)$. The thermal averaging over the electrons leads to the appearance of a Boltzmann distribution (normalized to unity)

$$w_l(p_z) = w_l w(p_z), \quad (1.23)$$

where the distribution at the Landau level is

$$w(p_z) = \frac{1}{\sqrt{\pi}\,p_T}\exp\left\{-\left(\frac{p_z}{p_T}\right)^2\right\} \quad (1.24)$$

and the distribution over these levels is

$$w_l = 2\sinh\left(\frac{\omega_H}{T}\right)\exp\left\{-\frac{\omega_H}{T}\left(l+\frac{1}{2}\right)\right\}. \quad (1.25)$$

As a result we obtain

$$K(\omega) = K^+(\omega)+K^-(\omega), \quad (1.26)$$

where K^+ and K^- are the contributions made to the absorption by the processes with emission and absorption of a phonon, respectively. They are given by

$$K^\pm(\omega) = \sum_{l,l'=0}^{\infty} K_{ll'}^\pm(\omega), \quad (1.27)$$

where $K_{ll'}$ is the contribution from the transitions of the electron from the Landau level l to the level l'. Here

$$K_{ll'}^\pm(\omega) = A(\omega)\,w_l\int_{-\infty}^{+\infty}dp_\parallel w(p_\parallel)\int_{-\infty}^{+\infty}dq_\parallel \times\int_0^\infty dq_\perp q_\perp\left[N_T(q)+\frac{1}{2}\pm\frac{1}{2}\right] \times B(q)\,q_\perp^2\left|Q_{ll'}\left(\frac{q_\perp}{p_H}\right)\right|^2\delta\left[(l-l')\omega_H+\omega\mp\omega(q) -\frac{1}{2m}(q_\parallel^2\mp 2p_\parallel q_\parallel)\right]. \quad (1.28)$$

In the integration $q^2 = q_\parallel^2 + q_\perp^2$. The factor $A(\omega)$ is a slowly varying function of the frequency, and does not contain singularities anywhere except the point $\omega = \omega_H$, which is of no interest to us:

$$A(\omega) = \frac{1}{4}(2\pi)^3 n a_R\frac{1}{m^2\omega}\left[\frac{1}{(\omega+\omega_H)^2}+\frac{1}{(\omega-\omega_H)^2}\right]. \quad (1.29)$$

The coupling constant of the electrons with the high frequency field is

$$a_R = e^2/c\sqrt{\varepsilon(\omega)}. \quad (1.30)$$

2. FORM, INTENSITY, AND LOCATION OF ABSORPTION PEAKS

In accordance with the conditions indicated in the introduction for observing cyclotron-phonon resonance, let us consider the optical phonons without dispersion, i.e., $\omega(q) = \omega_0$ and low temperatures $T \ll \omega_0, \omega_H$. Then integration with respect to $q_\perp$ is not connected with temperature and leads to the function

$$\Phi_{ll'}(q_\parallel) = \int_0^\infty dq_\perp q_\perp^3 B(q)\left|Q_{ll'}\left(\frac{q_\perp}{p_H}\right)\right|^2. \quad (2.1)$$

$B(q)$ is usually a power-law function and therefore does not have a characteristic variation interval. The characteristic interval of variation of Φ is determined by the function Q, and is therefore equal to p_H.

The formula for the absorption coefficient can be represented now in the form

$$K_{ll'}^\pm(\omega) = A(\omega)\left[N_0+\frac{1}{2}\pm\frac{1}{2}\right]w_l \times\int dp_\parallel w(p_\parallel)\int dq_\parallel\Phi_{ll'}(q_\parallel) \times\delta\left[\omega-\omega_{ll'}^\pm-\frac{1}{2m}(q_\parallel^2-2p_\parallel q_\parallel)\right], \quad (2.2)$$

where we have introduced the resonant frequencies

$$\omega_{ll'}^\pm = (l'-l)\omega_H\pm\omega_0. \quad (2.3)$$

Owing to the δ function, the absorption has singularities at the points $\omega_{ll'}^{\pm}$. The distances between these singularities are of the order of ω_H (or ω_0, which we assume to be of the same order). The character of the singularity can be investigated by assuming that the deviation from resonance is $|\Delta\omega| = |\omega - \omega_{ll'}^{\pm}| \ll \omega_H$. Then, owing to the factors w and δ, the only essential region of integration in the double integral (2.2) with respect to $p_{||}$ and $q_{||}$ is the one with dimensions determined by the larger of the quantities p_T or $(2m\Delta\omega)^{1/2}$. Since both these quantities are much smaller than p_H, we can assume in the investigation of the character of the singularity that the function Φ is constant, and put in it $q_{||} = 0$.

