ELECTRON DETACHMENT IN SLOW COLLISIONS BETWEEN NEGATIVE IONS AND ATOMS. II. ACCOUNT OF THE FINITE SIZE OF THE SYSTEM

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A reaction of the $A + A^- \rightarrow A + A + e$ type is considered. The effective potential of system A_2, in the field of which a weakly bound electron moves, is represented by two short-range potential wells. It is assumed that the bound state of A_2^- vanishes and disintegrates when the nuclei approach each other. The angular and energy distributions of the emitted particles are calculated for symmetric and antisymmetric states. It is shown that account of the finite size of the system produces an unexpectedly small change in the energy spectrum of the electrons. A general system of equations is deduced for the motion of a particle in the field of moving short-range potential wells of variable depth.

1. INTRODUCTION

IN our preceding paper [1] we considered the detachment of an electron in collisions of the type $A^- + B$ under the assumption that when the atom and the ion come closer together the Σ term AB^- merges with the molecule term AB, the weakly bound electron becomes unstable, and decay takes place. We have obtained the momentum distribution of the outgoing electrons and the decay probability, but the region of applicability of the theory was limited primarily by the fact that the system AB was replaced by an effective potential well, whose radius was assumed small compared with the wavelength of the outgoing electrons. Yet the distance R_0 between the atom and the ion, at which the binding energy $E(R)$ vanishes, can be quite large. At the same time, the momentum of the outgoing electrons k decreases very slowly with decreasing velocity V of the colliding particles (like $V^{1/3}$), so that the criterion of applicability of the theory $kR_0 \ll 1$ is satisfied only for very small velocities V.

In the present investigation we succeeded in getting rid of this limitation, replacing the system AB by an effective potential in the form of two small-radius wells—an approximately recently used by Firsov and Smirnov [2]. We are considering the collision of an ion with its parent atom, i.e., a reaction of the type

$$A + A^- \rightarrow A + A + e.$$

It turns out that, in the decay of the symmetrical and antisymmetrical states, the approximation with zero effective radius gives reasonable results for the outgoing electrons far beyond the limits of the criterion written out above.

The antisymmetrical state, which leads to the p state for the weakly bound proton in the case when the binding energy E tends to zero, yields essentially a different spectrum from that obtained from the decay of the symmetrical state corresponding to $l = 0$ (s state) and considered in [1]. It is possible to generalize the results to include states that correspond to higher values of l in the limit as $E \rightarrow 0$.

The initial, power-law part of the low-energy electron spectrum does not depend on the explicit form of the effective potential well and is determined completely by the first terms of the expansion of $E(R)$ in the vicinity of R_0. Actually, the spectrum is determined only by such quantities as the volume of the phase space etc. (similar, for example, to the behavior of the spectrum of the low-energy neutrino in β decay).

It must be noted that whereas problems involving excitation, charge exchange, etc. can be reduced after suitable schematization to a system of several differential equations, the interaction with the continuous spectrum leads even after simplification to partial differential equations, i.e., to the solution of diffraction problems, to the solution of equations of the heat conduction type, etc.

2. EQUATION OF MOTION OF A PARTICLE IN THE FIELD OF MOVING POTENTIAL WELLS OF SMALL RADIUS

Assume that there are N small-radius potential wells, the positions of which are characterized by vectors $\mathbf{R}_i(t)$, $i = 1, 2, \ldots, N$. Going over to the zero-effective-radius approximation (see, for example, [1,2]), we find that the wave function $\psi(\mathbf{r}, t)$ of the particle should have in the vicinity of each well the form

$$\psi(\mathbf{r}, t) = A_i(t)\left[\frac{1}{|\mathbf{r}-\mathbf{R}_i|} + f_i(t)\right] + O(|\mathbf{r}-\mathbf{R}_i|). \quad (1)$$

The functions $f_i(t)$ characterize the depth of each well, which can depend on the time. If $f_i < 0$ then a bound state exists in the isolated well, but if $f_i > 0$ the well is shallow and there is no such bound state. The transition $f_i \to +\infty$ corresponds to a well depth tending to zero—the wave function then becomes regular at the point $\mathbf{R}_i$.

