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ELECTRON-PHONON INTERACTION PROPORTIONAL TO THE EXTERNAL APPLIED FIELD AND SOUND AMPLIFICATION IN SEMICONDUCTORS

S. I. PEKAR

Institute of Semiconductors, Academy of Sciences, Ukrainian S.S.R.

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As a result of the dependence of the dielectric constant on the deformation of a semiconductor, an additional electron-phonon interaction arises on application of a static electric field. It is taken into account, along with the deformation potential and the piezo-electric field, in a study of sound waves in the presence of current carriers drifting in an external field. It is shown that four types of waves exist in an isotropic semiconductor with current carriers of a given sign: (1) A wave which is polarized perpendicular to the direction of propagation and applied field and which is not affected by the electron-phonon interaction, (2) a second transverse wave, (3) a longitudinal wave, and (4) a drift wave accompanied by displacements of the medium. The last three types of waves may be amplified by the applied field and possess almost equal amplification coefficients. For amplification to be possible the carrier drift must be faster than sound in cases (2) and (3) and slower in case (4).

In substances with an anomalously high dielectric constant, of the order of 2000 (e.g., barium titanate, rutile, etc.), the above-mentioned electron-phonon interaction, which is proportional to the external field, predominates and makes possible a gain of 1500 dB/cm at 45 Mc and 3.3 dB/cm at 10^5 cps. The gain is proportional to the frequency, as in the piezoelectric case.

IN 1956, Tolpygo and Uritskiĭ[1a] predicted theoretically the possibility of generating hypersound in a crystal by electron-phonon interaction, when the carrier drift velocity in the external field exceeds the speed of sound. Independently, Weinreich developed a detailed theory of this phenomenon[1b]. In his paper, and also in the more detailed analyses by Spector[2], Kazarinov and Skobov, and others, the electron-phonon interaction was determined by the deformation potential. The first amplification of hypersound was experimentally realized by Hutson, McFee, and White[3a], in a piezoelectric crystal—cadmium sulfide, in which the large effect was due to the strong interaction of the carriers with the piezoelectric field of the phonon, which exceeds greatly the field of the deformation potential (see also [3b]).

In non-piezoelectric crystals, in which the electron-phonon interaction is determined by the deformation potential, the effect is much smaller and could be observed only in rare exceptions. Such exceptional cases include, for example, n-germanium, in which Pomerants[4] obtained large amplification of transverse sound waves, because half of the valleys of the conduction band drop during shear deformation, dragging the electrons to the location of the deformation, whereas the other half of the valleys rises, repelling the electrons; as a result, the deformation is not accompanied by the appearance of space charge[5].

In this paper we consider an electron-phonon interaction mechanism connected with the dependence of the dielectric constant of the crystal on the deformation. It exists both in piezoelectric and in non-piezoelectric crystals. For longer hypersonic waves and at larger dielectric constants, this mechanism dominates over the deformation potential and should be taken into account in many phenomena, particularly in the absorption and amplification of sound waves. A distinguishing feature of the electron-phonon coupling considered here is the proportionality of the coupling constant to the external applied electric field and to the length of the sound wave.

1. FORCES ACTING ON THE DEFORMED CRYSTAL AND ON THE CARRIERS

Let us consider an anisotropic but non-piezoelectric crystal. Its macroscopic potential energy W can be expressed in terms of two independent macroscopic continual degrees of freedom: the elastic displacement vector of the medium $\mathbf{u}(\mathbf{r}, t)$ and the space-charge density $\rho(\mathbf{r}, t)$, produced by the carriers. The latter is due to the change in the electron concentration not only in the conduction band, but also at the local levels, with which the band electrons have time to attain equilibrium at the frequencies considered below.