Thus, the character of the singularity of all the peaks is the same and is determined by the integral

$$\int_{-\infty}^{+\infty} dp\, w(p) \int_{-\infty}^{+\infty} dq\, \delta\left[\Delta\omega - \frac{1}{2m}(q^2 - 2pq)\right]. \qquad (2.4)$$

This integral can be calculated in general form by means of the Bessel function K_0, leading to the following relation:

$$e^{\Delta\omega/2T} K_0(|\Delta\omega|/2T). \qquad (2.5)$$

Using the asymptotic behavior of the function K_0, we can readily clarify the character of the singularity. To the right of the resonant point $\Delta\omega > 0$, and far away from it, at distances $\Delta\omega \gg T$, the singularity has a root-like character $\sim\Delta\omega^{-1/2}$, going over at short distances when $\Delta\omega \ll T$ into a logarithmic singularity, $\sim\ln\Delta\omega$. To the left of the resonant point $\Delta\omega < 0$, and near it, where $|\Delta\omega| \ll T$, the singularity is also logarithmic, but far from it, at $|\Delta\omega| \gg T$, an exponential decrease $|\Delta\omega|^{-1/2}\exp(-|\Delta\omega|/T)$ is superimposed on the root singularity. This is perfectly understandable, for when the photon energy is "insufficient," $\Delta\omega < 0$, the transition occurs as a result of the Maxwellian tail of the electrons on the initial Landau level.

The logarithmic singularity becomes smeared out because of the collisional width of the level τ^{-1}, i.e., over distances $|\Delta\omega| \sim \tau^{-1}$ which, by assumption, are much smaller than T. Another cause of the smearing of the singularity is the dispersion of the optical phonons.

We now proceed to estimate the intensity of the resonant peaks. To this end it is necessary to calculate $\Phi_{ll'}(0)$, which depends on the scattering mechanism. For polarization scattering by optical phonons (PO) we have

$$B(q) = \frac{1}{(2\pi)^3}\pi\frac{1}{\tau_0}\frac{p_0}{m}\frac{1}{q^2}; \qquad (2.6)$$

for deformation scattering (DO)

$$B(q) = \frac{1}{(2\pi)^3}\pi\frac{1}{\tau_0}\frac{1}{mp_0}. \qquad (2.7)$$

Here $p_0 = (2m\omega_0)^{1/2}$ and τ_0 is the characteristic time (the relaxation time at low temperatures per phonon). The dimensionless constant for the coupling between the electrons and the lattice, α_L, can be introduced in accordance with the relation $\tau_0^{-1} = 2\alpha_L\omega_0$. We then obtain

$$\begin{aligned} &PO:\ \Phi_{ll'}(0) = (2\pi)^{-3}\pi 2^{3/2} m^{1/2}\alpha_L\omega_0^{3/2}\omega_H, \\ &DO:\ \Phi_{ll'}(0) = (2\pi)^{-3}\pi 2^{3/2} m^{1/2}\alpha_L\omega_0^{1/2}\omega_H^2(l + l' + 1). \end{aligned} \qquad (2.8)$$

It is clear from this that Φ, as well as the factor A, influences little the dependence of the intensity of the peak on l or l'.

Thus, the dependence of the intensity on l' is in general weak, and the dependence on l is determined by the number of electrons at the initial level w_l. Leaving out common and slowly-varying factors, we have

$$K_{ll'}^{+} \sim e^{-l\omega_H/T}, \qquad K_{ll'}^{-} \sim e^{-\omega_0/T}e^{-l\omega_H/T}. \qquad (2.9)$$

The same factors determine the most essential dependence on the temperature and on the magnetic field.

Let us consider the locations and the intensities of the resonant peaks. If $\omega_H > \omega_0$, then the emission peaks are situated at the points $\omega = m\omega_H + \omega_0$, $m = 0, 1, 2, \ldots$, and the absorption peaks are located at the points $\omega = m\omega_H - \omega_0$, $m = 1, 2, \ldots$ The transitions that make the principal contribution to the intensity will be those from the lower level, i.e., $l = 0$, $l' = m$. The absorption satellites have an intensity which is $\exp(-\omega_0/T)$ times smaller. The lines and the decisive transitions are shown in Fig. 4.