We write down the nonstationary Schrödinger equation for this problem:

$$\left(-\frac{1}{2}\nabla^2 - i\frac{\partial}{\partial t}\right)\psi = 2\pi\sum_{j=1}^{N} A_j(t)\,\delta(\mathbf{r}-\mathbf{R}_j(t)). \quad (2)$$

Using a Green's function, we can write a formal solution of this equation

$$\psi(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi i}}\sum_{j=1}^{N}\int_{-\infty}^{t} A_j(t')\exp\left[i\frac{|\mathbf{r}-\mathbf{R}_j(t')|^2}{2(t-t')}\right]\frac{dt'}{(t-t')^{3/2}}, \quad (3)$$

from which we see that the wave function can be constructed if one knows N time-dependent functions $A_j(t)$.

Let us go in (3) to the limit as $\mathbf{r} \to \mathbf{R}_i(t)$, subtract the diverging term from the right and left sides, and take formula (1) into account. We obtain a system of integral equations for the functions A_i

$$f_i(t)A_i(t) = \frac{1}{\sqrt{2\pi i}}\sum_{j=1}^{N}\int_{-\infty}^{t}\left[A_j(t')\exp\left(i\frac{|\mathbf{R}_i(t)-\mathbf{R}_j(t')|^2}{2(t-t')}\right) - A_i(t)\delta_{ij}\right]\frac{dt'}{(t-t')^{3/2}}. \quad (4)$$

The correctness of the obtained system can be readily verified for different particular cases. Thus, if there is one stationary well of valuable depth, then the system (4) can be readily transformed into the integral equation obtained in [1]. If all the wells are stationary and of constant depth, then, assuming that all the $A_i(t)$ depend harmonically on the time, we arrive at an algebraic system of linear equations for A_i, which makes it possible to find the stationary states. This system can be readily obtained also directly from condition (1). Finally, if there is one uniformly moving well of constant depth, then the wave function for it can be written immediately by using the Galileo principle. It is easy to verify that this function agrees with (4).

Thus, the solution of the partial differential equation has been reduced to a solution of a system of homogeneous integral Volterra equations for functions of one variable t. This has greatly simplified the problem. These equations can be solved also numerically, for example by the method of steps. It can be assumed that the model considered here will be useful for a great variety of problems.

3. APPROXIMATION OF TWO WELLS OF SMALL RADIUS

We now replace the real problem of the collision between the atom and the ion by the problem of the motion of the electron in the field of two wells of small radius. Such an approximation was used earlier for this problem by Smirnov and Firsov [2] to estimate the value of R_0, and by the same token the cross section for electron detachment. The dynamics of the process was not considered in [2].

Strictly speaking, we would have to consider two moving wells of constant depth and to specify the trajectory of their relative motion. However, the problem cannot be solved exactly even in the simplest approximation of straight-line travel. In addition, we assume that the nuclei move slowly, so that the process occurs essentially near $R \approx R_0$. It is therefore permissible to consider two stationary wells located at a distance $R = R_0$ from each other, but of variable depth, and to choose the rate of change of the well depths such that the binding energy varies with time in the same manner as in the real motion. Further, inasmuch as only the behavior of the terms in the region where the electron ceases to be bound is of importance, we can, just as in [1], assume that f_1 and f_2 are linear functions of the time and pay no attention to their behavior as $t \to \pm\infty$, which does not influence the spectrum of the emitted electrons.