If the elastic deformation tensor is denoted by $e_{ij} \equiv \frac{1}{2}(\partial u_j/\partial x_i + \partial u_i/\partial x_j)$, then the crystal energy is

$$W = \int \left\{ \frac{1}{2} \lambda_{ijkl} u_{ij} u_{kl} + \frac{1}{e} b_{ij} u_{ij} \rho + \frac{1}{8\pi} \mathbf{D} \varepsilon^{-1} \mathbf{D} \right\} dV. \quad (1)$$

Here λ_{ijkl} are the adiabatic moduli of elasticity[1], e the carrier charge, b_{ij} the deformation-potential tensor (some average value for the band electrons and the localized electrons in equilibrium with them), D the electrostatic induction of the field produced by the charge $\rho(\mathbf{r}, t)$ and by the sources of the external electric field, and ϵ^{-1} the reciprocal dielectric tensor. The first term in (1) is the potential energy of elastic deformation of the neutral crystal in the absence of an electric field, and the second is the change of energy of the electrons in the band and at the local levels as a result of deformation. The third term is the work consumed in producing the electric field in an already deformed crystal, in which ϵ^{-1} depends on the deformation and consequently on the coordinates.

If the local levels of the electrons can be divided into two sharply delineated groups—levels with thermal ionization time considerably smaller and considerably larger than the period of oscillations—then the passage of a sound wave will cause the occupation of the levels of the first group to be in thermal equilibrium with the conduction band, while the level occupation in the second group will remain constant. If the levels of the first group exceed the chemical potential by much more than kT, then the electron concentration n_e in each level will remain proportional to the concentration of the electrons in the conduction band n:

$$n_s = \nu_s(T) n, \qquad \rho = e \left[\sum_s \nu_s + 1 \right] (n - n_0),$$
$$b_{ij} = \left(\sum_s b_{ij}^{(s)} \nu_s + b_{ij}^{(0)} \right) \Big/ \left(\sum_s \nu_s + 1 \right). \quad (2)$$

Here n_0 is the concentration of the conduction electrons in the neutral semiconductor, $b_{ij}^{(0)}$ the deformation-potential constant of the band electron, and $b_{ij}^{(s)}$ the deformation-potential constants of the localized electrons. Thus, the quantity b_{ij} which will be dealt with below can differ from the tensor $b_{ij}^{(0)}$ contained in the formula for the mobility. In the frequency band in which the aforementioned level grouping is possible, ν_s and b_{ij} do not depend on the frequency, but can depend on the temperature.

To calculate the potential $V(\mathbf{r}, t)$ of the macrofield acting on the conduction electrons it is necessary to take the variational derivative of W with respect to ρ at a fixed value of $\mathbf{u}$. Recognizing that the electric field intensity $\mathbf{E}$, its potential φ, and the induction $\mathbf{D}$ satisfy the equations

$$\mathbf{E} = \varepsilon^{-1} \mathbf{D} = -\nabla \varphi, \qquad \operatorname{div} \delta \mathbf{D} = 4\pi \delta\rho, \quad (3)$$

we obtain

$$V = (\delta W / \delta\rho)_{\mathbf{u}} = e^{-1} b_{ij} u_{ij} + \varphi. \quad (4)$$

In calculating the variation of W with respect to $\mathbf{u}$ at fixed ρ it is necessary to take into account the fact that, according to (3), $\operatorname{div} \delta\mathbf{D} = 0$, and therefore in the last term of (1) we need vary only ϵ^{-1}; further, from $\epsilon^{-1}\epsilon = 1$ it follows that $\delta\epsilon^{-1} = -\epsilon^{-1} \cdot \delta\epsilon \cdot \epsilon^{-1}$. As a result

$$(\delta W)_\rho = - \int \left\{ \lambda_{ijkl} \frac{\partial u_{ij}}{\partial x_k} \delta u_l + \frac{1}{e} b_{il} \frac{\partial \rho}{\partial x_i} \delta u_l + \frac{1}{8\pi} E \delta\varepsilon E \right\} dV. \quad (5)$$