If $\omega_H < \omega_0$, then the picture is more complicated. Figure 5 shows the case $2\omega_H < \omega_0 < 3\omega_H$. The emission peaks are situated at the points $\omega = m\omega_H + \omega_0$, $m = -2, -1, 0, +1, +2, +3, \ldots$ When $m < 0$, the decisive transitions are those from the upper levels, i.e., $l = |m|$, $l' = 0$. The intensity of these peaks decreases with increasing $|m|$ like $\exp(-|m|\omega_H/T)$. When $m \geq 0$, the decisive transitions will be those from the lower level, i.e., $l = 0$, $l' = m$. The absorption peaks will be at $\omega = m\omega_H - \omega_0$, $m = 3, 4, \ldots$ The decisive transitions will be from the lower level. Because of the factor $\exp(-\omega_0/T)$, they are lower than even the emission peaks connected with the transitions from the upper levels.

In both cases, at large values of m the intensities of the peaks decrease like m^{-3} for PO and m^{-2} for DO.

3. DISCUSSION OF RESULTS AND POSSIBILITIES OF EXPERIMENTAL OBSERVATION

The singularity of the absorption peak of cyclotron-phonon resonance has the same origin as in magnetophonon resonance, i.e., it is connected with the presence of singularities in the density of the initial and final states [2]. To verify this, we can represent the absorption coefficient schematically in the same form as the magnetoconductivity σ_{xx} in [2], namely

$$K \sim \int d\varepsilon g(\varepsilon) f(\varepsilon) \int d\varepsilon' g(\varepsilon') W(\varepsilon, \varepsilon') \delta(\varepsilon' - \varepsilon - \omega \pm \omega_0) = \int d\varepsilon g(\varepsilon) f(\varepsilon) g(\varepsilon + \omega \mp \omega_0) W(\varepsilon). \quad (3.1)$$

Here $f(\epsilon)$ is the population of the initial level ϵ, $W(\epsilon, \epsilon')$ is a regular function connected with the matrix element of the transition to the final state ϵ', and $g(\epsilon)$ is the density of the state in the magnetic field, which has root singularities at the points $\epsilon_l = \omega_H(l + \frac{1}{2})$. If ω is close to one of the frequencies $\omega_{ll'}^{\pm}$, then the integrable singularities of $g(\epsilon)$ in ϵ_l and of $g(\epsilon')$ in $\epsilon_{l'}$ coincide; the product $g(\epsilon)g(\epsilon')$ already has a non-integrable singularity, and this leads to a singularity in the absorption coefficient. Its character can be determined by disregarding the regular factors $f(\epsilon)$ and $W(\epsilon)$, so that

$$K \sim \int d\varepsilon [(\varepsilon - \varepsilon_l)(\varepsilon - \varepsilon_l + \Delta\omega)]^{-1/2} \sim \ln|\Delta\omega|. \quad (3.2)$$

However, as $T \to 0$ the population $f(\epsilon)$ also becomes singular. More accurately speaking,

$$g(\varepsilon) f(\varepsilon) \to \delta(\varepsilon - \varepsilon_l). \quad (3.3)$$

Therefore in this case

$$K \sim \int d\varepsilon \delta(\varepsilon - \varepsilon_l)(\varepsilon - \varepsilon_l + \Delta\omega)^{-1/2} = \begin{cases} 0, & \text{if } \Delta\omega < 0, \\ \Delta\omega^{-1/2}, & \text{if } \Delta\omega > 0. \end{cases} \quad (3.4)$$

This is the meaning of the transformation of the logarithmic singularity into a root singularity on moving away from the resonance point. In other words, when $T \to 0$ the singularity of the absorption coefficient duplicates the singularity of the state density.

Unlike static magnetophonon resonance, cyclotron-phonon resonance does not contain the small factor $\exp(-\omega_0/T)$ for the emission lines. This is connected with the fact that in a static field at low temperatures, the interaction between the electron and the lattice must begin from the absorption of the phonon (after which follows instantaneous re-emission). The probability of such a composite scattering is determined by the first stage, and is therefore exponentially small. The high frequency field throws the electrons into the region $\epsilon > \omega_0$, and preliminary absorption of the phonon becomes unnecessary—emission takes place immediately. However, the cyclotron-phonon resonance contains another small factor α_R, which is missing in magnetophonon resonance.