Finally, we confine ourselves to the case of two identical wells, i.e., we assume that $f_1 = f_2$ (the reaction $A + A^- \to A + A + e$). This case is of fundamental interest and admits of an exact solution. Putting

$$f_1 = f_2 = \beta t, \quad A^{\pm} = A_1 \pm A_2, \quad \mathbf{R}_2 - \mathbf{R}_1 = \mathbf{R} = \text{const},$$

we obtain from the system (4) an equation for $A^{\pm}$:

$$\beta t A^{\pm}(t) = \frac{1}{\sqrt{2\pi i}} \int_{-\infty}^{t} \left[A^{\pm}(t') - A^{\pm}(t) \pm A^{\pm}(t') \exp\left(i \frac{R^2}{2(t-t')} \right) \right] \frac{dt'}{(t-t')^{3/2}}. \quad (5)$$

Since the kernel of the integral equation depends only on the difference t − t′, and in the left side the coefficient of $A^{\pm}$ is linear, we solve the equation by the Laplace method. We obtain

$$A^{\pm}(t) = B \int_C \exp\left\{ \frac{i}{\beta} \left[\frac{v^2}{2} \beta t + \frac{v^3}{3} \pm e^{-vR} \left(\frac{1}{R^3} + \frac{v}{R^2} \right) \right] \right\} v\, dv. \quad (6)$$

An investigation of the transition to the limit as $t \to -\infty$, similar to that in [1], enables us to determine the contour whose branches go off to infinity in the directions $\arg v = -\pi/2$ and $\arg v = \pi/4$.

Substituting the obtained expression for A in (3), we obtain the wave function

$$\psi(\mathbf{r}, t) = B \int_C \exp\left[\frac{i}{\beta} \left(\frac{v^3}{3} \pm \frac{e^{-vR}}{R^3} (1 + vR) \right) + \frac{iv^2}{2} t \right] \times \left[\frac{\exp(-v|\mathbf{r} - \mathbf{R}_1|)}{|\mathbf{r} - \mathbf{R}_1|} \pm \frac{\exp(-v|\mathbf{r} - \mathbf{R}_2|)}{|\mathbf{r} - \mathbf{R}_2|} \right] v\, dv. \quad (7)$$

It follows directly from this formula that ψ is the solution of the Schrödinger equation for a free particle everywhere except the points $\mathbf{r} = \mathbf{R}_1$ and $\mathbf{r} = \mathbf{R}_2$.

Let us go over to the momentum representation. Then

$$\varphi(\mathbf{k}, t) = \frac{2B}{\sqrt{2\pi}} [\exp(-i\mathbf{k}\mathbf{R}_1) \pm \exp(-i\mathbf{k}\mathbf{R}_2)] \times \int_C \exp\left[\frac{i}{\beta} \left(\frac{v^3}{3} \pm \frac{e^{-vR}}{R^3} (1 + vR) \right) + \frac{iv^2}{2} t \right] \frac{v\, dv}{v^2 + k^2}. \quad (8)$$

Finally, repeating verbatim the reasoning of [1], we find that as $t \to +\infty$ the momentum distribution for the outgoing electrons is determined by the residue of the integrand at $v = -ik$. We obtain the distribution

$$w(k) = \frac{k^2}{2\pi\beta} [1 \pm \cos(kR\cos\theta)] \times \exp\left[-\frac{2}{\beta} \int_0^k \left(1 \pm \frac{\sin k'R}{k'R} \right) k'^2\, dk' \right], \quad (9)$$

or, after averaging over the angles,

$$w(k) = \frac{2k^2}{\beta} \left(1 \pm \frac{\sin kR}{kR} \right) \times \exp\left[-\frac{2}{\beta} \int_0^k \left(1 \pm \frac{\sin k'R}{k'R} \right) k'^2\, dk' \right]. \quad (10)$$

The undetermined factor B in formula (8) has been replaced in (9) and (10) by the factor that follows from the normalization condition.

4. DISCUSSION

We first express the parameter β, which enters in the final result, in terms of the initial parameters of the problem: the rate of approach of the nuclei V_R and the behavior of the terms in the vicinity of the coalescence point. The equation for the energy of the stationary state of the particle in the field of two identical wells of smaller radius with effective depth f is of the form (see [2])

$$f + \gamma = \pm e^{-\gamma R}/R, \qquad E = -\gamma^2/2. \quad (11)$$

At the point where γ and E vanish, we have for the symmetrical case

$$f_0 = \frac{1}{R_0}, \quad \left(\frac{\partial \gamma}{\partial f} \right)_0 = -\frac{1}{2}, \quad \left(\frac{\partial \gamma}{\partial R} \right)_0 = -\left| \frac{d^2E}{dR^2} \right|_0^{1/2} = -(E_0'')^{1/2}. \quad (12)$$