To calculate $\delta\epsilon$ we confine ourselves to the case when the undeformed semiconductor is isotropic, and we put[6]

$$\varepsilon_{ik} = \varepsilon_0 (\delta_{ik} - g_1 \delta_{ik} \operatorname{div} \mathbf{u} - g_2 u_{ik}), \quad (6)$$

where g_1, g_2, and ϵ_0 are constants. Then the last term in (5) can be represented in the form

$$-\frac{1}{8\pi} \int \mathbf{E} \delta\varepsilon \mathbf{E} dV = -\frac{\varepsilon_0}{8\pi} \int \left[g_1 \frac{\partial \mathbf{E}^2}{\partial x_l} + g_2 \frac{\partial E_i E_l}{\partial x_i} \right] \delta u_l dV. \quad (7)$$

From (5) and (7) it follows that the force acting on a unit volume of the crystal is

$$F_l = -\left(\frac{\delta W}{\partial u_l} \right)$$
$$= \lambda_{lkij} \frac{\partial u_{ij}}{\partial x_k} + \frac{1}{e} \frac{\partial \rho}{\partial x_i} b_{il} + \frac{\varepsilon_0}{8\pi} \left[g_1 \frac{\partial \mathbf{E}^2}{\partial x_l} + g_2 \frac{\partial E_i E_l}{\partial x_i} \right]. \quad (8)$$

In an isotropic medium the only nonvanishing elements of the tensor λ_{lkij} are

$$\lambda_{xxyy} = \lambda, \qquad \lambda_{xyxy} = \mu, \qquad \lambda_{xxxx} = \lambda + 2\mu \quad (9)$$

and the elements equal to the foregoing from symmetry considerations (i.e., obtained by permuta-

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[1] Adiabatic temperature oscillations of the crystal during the passage of a sound wave are accompanied by electron transitions between the band and local levels, which leads to an additional dependence of λ_{ijkl} on the frequency and on $\rho(r, t)$. Below we shall neglect these effects.

tion of the first pair of indices, or of the second pair of indices, or of the first pair with the second). Taking (9) into account, and also assuming that $b_{ij} = b\delta_{ij}$ in an isotropic medium, we can rewrite (8) in the form

$$\mathbf{F} = (\lambda + \mu)\nabla(\nabla\mathbf{u}) + \mu\Delta\mathbf{u} + \frac{b}{e}\nabla\rho$$
$$+ \frac{\varepsilon_0}{8\pi}[g_1\nabla\mathbf{E}^2 + g_2(\mathbf{E}(\nabla\mathbf{E}) + (\mathbf{E}\nabla)\mathbf{E})]. \tag{10}$$

Here λ and μ are Lame coefficients.

The potential (4) corresponds, in the case of an isotropic body, to the following total effective field acting on the conduction electrons

$$\mathbf{E}^* = -\nabla V = -be^{-1}\nabla(\nabla\mathbf{u}) + \mathbf{E}. \tag{11}$$

The electric field $\mathbf{E}$ consists of three parts: the field of the external sources in the undeformed crystal $\mathbf{E}_0$, a correction $\mathbf{E}_u$ due to the change of ϵ upon deformation, and the field $\mathbf{E}_\rho$ corresponding to the space charge. The values of $\mathbf{u}$, ρ, $\mathbf{E}_u$, and $\mathbf{E}_\rho$ are assumed to be of first order of smallness, while $\mathbf{E}_0$ and n_0 are of zeroth order. All three fields are assumed to be potential. $\mathbf{E}_0$ is assumed specified, and $\mathbf{E}_u$ and $\mathbf{E}_\rho$ are determined by the following equations, in which only the first-order terms have been retained:

$$\nabla\mathbf{E}_u = (g_1 + g_2/2)(\mathbf{E}_0\nabla)(\nabla\mathbf{u}) + {}^1\!/_2 g_2\mathbf{E}_0\Delta\mathbf{u}, \tag{12}$$

$$\nabla\mathbf{E}_\rho = 4\pi\rho/\varepsilon_0. \tag{13}$$

In expressions (10) and (8), the terms quadratic in E_0 constitute the usual electrostriction force, which produces a small static deformation of the body without any practical change in its properties. We shall henceforth neglect these terms. The terms with the product $\mathbf{E}_0\mathbf{E}_u$ have likewise no bearing on the electron-phonon interaction; they are linear in $\mathbf{u}$, and can be combined with the elastic force by slightly changing the elastic moduli, the changes being proportional to $\mathbf{E}_0$. These terms will also be discarded. The aforementioned electron-phonon interaction, which is proportional to the external field, is represented by the term $\mathbf{E}_0\mathbf{E}_\rho$ in (8) and (10), and also by the term $\mathbf{E}_u$ in (11).