The method employed for calculating K is connected with the limitation $\omega\tau \gg 1$ [6] and it is therefore incorrect to make the transition $\omega \to 0$ in order to obtain the static conductivity σ_{xx} in a magnetic field. In spite of this fact, putting formally $\omega = 0$, we obtain the main qualitative singularities of the static magnetophonon resonance (the character of the singularity). The point is, apparently, that since the resonant frequencies are determined from the energy conservation law, and the character of the singularity at these frequencies is determined from the state density, these qualitative singularities are in general independent of the method of calculation. This does not pertain, however, to the amplitude of the singularity, which, as can be seen from (1.29), becomes infinite when the method is used in conjunction with the invalid transition $\omega \to 0$.

We now consider the conditions for an experimental observation of cyclotron-phonon resonance. It should be observed in the form of oscillations with frequency ω_H for the function $K(\omega)$ against the background of ordinary cyclotron resonance with an absorption coefficient

$$K_C(\omega) = \frac{1}{c\sqrt{\varepsilon(\omega)}} \frac{ne^2}{m} \frac{\tau}{1 + (\omega - \omega_H)^2\tau^2} \quad (3.5)$$

and lattice absorption by transverse optical phonons

$$K_L(\omega) = \frac{1}{c\sqrt{\varepsilon(\omega)}} \omega_t^2(\varepsilon_0 - \varepsilon_\infty) \frac{\omega^2\gamma}{(\omega^2 - \omega_t^2)^2 + \omega^2\gamma^2}. \quad (3.6)$$

Here $\omega_t = \omega_0(\epsilon_\infty/\epsilon_0)^{1/2}$ is the frequency of the transverse phonons, ϵ_0 and ϵ_∞ are the static and high-frequency dielectric constants, and γ is the level width connected with the anharmonicity. Since the frequencies of the cyclotron-phonon resonance are shifted from the cyclotron-resonance frequency by at least ω_0 (when $m = 1$), and since $\omega\tau_0 \gg 1$, the peak of the cyclotron-phonon resonance will be observed in the far region of the wings of the cyclotron resonance.

Let us compare now the intensity of the principal line K_0^+ with K_C at the frequency $\omega = \omega_H + \omega_0$. Assuming that $|\omega_H - \omega_0| \sim \omega_H \sim \omega_0$ and that the dimensionless factor (2.5) is of the order of unity, we obtain from (2.2)

$$K_{01}^+ \sim \alpha_R\alpha_L \frac{n}{m\omega_0}\left(\frac{\omega_0}{T}\right)^{1/2}. \quad (3.7)$$

For cyclotron resonance we have when $|\omega - \omega_H|\tau \gg 1$

$$K_C \sim \alpha_R \alpha_L n / m\omega_0. \quad (3.8)$$

Thus, the ratio K_{01}^+/K_C, (and of course also $K_{0l'}^+/K_C$) contains the large parameter $(\omega_0/T)^{1/2}$. Therefore the absorption oscillations connected with the cyclotron-phonon resonance will be small against the background of cyclotron resonance.

It must be noted here that the presence of the factor α_L and K_C is connected precisely with the fact that we are considering absorption on the wings; at the maximum, i.e., when $|\omega - \omega_H|\tau \ll 1$, we have

$$K_C \sim \alpha_R \alpha_L^{-1} n / m\omega_0. \quad (3.9)$$

Let us make some numerical estimates, using InSb as an example. We have in accordance with [10]

$$\omega_t = 3.5 \cdot 10^{13}\ \text{sec}^{-1}(54\ \mu), \quad \gamma = 1.3 \cdot 10^{12}\ \text{sec}^{-1};$$
$$\varepsilon_0 = 17.5 \quad \varepsilon_\infty = 16,$$
$$\omega_0 = \omega_l = 3.7 \cdot 10^{13}\ \text{sec}^{-1}(50\ \mu), \quad \alpha_L = 0.014.$$

Using $m = 0.015m_0$ we find that $\omega_H = \omega_0$ when $H = 32$ kG. Thus, the situation shown in Fig. 5, where $\omega_H < \omega_0$, can be readily attained; the situation on Fig. 4, where $\omega_H > \omega_0$, is also feasible although much more difficult to obtain.