Equating the changes of γ resulting from the motion of the nuclei and from the variation of the depths of the wells, we obtain

$$\left(\frac{\partial \gamma}{\partial f} \right)_0 \left(\frac{df}{dt} \right)_0 = \left(\frac{\partial \gamma}{\partial R} \right)_0 \left| \frac{dR}{dt} \right|_0, \qquad \beta = 2(E_0'')^{1/2} (V_R)_0. \quad (13)$$

This expression for β does not contain R_0 and differs only by a factor of 2 from the analogous parameter in [1].

If we assume that kR is small and expand the exponent and the pre-exponential factor in a series, retaining one term in each, we obtain

$$w(k) = \frac{4k^2}{\beta} \exp\left(-\frac{4k^3}{3\beta} \right), \quad (14)$$

which, with account of (13), coincides exactly with formula (13) from [1]. This confirms by the same token the assumption made in [1] that formula (14) is valid for sufficiently small k even when kR_0 ($\bar{k}$ is the average momentum) is not small, but is comparable with or even larger than unity. On the other hand, if $\beta R_0^3 \ll 1$, then formula (14) is applicable practically for all k, and this condition coincides with the condition (20) in [1].

It must be noted that the symmetrical bound state vanishes not when the two potential wells come closer together, but when they move apart ($\partial\gamma/\partial R$) < 0, and this vanishing takes place only when f > 0, i.e., there is no bound state for an isolated well. We can therefore no longer employ in this case the approximation of two moving wells

of constant depth, and the approximation of stationary wells of variable depth considerd here is more suitable for the description of the phenomenon.

For the antisymmetrical case we obtain from (11)

$$\left(\frac{\partial E}{\partial f}\right)_0 = -\frac{1}{R_0};$$

$$f_0 = -\frac{1}{R_0}, \quad \left(\frac{\partial E}{\partial f}\right)_0\left(\frac{df}{dt}\right)_0 = \left(\frac{\partial E}{\partial R}\right)_0\left|\frac{dR}{dt}\right|_0. \qquad (15)$$

Thus the parameter β has in this case the form

$$\beta = R_0\left|\frac{\partial E}{\partial R}\right|_0\left|\frac{\partial R}{dt}\right|_0 = (V_R)_0 E_0' R_0. \qquad (16)$$

In the case when $kR \ll 1$ we obtain the distribution

$$w(k) = \frac{k^4R^2}{3\beta}\exp\left(-\frac{k^5R^2}{15\beta}\right). \qquad (17)$$

Thus, the approximation used in [1] is not applicable to this case. The wave function of the weakly bound electron retains the antisymmetry as $R \to R_0$, and depends on the angle like $\cos\theta$ at large distances from the nuclei, i.e., it is a p-function. Then the term corresponding to the bound state does not touch the boundary of the continuous spectrum, but, as can be seen from (15) and (16), crosses the boundary at a certain angle. This term is further continued in the continuous spectrum and corresponds to the quasi-stationary state which is produced as a result of the presence of a centrifugal potential barrier when $l \neq 0$ (see, for example, [3]). This leads to a certain stabilization of the state of the electron and to an increase of the average momentum of the emitted electrons, as can be seen from (17). For states with $l \neq 0$, the zero-effective-radius approximation is not applicable directly, but we see from (17) that such a transition can still be made if we let β approach zero simultaneously with R_0, such that the ratio R^2/β remains finite.

