SHOCK WAVES IN LAYERED SYSTEMS

E. I. ZABABAKHIN

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Unrestricted cumulation is observed which is not related to the centripetal motion of a gas but is caused by the special periodic structure of the matter in which the shock wave moves. The motion of such a wave is a periodic self-similar one. Some properties of this new type of self-similarity are described.

INTRODUCTION

THE unbounded cumulation of energy is as a rule associated with centripetal motion, for example in spherically converging shock waves, in the collapse of bubbles in a liquid, and also in converging shock waves of a field. The question naturally arises as to whether this geometric circumstance (convergence to an axis or to a point) is a necessary condition for unbounded cumulation. By considering this problem, it was possible to construct an example of plane shock-wave motion with spontaneous, unbounded growth of pressure on its front; that is, the answer to the problem was shown to be negative.[1)]

Keeping in mind that, under strong shock compression, any material (cold prior to compression) behaves as an ideal gas, we shall consider the motion of the wave just in ideal gases.

Let the system consist of alternating layers of light and heavy gases, and let it be set in motion by a piston so that a shock wave whose front is parallel to the layer travels in it. Further, let the thickness of each successive heavy layer be less than the foregoing, and let the same be true for the light layers. As the wave moves in such a system, one can expect an increase in the shock wave on the basis of the following qualitative consideration.

If the density of the light layers is very small, then in the motion they will be strongly compressed between the layers of heavy material and all of the motion will become similar to a series of collisions of the heavy layers through elastic layers. If in this case the succeeding layer is lighter than the preceding one, while the losses in energy in the collisions are not very great, then it can recoil with great velocity. The same circumstance is repeated for the next shock and so forth.

By establishing a definite ratio of thickness of neighboring heavy layers (and the same for light layers), we get the layered system shown in Fig. 1. The ratios of the thicknesses of all light layers are identical:

$$b_i / x_i = \varepsilon_1 = \text{const}.$$

The same holds for the heavy layers:

$$\varepsilon_h = \text{const}.$$

The boundary of the system is located at x = 0, the number of layers in it is infinitely large. The wave moves from right to left. The real picture of its motion is much more complicated than is described qualitatively, and therefore the question of its unbounded increase is still unclear and must be investigated separately.

Our system is self-similar: when its dimentions change by a factor $(1 - \epsilon_l)(1 - \epsilon_h)$, the system coincides with the original system.

It is clear that such systems can be constructed not only from pairs of layers but also from triplets, quadruplets, and so forth, and also from a continuously repeating profile density, for ex-

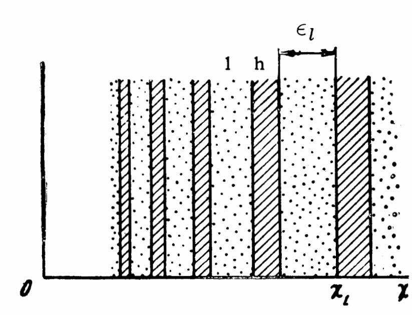

FIG. 1. Self-similar layered system: l = light layer, h = heavy layer.

1)We note that the previously investigated singularity [1] arising upon emergence of a shock wave to the free surface of the atmosphere is not cumulation. In this case, only the velocity of the wave and the temperature behind it increase without limit; the pressure and the energy density do not cumulate, but rather tend to zero.

ample, of the form

$$\rho = A + B \sin \ln x$$

or of the more general form

$$\rho = x^k (A + B \sin \ln x).$$

When such a system is continuously increased and its density scale changed accordingly the distribution of $\rho(x)$ will periodically repeat the initial distribution.

1. QUALITATIVE PICTURE OF THE MOTION

Let us first consider the simplest picture of motion of a plane wave in a periodic layered system with constant absolute thicknesses of light and solid layers. If the piston producing the motion moves for a sufficiently long time with unchanged velocity, then a periodic change of pressure is established in the forward moving wave. The character of the motion that is established is shown in Fig. 2 (only the first wave is shown). The pressure p of the front changes jumpwise upon passage from layer to layer, and also in places where the secondary waves—the results of reflections from the boundaries of the layers—catch up with the first front.

In a self-similar system having a limit, that is, of the type shown in Fig. 1, the periodicity of the pressure on the front can be accompanied by its general increase by a constant factor at each pair of layers. Therefore, for corresponding values of x, for example, on the right sides of the heavy layers, the pressure at the front will have the form

$$p = a / x^n.$$

For other values of x (for example, the centers of the light layers) the law will be the same but the value of a will be different. The character of such a system is shown in Fig. 3.

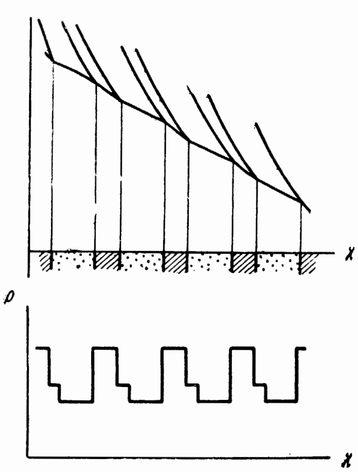

FIG. 2. Steady motion of a shock wave in a simple layered system; p = pressure on the front of the first wave.

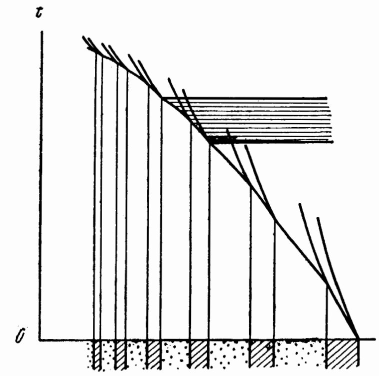

FIG. 3. Motion of a wave in a self-similar layered system.