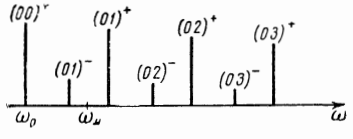

FIG. 4. Location and relative magnitude of the absorption peaks when $\omega_H > \omega_0$. Each peak is marked by the transition $(ll')^{\pm}$, which makes the main contribution to its intensity. Here l and l' are the numbers of the initial and final Landau levels, and the $\pm$ sign indicates that the transition is accompanied by emission (absorption) of a phonon.

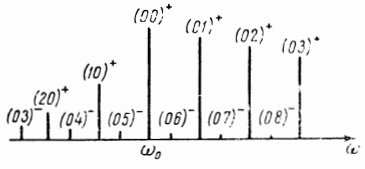

FIG. 5. Location and relative magnitude of the absorption peaks for $2\omega_H < \omega_0 < 3\omega_H$. The notation is the same as in Fig. 4.

The intensity of the cyclotron-phonon resonance is low even at its maximum, since it does not become infinite when $\alpha_L \to 0$. This is connected with the logarithmic character of the singularity; thus, $\alpha_L \ln \alpha_L \to 0$, whereas for cyclotron resonance $\alpha_L(\alpha_L)^{-2} \to \infty$. Therefore, apparently, to observe cyclotron-phonon absorption it is necessary to use samples with appreciable electron concentration. Suitable for this purpose are samples with $n = 10^{16}$ cm^{-3} and mobility $\mu = 10^5$ cm^2/V-sec at $T = 80$°K. Then $\omega_0\tau \sim 40$, so that even fields of 10 kG are strong in the classical sense. To be sure, the quantizing properties will not be too good: $\omega_0/T \sim 3$ at 80°K, so that $\omega_H/T \sim 1$ at 10 kG. This difficulty is in fact encountered also in experiments on magnetophonon resonance at nitrogen temperatures. The transition to hydrogen temperatures $T = 20$°K would make the strong-field criterion somewhat worse, but would improve greatly the quantization criterion.

Assuming that $\alpha_R \sim 10^{-3}$ and $\alpha_L \sim 10^{-2}$, we obtain for such samples $K_{01}^+ \sim 10$ cm^{-1}. This is small compared with the lattice absorption, for which $K_L \sim 10^4$ cm^{-1} at the maximum and $K_L \sim 10^2$ cm^{-1} on the wings at $|\omega - \omega_t| \sim \omega_0$. However, K_L gives a monotonic background, which apparently can be excluded with the aid of (3.6).

If the scattering of the electrons by the optical phonons is not dominating, and the scattering by acoustical phonons is also present, then the latter will make a contribution to the absorption coefficient; this contribution can be calculated with the aid of the general formula (1.28). However, this contribution will be a monotonic function of ω. It must be borne in mind that when comparing the role of the acoustic and optical scatterings, the latter must be taken without the small factor $\exp(-\omega_0/T)$. Therefore, the lowering of the temperature will not weaken the cyclotron-phonon absorption, unlike the magnetophonon effect.

We note also that the cyclotron-phonon resonance can be observed also for the phonons (optical or acoustical) responsible for the interline scattering in semiconductors with several energy minima. In this case, resonance absorption is accompanied by electron transfer from one Landau level in one minimum to another Landau level in another minimum. If we assume both minima to be spherical with identical effective masses, then the corresponding contribution to the absorption can be calculated by means of formula (3.2) in the same manner as for DO scattering, for the matrix element of the transition $B(q)$ for intervalley transitions can be regarded as independent of q [11]. The meaning of ω_0 and τ_0 changes accordingly, but their order of magnitude remains the same. In the case of anisotropy of

the minima, the effect can become complicated by different values of ω_H at these minima for a common orientation of the magnetic field, but qualitatively its character remains the same. The formulas obtained then lead to an order-of-magnitude estimate[2)].

In a more complete theory it would be necessary to take into account the spin of the electron, its orientation in the constant magnetic field (the Zeeman splitting of the Landau levels), and the spin-orbit coupling. Transitions in which phonons participate could then lead to flipping of the electron spin, which is similar in some degree to Rashba's combined resonance [13].

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