The dependence on the angle θ is contained in the obtained distribution only in the pre-exponential factor. It has a typical interference character and is the result of superposition of two spherical outgoing waves e^{ikr}/r, shifted a distance R relative to each other. The waves are in phase for the symmetrical state, and out of phase for the antisymmetrical state. The dependence on the angle is symmetrical about $\theta = \pi/2$. The symmetrical state gives a maximum of momentum distribution at $\theta = \pi/2$, while the antisymmetrical state gives zero. If $kR < \pi/2$, then the distribution varies monotonically in the intervals $0 < \theta < \pi/2$ and $\pi/2 < \theta < \pi$; on the other hand, interference maxima and minima that reach zero appear if $kR > \pi$. In fact, this picture will become smeared as a result of the motion of the atom and the ion and because of their finite dimensions, which we have neglected. In addition, the angle θ is measured from the axis joining the nuclei at $R = R_0$, which does not coincide with the direction of motion of the incoming particle, so that averaging over the impact parameters results in additional smoothing of the angular distribution. The latter averaging can be eliminated by recording simultaneously the angle of inclination of the heavy particle and the direction of the emitted electron, i.e., by using a coincidence procedure, but this is very difficult to do in our case. We shall therefore consider only the averaged cross sections.

5. AVERAGING OVER THE IMPACT PARAMETERS. EFFECTIVE CROSS SECTION

We introduce in lieu of the parameter β and the momentum k the dimensionless quantities $s = \beta R_0^3$ and $q = kR_0$. Then, for the normalized distribution $w(s, q)$ we obtain

$$w(s, q) = f'(q)s^{-1}\exp(-f(q)/s); \qquad (18)$$

$$f(q) = {}^2/_3 q^3 \pm (\sin q - q\cos q). \qquad (19)$$

If we now take into account the collision geometry and assume that the ion travels past the atom along a straight line with constant velocity V, then the quantity $(V_R)_0$, which enters in β and s, is equal to $V\cos\alpha$, where α is the angle between the vector R_0 and the direction of ion motion.

Introducing the parameter s_0 defined by the formula $s_0 = s/\cos\alpha$, which characterizes the collision as a whole, and assuming that formula (18) is valid for all the impact parameters ρ, we obtain, after averaging over ρ, the effective cross section

$$\sigma(s_0, q) = 2\pi\int_0^{\pi/2} w(s_0\cos\alpha, q)R_0\sin\alpha R_0\cos\alpha\, d\alpha; \qquad (20)$$

$$\frac{\sigma(s_0, q)}{\pi R_0^2} = 2\frac{f'(q)}{s_0}\int_0^{\infty}\exp\left(-\frac{f(q)}{s_0 x}\right)dx$$

$$= 2\frac{f'(q)}{s_0}F\left(\frac{f(q)}{s_0}\right); \qquad (21)$$

$$F(y) = e^{-y} + y\,\mathrm{Ei}(-y), \qquad (22)$$

with the function $\sigma(s_0, q)/(\pi R_0^2)$ normalized to unity with respect to q.