2. SOLUTION OF THE WAVE EQUATIONS

The current density in the crystal is

$$\mathbf{J} = \sigma[\mathbf{E} - be^{-1}\nabla(\nabla\mathbf{u})] + q\mathbf{v}\rho - qD\nabla\rho, \tag{14}$$

where σ is the electric resistivity of the neutral semiconductor, $\mathbf{v}$ the drift velocity in the field $\mathbf{E}_0$, D the diffusion coefficient of the band electrons, and q the constant coefficient in the relation $e(n - n_0) = q\rho$; if formulas (2) are valid we have

$$q = \Big[\sum_s \nu_s + 1\Big]^{-1}.$$

We assume that $\mathbf{E}_0$ is a homogeneous static field. $\mathbf{u}$ and ρ are determined from the equations

$$\gamma\ddot{\mathbf{u}} = \mathbf{F}, \qquad \dot{\rho} + \nabla\mathbf{J} = 0$$

(γ is the density). If we set $\mathbf{u}$ and ρ proportional to $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ and take into account (10) and (12)—(14), these equations become

$$(\lambda + \mu)(\mathbf{ku})\mathbf{k} + \mu k^2\mathbf{u} - \gamma\omega^2\mathbf{u}$$
$$= [(g_1 + g_2/2)(\mathbf{E}_0\mathbf{k})\mathbf{k}/k^2 + {}^1\!/_2 g_2\mathbf{E}_0 + ib\mathbf{k}/e]\rho, \tag{15}$$

$$\{qDk^2 + 4\pi\sigma/\varepsilon_0 + i[q\mathbf{vk} - \omega]\}\rho$$
$$= \sigma[(g_1 + g_2/2)(\mathbf{kE}_0)(\mathbf{ku})$$
$$+ {}^1\!/_2 g_2 k^2\mathbf{E}_0\mathbf{u} - ibe^{-1}k^2\mathbf{ku}]. \tag{16}$$

The projection of the vector equation (15) on the direction of $\mathbf{p}$, which is perpendicular to $\mathbf{k}$ and $\mathbf{E}_0$, leads to the equation of the ordinary transverse sound wave

$$(\mu k^2 - \gamma\omega^2)u_p = 0. \tag{17}$$

This wave does not interact with ρ and $\mathbf{E}_0$, nor is it involved in (16). Therefore it will be no longer mentioned. The remaining projections of $\mathbf{u}$ are best determined by projecting (15) on directions parallel and perpendicular to $\mathbf{k}$ and $\mathbf{p}$. As a result we obtain, respectively,

$$[(\lambda + 2\mu)k^2 - \gamma\omega^2]u_{\|} = [(g_1 + g_2)E_{0\|} + ib|k|/e]\rho, \tag{18}$$

$$(\mu k^2 - \gamma\omega^2)u_\perp = {}^1\!/_2 g_2 E_{0\perp}\rho. \tag{19}$$

From (16), (18), and (19) we determine the amplitudes of $u_{\|}$, $u_\perp$, and ρ. The conditions for the solvability of these homogeneous equations determines the connection between ω and $\mathbf{k}$, i.e., the wave dispersion law.