Evidently the dependence of the pressure on x for the self-similar motion under consideration can be conveniently represented in logarithmic coordinates, where the periodic system with continuously decreasing layers will have a constant pitch, while the dependence of ln p on ln x is shown by a periodic broken line of the type shown in Fig. 4.

The self-similar motions that have been described are much more complicated than the cases of motion of the wave in a homogeneous medium considered earlier. In the latter the solution reduced to finding profiles of pressure density and velocity (their distribution in x) at a single instant of time. The distributions at all other instants were similar and were obtained by means of a scale transformation. Thus the problem was reduced to a single independent variable, that is, to ordinary differential equations. In our case, for the description of the entire solution one needs not the single profile $\rho(x)$ [and also $p(x)$ and the velocity $u(x)$], but all the profiles for a single period, i.e., it is necessary to know p as a function of two variables over the entire section of plane (x, t), for example, that shown in Fig. 3 by the horizontal hatching. Therefore, the new problems do not reduce to a single argument and to ordinary differential equations and solution of

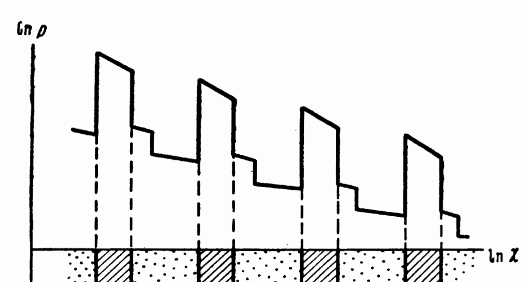

FIG. 4. Pressure at the front of a periodic self-similar wave.

partial differential equations becomes necessary.

One method of solution can be the numerical calculation of the motion of the waves through a sufficiently large number of layers, in the process of which self-similarity is produced. Evidence of this will be the appearance of repeated profiles in the dependence of ln p on ln x. It is not necessary to consider the subsequent motion of the wave; it can be constructed from the already obtained profiles of ρ, p, and u by means of the index n that has been evaluated and by virtue of the fact of self-similarity (in our case, periodicity).

We note that in principle the appearance of a solution with a period corresponding not to one period of the system but to two, three, and so forth has not been excluded. Some of these can be unstable, that is, they can be destroyed by small perturbations which do not destroy the symmetry of the motion (planarity of the wave).

The result of the numerical calculation of a single specific case is given in the next section. The chief purpose of the calculation was to determine the exponent of cumulation n in the law of $p \sim x^{-n}$ or $E \sim x^{-n}$, where E is the energy volume density on the wave front. In our case $E = p/\rho(\gamma - 1)$ where γ is the ratio of specific heats. In a system consisting of alternating layers, the ρ are identical at corresponding points, that is, E and p increase according to the same power law on the front.

It is not difficult to establish the limit for possible values of n. If the energy were transferred from layer to layer as a whole (not remaining in the layers either in the form of kinetic energy or in the form of heat), then its density would be inversely proportional to the thickness of the layer or to the distance from the edge x, i.e., we would have $E \sim x^{-1}$. Thus the upper limit for the exponent of cumulation of a plane wave is $n = 1$. For a plane wave in a homogeneous gas, there is in general no cumulation, that is, $n = 0$.

2. RESULT OF NUMERICAL CALCULATION

Starting from qualitative notions concerning the character of the motion, it can be expected that the cumulation will contribute to a large difference in the densities of the heavy and light layers; therefore, the calculation was made for a system with a very large density ratio, equal to 25. The relative thicknesses of the layers were chosen to be $\epsilon_h = 0.1$ and $\epsilon_l = 0.2$. Numerical calculation of the motion of the plane wave showed that as it approaches the boundary the wave is amplified, that is, an unrestricted cumulation actually takes place; here the exponent is $n = 0.23$. This is the first example of unrestricted increase in pressure taking place in one-dimensional motion, that is, without symmetric convergence of the wave to a center or an axis.

The exponent n is determined from the slope of the broken line which is the plot of the calculated dependence of ln p on ln x. This slope is small (the cumulation is weak) but significantly exceeds the various irregularities connected with the crudeness of the calculation and with the departure of the actual regime from the asymptotic condition.

3. TWO POSSIBLE MODES OF MOTION

We have proved the fact of amplification of a wave in a system with a periodic structure only for ideal gases (cold before the condensation) or for a sufficiently strong wave in real condensed substances, which behave in the strong wave like ideal gases. It is easy to see that the mode can be quite different for a weak initial wave. In fact, if the system consists of layers which have a finite sound velocity, i.e., which can conduct weak shock waves (acoustic), then the first wave in such a system, if it is weak, will become weaker by a finite factor in each pair of layers. The secondary fronts, moving at this time with the same velocity (sound velocity) will not overtake it, i.e., the first wave will be unrestrictedly damped.

Thus, for a layered system consisting of condensed materials, there is a critical strength of the initial wave. If the wave is weaker than critical, then it is unrestrictedly damped; if it is stronger, then it increases without restriction. The presence of a critical strength of the initial wave clearly distinguishes the layered systems from the homogeneous ones, in which any weak converging wave finally leads to a mode of unrestricted amplification.

In conclusion, I wish to thank L. A. Bunatyan, A. A. Bunatyan, K. K. Krupnikov and V. F. Kuropatenko who materially aided the completion of the present research.

[1] G. M. Gandel'man and D. A. Frank-Kamenetskiĭ, DAN SSSR **107**, 811 (1956), Soviet Phys. Doklady **1**, 223 (1957).

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