The curves plotted from (20)—(22) for the

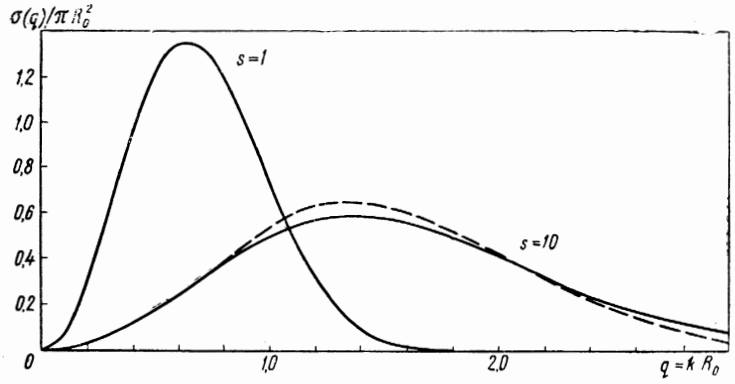

FIG. 1

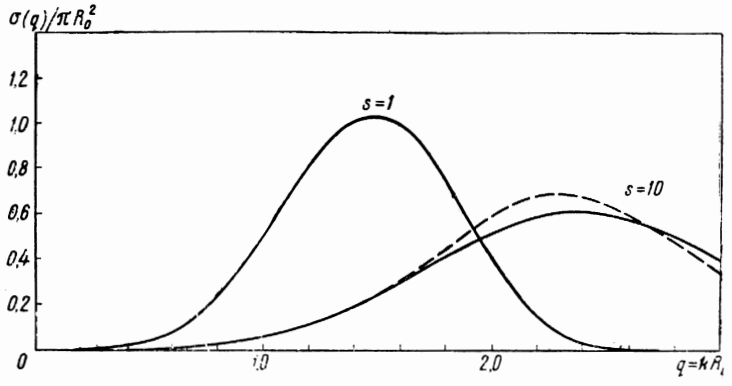

FIG. 2

symmetrical and antisymmetrical cases (for $s_0 = 1$, and $s_0 = 10$), are shown in Figs. 1 and 2 respectively. For $s_0 = 10$ the dashed lines show the curves corresponding to the approximate formulas (14) and (17), if we put approximately $f^+(q) = 4q^3/3$ and $f^-(q) = q^5/15$. Although this assumption is rigorously valid only when $s \ll 1$, we see from the plots that even when $s_0 = 10$ it gives good results everywhere except for large q, at which the cross section is already small. For $s_0 = 1$, within the scale of the drawing, the approximate curves almost coincide with the accurate ones. We see therefore that the approximation with zero effective radius for the forces turns out to be unexpectedly good even for rather large average outgoing-electron velocities, when the wavelength is comparable with the dimensions of the system ($q \sim 1$). It can be assumed that even a more accurate account of the finite dimension of the atoms will change the obtained results appreciably only when q is large.

In a real collision, the symmetrical and antisymmetrical states enter with equal weight, so that the spectrum of the outgoing electrons is obtained as a result of superposition of the two curves. In particular, at a certain ratio of the parameters E_0', E_0'', R_0^+ and R_0^- we can expect to obtain a curve with two maxima as a result of the superposition. Unfortunately we do not know the parameters E_0' and E_0'', so that we cannot employ the obtained formulas for collisions of concrete atom-ion pairs.

To estimate E_0' (but not E_0'') we can use the same approximation of two small-radius wells, assuming that their effective depth does not vary as they come closer together. Then [2]

$$f_0 = -R_2^{-1} = -\sqrt{2I}, \quad E_0' = R_0^{-3} = (2I)^{3/2},$$
$$q = k/k_0 = k/\sqrt{2I}, \quad s_0 = VR_0 = V/\sqrt{2I}, \qquad (23)$$

where I is the electron-affinity energy and k_0 the average momentum of the weakly bound electron. If, for example, I = 1 eV, then $s_0 = 1$ corresponds to a nuclear velocity $V_0 = 0.6 \times 10^8$ cm/sec, and $s_0 = 10$ corresponds to $V = 6 \times 10^8$ cm/sec. It is known, however, that this approximation is not good enough even for ions of alkaline metals. The determination of E_0' and E_0'' within any degree of accuracy apparently calls for rather cumbersome variational calculations. One can count more on being able to determine these quantities from experiment by comparing the experimental and the theoretical curves.