If we introduce the notation

$$\frac{1}{\tau} = \frac{4\pi\sigma}{\varepsilon_0}, \quad \frac{1}{\tau_k} = \frac{1}{\tau} + qDk^2, \quad k_1 = \frac{\omega}{s_1},$$

$$k_2 = \frac{\omega}{s_2}, \quad s_1 = \left(\frac{\lambda + 2\mu}{\gamma}\right)^{1/2}$$

is the velocity of the longitudinal sound in the substance, $s_2 \equiv \sqrt{\mu/\gamma}$ the velocity of the transverse sound, and

$$G_1 = \frac{(g_1 + g_2)E_{0\|} + ibk/e}{s_1\sqrt{\gamma}} \qquad G_2 = \frac{g_2 E_{0\perp}}{2s_2\sqrt{\gamma}}$$

then the dispersion law (which can be easily obtained by eliminating $u_{\|}$ and $u_{\perp}$ from (16), (18), and (19)), takes the form

$$\frac{\varepsilon_0}{4\pi}\frac{\tau_k/\tau}{1+i[q\mathbf{v}\mathbf{k}-\omega]\tau_k}\left\{\frac{|G_1|^2}{1-k_1^2/k^2}+\frac{|G_2|^2}{1-k_2^2/k^2}\right\}=1. \quad (20)$$

In the general case the function $\mathbf{k}(\omega)$ can be tabulated if we know the parameters entering in (20). On the other hand, if we assume that the electron-phonon interaction only weakly perturbs the waves existing in the medium without this interaction, i.e., that the right sides of (16), (18), and (19) are small compared with the individual terms of the left sides, then we obtain simple approximate solutions for all three types of waves defined by (20).

1. For the wave with $k \approx k_1$,

$$1-\frac{k_1^2}{k^2}=\frac{\varepsilon_0}{4\pi}\frac{\tau_k}{\tau}|G_1|^2\frac{1}{1+i[qv_{\|}/s-1]\omega\tau_k}, \quad (21)$$

where $s \equiv \omega/k$ is the phase velocity of the acoustic-charge wave. If, for example, we specify real ω, then the relative change in k due to the electron-phonon interaction is

$$\frac{\Delta k}{k_1}\equiv\frac{k-k_1}{k_1}=\frac{\varepsilon_0}{8\pi}\frac{\tau_k}{\tau}|G_1|^2\frac{1-i[qv_{\|}/s-1]\omega\tau_k}{1+[qv_{\|}/s-1]^2\omega^2\tau_k^2}; \quad (22)$$

$$u_{\|}=\frac{G_1\rho}{2k_1\Delta k s_1\sqrt{\gamma}}, \quad u_{\perp}=\frac{G_2\rho}{(k_1^2-k_2^2)s_2\sqrt{\gamma}}. \quad (23)$$

2. For the wave with $k \approx k_2$, putting $\Delta k \equiv k - k_2$, we obtain similarly

$$\frac{\Delta k}{k_2}=\frac{\varepsilon_0}{8\pi}\frac{\tau_k}{\tau}|G_2|^2\frac{1-i[qv_{\|}/s-1]\omega\tau_k}{1+[qv_{\|}/s-1]^2\omega^2\tau_k^2}; \quad (24)$$

$$u_{\|}=\frac{G_1\rho}{(k_2^2-k_1^2)s_1\sqrt{\gamma}}, \quad u_{\perp}=\frac{G_2\rho}{2k_2\Delta k s_2\sqrt{\gamma}} \quad (25)$$

In formulas (22) and (23) it is necessary to substitute k_1 for k in the expressions for G_1 and τ_k, while in formulas (24) and (25) it is necessary to substitute k_2. The approximation of weak electron-phonon coupling used above signifies that $\Delta k \ll k_2 - k_1$ in both waves in question. Then **u** is practically parallel to **k** in the first wave and practically perpendicular to **k** in the second wave.

3. For the wave with

$$k \approx k_3 \equiv \omega/qv_{\|} + i/qv_{\|}\tau_k,$$

the dispersion law takes the form

$$k=\frac{\omega}{qv_{\|}}+\frac{i}{qv_{\|}\tau_k}-\frac{i\varepsilon_0}{4\pi\tau qv_{\|}}\left\{\frac{|G_1|^2}{1-k_1^2/k_3^2}+\frac{|G_2|^2}{1-k_2^2/k_3^2}\right\}. \quad (26)$$

In order for the employed method of weak electron-phonon coupling to be valid, it is necessary that the third term in the right side of (26) be much smaller in magnitude than the sum of the first two terms. If we are interested in waves with growing amplitude (in space or in time), then it is necessary that the imaginary part of k be negative, i.e., that the third term be larger in modulus than the second. These conditions lead to the inequality $\omega\tau_k \gg 1$, which makes it possible to write (26) in the approximate form

$$k=\frac{\omega}{qv_{\|}}+\frac{i}{qv_{\|}\tau_k}-\frac{\varepsilon_0\omega\tau_k}{8\pi\tau qv_{\|}}\left\{|G_1|^2\beta_1^2\frac{1-{}^1\!/_2 i\omega\tau_k(1-\beta_1^2)}{1+[{}^1\!/_2\omega\tau_k(1-\beta_1^2)]^2}\right.$$
$$\left.+|G_2|^2\beta_2^2\frac{1-{}^1\!/_2 i\omega\tau_k(1-\beta_2^2)}{1+[{}^1\!/_2\omega\tau_k(1-\beta_2^2)]^2}\right\}, \quad (27)$$

where

$$\beta_1 \equiv s_1/qv_{\|}, \quad \beta_2 \equiv s_2/qv_{\|}.$$

If β_1 and β_2 are larger than unity, then the corresponding terms make a negative contribution to the imaginary part of k. In the wave considered here $u_{\|}$, $u_{\perp}$, and ρ are connected by the relations

$$u_{\|}=\frac{G_1\rho}{(k^2-k_1^2)s_1\sqrt{\gamma}}, \quad u_{\perp}=\frac{G_2\rho}{(k^2-k_2^2)s_2\sqrt{\gamma}}. \quad (28)$$

3. DISCUSSION OF RESULTS AND COMPARATIVE ESTIMATE OF THE ROLE OF TWO TYPES OF ELECTRON-PHONON INTERACTIONS

In an isotropic medium with carriers of one sign and with a homogeneous external applied electric field, all the equations of the acoustic-charge waves contain only the component of the drift velocity along the wave vector **k**; the transverse component of the carrier drift velocity does not enter in these equations. A wave polarized perpendicular to **k** and $\mathbf{E}_0$ [see (17)] remains purely acoustic, i.e., it does not interact with the carriers and with the field $\mathbf{E}_0$. The remaining three waves can, as a result of electron-phonon interaction, become additionally absorbed or amplified, depending on the sign of the imaginary part of k.

For a specified real ω, the intensity of these waves varies in space like $\exp[-2x\,\mathrm{Im}(k)]$. The variation of k due to the electron-phonon interaction is determined by formulas (22), (24), and (27) respectively for each of the waves. According to (22) and (24), amplification should set in when $qv_{\|}$ exceeds the phase velocity of the mixed acoustic-charge wave (if the amplification effect in question exceeds the absorption for various other reasons). On the other hand, in the case of (27), the condition for the imaginary part of k to be negative has a somewhat more complicated form but can also be satisfied.

If the Einstein relation between the mobility and the diffusion coefficient is satisfied, then τ_k/τ does not depend on the mobility. For amplification of the waves it is therefore advantageous to use semiconductors with minimum mobility, for according to (23) and (24) the amplification coefficient does not depend on the mobility, and in the case of (27) it even increases with decreasing mobility. On the other hand, in substances with low mobility, breakdown usually occurs at larger fields, and in order to produce the required drift velocity it is possible to increase E_0 and thereby increase the amplification coefficient.

Further, at large carrier densities n_0, we have $\tau_k/\tau \approx 1$, i.e., it does not depend on n_0. It is therefore advantageous to reduce n_0 until $4\pi\sigma/\epsilon_0$ becomes of the same order as qDk^2; then the Joule heat released decreases without adversely affecting the amplification coefficient, i.e., the energy efficiency increases and thermal breakdown of the device is prevented.