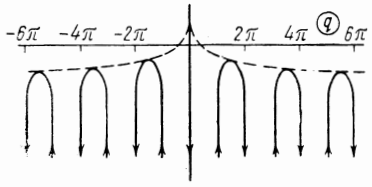

FIG. 3

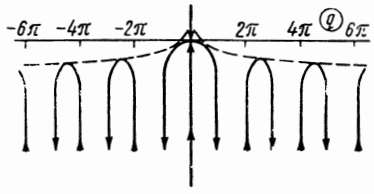

FIG. 4

6. MOTION OF THE POLES OF THE SCATTERING MATRIX AND THE QUASI-ADIABATIC APPROXIMATION

At each instant of time the stationary, quasistationary, and virtual states of our system are characterized by the poles of the S matrix or (which is the same) by the zeroes of the Jost function on the complex plane E or k (see, for example,[3,4]). The approximation which we are considering here and which can be called quasiadiabatic, consists in the fact that we are describing the process of the decay of a bound state from the point of view of the motion of these zeroes (poles) and schematize this motion in a definite manner. We have solved previously[1] the decay problem, where account was taken of only one zero that moved uniformly along the imaginary axis in the complex k plane, going over from the upper half plane (bound state) to the lower one (virtual state). The case considered here corresponds to an account of infinite number of zeroes on the complex plane of the variable $q = kR_0$. The trajectories of these zeroes can be readily obtained by solving Eq. (11) for different values of the parameter f, which is proportional to the time t, and by putting $q = i\gamma R_0$. For the symmetrical and antisymmetrical cases these trajectories are shown respectively in Figs. 3 and 4[1)]. For the antisymmetrical case, upon vanishing of the bound state, the two zeroes cross at the origin and then move apart to the right and to the left, forming, as already noticed, the quasistationary states. The zero-effective-radius approximation corresponds to an account of one of the entire infinite number of zeroes, in the symmetrical case and of two zeroes for the antisymmetrical case.

In order for either one of these approximations to be applicable, it is necessary that the assumed scheme for the motion of the zeroes correspond to the real picture in the vicinity of the origin and during the course of a definite time interval, after the lapse of which the decay has essentially already taken place. According to[1], this condition is analogous to the adiabatic Massey criterion and has respectively for the symmetrical and, analogously, the antisymmetrical case the form

$$E_0''(\delta R)^3/V \gg 1, \qquad E_0'(\delta R)^2/V \gg 1, \tag{24}$$

where δR is the region corresponding to the linear variation of f, and should obviously be in turn much smaller than R_0, i.e., $\delta R \ll R_0$. It is possible, however, that the formulas are applicable in a somewhat broader interval, so that both conditions can be combined and we can write in lieu of (24)

$$V \leqslant E_0''R_0^3, \qquad V \leqslant E_0'R_0^2. \tag{25}$$

The method developed here can be employed to carry out the calculations also for two wells of unequal depth, and this would make it possible to simulate the process of collision of different atoms and ions, but then the problem could not be solved in analytic form and would call for rather laborious numerical calculations.

In addition to conditions (24) and (25), which limit the applicability of the theory from the high-velocity side, there remains in force also the limitation from the low-velocity side[1], which is connected with the fact that we are considering the nuclei classically, specifying their motion beforehand, i.e., we regard them as unlimited sources of energy. This is precisely why we have obtained for the outgoing electrons an energy spectrum which is not bounded from above, whereas in fact it is bounded by the conservation law. In the case when the energy of the colliding particles is much larger than the electron-affinity energy (i.e., when classical treatment of the nuclei is permissible), the probability of large energy transfer to the electron is so small in our case (the far exponential region of the spectrum), that it is practically indistinguishable from zero. In the case, however, when the energy of the colliding particles is comparable with the energy of the electron affinity (near-threshold region),

[1)]All the branches are tangent to the curve $\mathrm{Im}\, q = -\ln|\mathrm{Re}\, q|$, which is shown dashed in the figures.

we must, unless we are especially interested in large energy transfer, consider explicitly the energy exchange between the moving nuclei and the electron, i.e., a quantum description of the nuclear motion is essential, and the theory considered here is not valid.

If a quasistationary state with relatively large lifetime is produced as the nuclei come closer together in the region $R < R_0$, then for the antisymmetrical case the zeroes which diverge from the origin to the right and to the left will move essentially closer to the real axis than on Fig. 4. For the symmetrical case (Fig. 3), the zero that moves downward can also encounter another zero on the imaginary semi-axis in the lower half plane near the origin, after which quasistationary states are also formed. In these cases the region δR will be small and the formulas obtained by us will be valid only for sufficiently small V and s_0.

In place of the analytic properties of the Jost function, we can consider this same problem from the point of view of the analytic properties of the energy terms $U(R)$, which are different values of one multiple-valued analytic function of the complex variable R. An investigation of the spectrum of the electrons emitted during the decay enables us to investigate the analytic properties of the binding energy $E(R)$ near the real root R_0, which is the edge of the cut of the function $E(R)$, so that when $R < R_0$ the function $E(R)$ assumes now, generally speaking, complex values on the real axis.

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