If we exclude the dependence of the dielectric constant on the deformation, letting g_1 and g_2 tend to zero and leaving in the electron-phonon interaction only the deformation potential, then the solution (22) and (23) for an almost-longitudinal wave should go over to that obtained by Weinreich[1]. This actually takes place if we neglect in our expression for $1/\tau_k$ the term $4\pi\sigma/\epsilon_0$ compared with the term qDk^2 (which is not always possible). It is also necessary to eliminate carrier recombination from both theories, since the assumptions made concerning this recombination differ in these theories; to this end it is necessary to put q = 1 and let the recombination time approach infinity in Weinreich's formulas.

The results obtained above can be generalized to include piezoelectric crystals. For such crystals

$$D_i = D_{0i} + \varepsilon_{ik}E_k + \gamma_{ikl}\sigma_{kl} \;=\; D_{0i} + \varepsilon_{ik}E_k + d_{ijk}u_{jk},$$

$$d_{ijk} \approx \gamma_{imn}\lambda_{mnjk}. \tag{29}$$

Here γ_{ikl} are the piezo-moduli, which make up a third-rank tensor symmetrical in the second and third indices, and σ_{kl} is the stress tensor; summation over the repeated indices is implied, $\mathbf{D}_0$ is the constant spontaneous induction. Now the deformation produces, besides the field $\mathbf{E}_u$ considered above, also a piezoelectric field $\mathbf{E}_p$, defined by the equations

$$\frac{\partial}{\partial x_i}(\varepsilon_0 E_{pi} + d_{ijk}u_{jk}) = 0 \text{ or } \mathrm{E_p} = -\frac{\mathrm{k}}{\varepsilon_0 k^2} d_{ijk}k_i u_{jk}. \tag{30}$$

The field $\mathbf{E}_p$ should be added to the right sides of (11) and (15) or (16). In all other respects the previous calculations remain in force. The expressions for G_1 and G_2 are generalized in the following manner:

$$G_1 = \frac{1}{s_1\sqrt{\gamma}}\left[(g_1 + g_2)E_{0\parallel} - \frac{d_{\parallel,\parallel,\parallel}}{\varepsilon_0} + ibe^{-1}k\right],$$

$$G_2 = \frac{1}{s_2\sqrt{\gamma}}\left[\frac{g_2}{2}E_{0\perp} - \frac{d_{\parallel,\parallel,\perp}}{\varepsilon_0}\right]. \tag{31}$$

We now compare tentatively the amplification coefficients in the case of electron-photon interaction proportional to the external field, considered above, and in the case of the interactions considered in the earlier papers. If we estimate the order of magnitude of g_1 by means of the Lorentz-Lorenz formula in generalized form

$$(\varepsilon - 1) / (\varepsilon + 2) = \text{const}\cdot\gamma, \tag{32}$$

we obtain

$$g_1 = (\varepsilon_0 + 2)(\varepsilon_0 - 1) / 3\varepsilon_0 \approx \varepsilon_0 / 3. \tag{33}$$

There are no grounds for assuming that g_2 can exceed this order of magnitude. Formulas (22), (24), and (31), (33) show that to obtain maximum amplification with an interaction proportional to $\mathbf{E}_0$ it is necessary to use substances with maximum ϵ_0; in the case of piezoelectric interaction it is necessary to use substances with maximum $d_{ijk}/\sqrt{\epsilon_0}$.

The sound absorption coefficient is

$$\alpha_j = -K_j\frac{\tau_k}{\tau}\frac{(qv_\parallel/s - 1)\omega\tau_k}{1 + (qv_\parallel/s - 1)^2\omega^2\tau_k^2},$$

$$K_j \equiv k_j\frac{\varepsilon_0}{4\pi}|G_j|^2, \quad j = 1, 2. \tag{34}$$

Here the last factor (fraction) varies, depending on $v_\parallel$, from $-1/2$ to $1/2$, and τ_k/τ is assumed, for estimating purposes, to be optimal, i.e., ≈ 1. Thus, it remains to compare the values of the coefficient K.

If piezoelectric amplification in CdS is employed, the value of K corresponds to 220 dB/cm at 45 Mc.[3] In the case of amplification in an interaction proportional to the external field, using a substance with $\epsilon_0 = 2{,}000$, such as barium titanate or similar substances (solution of lead titanate in barium titanate, etc.), we obtain for $s_1 = 2 \times 10^5$ cm/sec, $\sqrt{\gamma} = 2$, $E_0 = 10^4$ V/cm, and the same sound frequency

$$K_1 \sim \omega\varepsilon_0^3 E_{0\parallel}^2 / 36\pi s^3\gamma = 675 \text{ cm}^{-1},$$

corresponding to 3,000 dB/cm. In both types of exciton-phonon interaction K is proportional to the frequency, i.e., the relation obtained for K remains the same for all frequencies.

Thus, the type of electron-phonon coupling considered in the present paper, in substances with anomalously large dielectric constant, can result in no less a sound amplification than piezoelectric coupling. As explained above, it is possible to use here poor semiconductors with low mobility, either amorphous or polycrystalline with microcrystal dimension much smaller than the wavelength of sound.

If $v_{\|}$ is such that in the expression for α_j in (34) the last factor is equal to $1/2$, then the amplification obtained in the preceding numerical example is 1500 dB/cm. It is possible that in the future it will be possible to approach the region of audio frequencies. Thus, at the same material parameters, but at a frequency 10^5 cps, a gain of 3.3 dB/cm is obtained. The wavelength is then equal to 2 cm, making it possible to construct resonators, to circulate sound through a closed waveguide, etc.

The solution (27) and (28) represents a wave whose phase velocity is approximately equal to $qv_{\|}$ at all frequencies. It can therefore be called a drift wave, to distinguish it from the previously considered sound wave. The third term in (27) makes a negative contribution to the imaginary part of k, i.e., amplification when $qv_{\|}$ is smaller than s_1 or s_2 respectively in the first and second term of the curly bracket. The imaginary part of each of these terms, as a function of $v_{\|}$, has a sharp resonant maximum at

$$qv_{\|} = s_j(1 - 1/\omega\tau_k) \quad (j = 1,\ 2; \quad \omega\tau_k \gg 1)$$

and a relatively small half-width

$$q\Delta v_{\|} = 2\sqrt{3}\, s/\omega\tau_k.$$

Thus, the resonance regions do not overlap in practice, and the amplification from each of the terms in the curly bracket of (27) can be regarded independently. The maximum of each term is $|G_j|^2/2$, and the absorption coefficient at the maximum is equal to

$$\alpha_j^{max} = 2/s_j\tau_k - K_j/2. \tag{35}$$

Here K_j is determined by the same formula (34). Consequently, the second term in (35) corresponds to the same amplification as obtained for the previously considered acoustical waves. However, the first term always corresponds to absorption. In order to obtain amplification, it is necessary that the first term be smaller than the second. This is indeed the case at appropriate values of the parameters. By way of an example let us consider the already discussed case with $K_1/2 = 338\ \text{cm}^{-1}$ at 45 Mc. We have assumed here that $E_{0\|} = 10^4$ V/cm and $qv_{\|} \approx s_1 = 2 \times 10^5$ cm/sec, from which it follows that at room temperature $qD = 0.5\ \text{cm}^2/\text{sec}$ and $1/\tau_k = 10^6\ \text{sec}^{-1}$. In this case $2/s_1\tau_k \sim 10\ \text{cm}^{-1}$, i.e., much smaller than $K_1/2$, and the amplification of the drift wave will be the same as that of the acoustic wave, i.e., 1500 dB/cm.

In (35) the first term is proportional to ω^2 and the second to ω, therefore on going over to lower frequencies the relative role of the first term is still smaller and amplification of the drift wave can be obtained even in a substance with a larger diffusion coefficient.

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