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CONFINEMENT OF SLOW IONS OF A PLASMA WITH POSITIVE POTENTIAL IN A MIRROR TRAP

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The effective mirror ratio for comparatively slow ions (with energies of the order of T_e) confined in a magnetic trap with a small mirror ratio ($R \approx 1.5-2.0$) is determined for the case when the plasma is charged to a considerable positive potential ($\varphi_0 \gg T_e$). It is assumed that the ion component of the plasma consists almost wholly or partially of ions with a high-energy which significantly exceeds the plasma potential. The potential distribution along the trap, the minimum energy of the confined ions, the magnitude of the effective mirror ratio for ions with an energy near the minimal value, and the limiting possible density of such ions (compared with that of fast ions) are considered. It is shown that owing to concentration of the largest potential drop in a narrow region near the mirror traps, the minimum energy of the confined ions does not depend on φ_0 and is of the order of T_e. In this case the effective mirror ratio for ions with an energy above the minimum value may not differ strongly from the initial ratio, whereas the critical density of the accumulated slow ions may exceed by several times the fast-ion density. It is concluded that the presence of a group of fast ions and the consequent small potential drop in the main volume of the trap may be a favorable factor in the ambipolar plasma losses through the magnetic mirror traps.

1. INTRODUCTION

In experimental investigations of the behavior of a plasma in a trap with mirrors it is frequently observed that the confined plasma acquires an appreciable positive potential. In different cases this potential may be due to different causes, for example to flute or cyclotron instability [1,2] or to some other processes not yet fully understood [3,4].

In the presence of a potential, the ions of the confined plasma, experience an additional acceleration along the magnetic field as they move towards the mirrors, and can escape more readily. If the ion acquires outside the central plane of the trap a longitudinal energy $e\varphi_0$ (φ_0—potential difference between the center of the trap and the mirrors), then it is easy to show, using the adiabatic invariance of the magnetic moment of the particle, that the actual mirror ratio R_{eff} decreases in this case in accordance with the formula

$$R_{\text{eff}} = \frac{R}{1 + e\varphi_0/E_0}, \qquad (1)$$

where R is the mirror ratio for zero plasma potential and E_0 the kinetic energy of the ion on passing through the central plane of the trap. From formula (1) it follows that in the case of small mirror ratios ions with energies $\lesssim e\varphi_0$ will not be retained in the trap. However, such a deduction is based on rather crude ideas, including the assumption that the potential drop occurs near the central plane. Actually the longitudinal distribution of the potential in the plasma can be different, and then the conditions for the confinement of the slow ions may turn out to be more favorable.

In the present paper we consider the special case of positively charged plasma, with ions whose energy spectrum includes a group of particles of energy much higher than the plasma potential. Taking into account the longitudinal distribution of the potential, an attempt is made to ascertain the minimum energy that can be possessed under the given conditions by the ions confined in the plasma, the effective mirror ratio for ions with energy near the minimum, and the possible density of such ions compared with the fast ions. These questions are of interest in many experimental problems, for example in connection with a determination of the character of the cold part of the ion spectrum, which cannot always be measured experimentally. In addition, the singularities of the confinement of the slow ions, brought about by an account of the longitudinal distribution of the potential, have a bearing on the problem of ambipolar loss of ions from the trap, something which will be discussed in detail in the conclusion.

The quantitative calculations were made for small mirror ratios (R = 1.5 and 2.0), since this case is closest to the experimental practice and is most unfavorable with respect to the influence of the plasma potential on the escape of the ions from the trap.

2. LONGITUDINAL DISTRIBUTION OF THE POTENTIAL IN THE TRAP

Before we proceed to consider the problems to be solved, we derive a relation which will be needed later and which yields the particle density in any point in the trap, provided the distribution function in the central cross section is given. It is assumed here that the collisions of the particles during the time between two reflections from the mirrors can be neglected.

We consider some isolated force tube, such that the distribution function of the particles, the plasma potential, and the magnetic field can be regarded as constant over its section. Assume that for the central section of the tube we know the distribution function f_0 of the particles of the species of interest to us in velocity space. We choose as the coordinates in this space the kinetic energy E_0 and the angle α_0 between the velocity vectors and the magnetic field (the subscript zero will denote henceforth all the quantities pertaining to the central section). We introduce as a longitudinal coordinate a quantity $\xi = H(z)/H_0$, where z is the distance measured from the center along the axis of the magnetic field. We also assume some longitudinal distribution of the potential $\varphi(\xi)$ inside the plasma, and take the value of the potential in the central section to be zero.

We separate a group of particles contained in intervals dE_0 and $d\alpha_0$ about certain values of E_0 and α_0. These particles form in the central section the density component $dn_0(E_0, \alpha_0)$. It is obvious that at the point ξ the particles in question correspond to a certain component of density $dn(E, \alpha)$, with $E = E_0 - e\varphi$, while the angles α and α_0 are related by

$$\sin^2\alpha = \xi \frac{E_0}{E_0 - e\varphi} \sin^2\alpha_0, \tag{2}$$

which follows from the conservation of the magnetic moment $E_\perp/H$ and of the total energy $E + e\varphi$.

To find the quantitative connection between dn and dn_0, we note that the same flux of the particles in question passes through the section of the tube at the point ξ and at the center, i.e.,

$$dnv_\parallel ds = dn_0 v_{\parallel 0} ds_0. \tag{3}$$

Here $v_\parallel$ denotes the longitudinal velocity component and ds the area of the section of the tube in question. Obviously,

$$dn_0 = f_0 \cdot 2\pi v_0^2 \sin\alpha_0 dv_0 d\alpha_0 \sim f_0 \sqrt{E_0} \sin\alpha_0 dE_0 d\alpha_0$$

(since the constant factors cancel out in the final result, we leave them out here and replace provisionally the proportionality sign by an equal sign). We note also that

$$ds_0 / ds = \xi, \qquad v_{\parallel 0} = E_0^{1/2} \cos\alpha_0,$$

$$v_\parallel = (E_0 - e\varphi)^{1/2} \cos\alpha = (E_0 - e\varphi - E_0 \xi \sin^2\alpha_0)^{1/2}.$$

Substituting these quantities in (3) and integrating with respect to E_0 and α_0, we obtain the sought-for relation:

$$n(\xi) = \xi \int\limits_{E_{0\,min}}^{E_{0\,max}} E_0^{1/2} dE_0 \int\limits_{\alpha_{0\,min}}^{\alpha_{0\,max}} f_0 \frac{\sin\alpha_0 \cos\alpha_0 d\alpha_0}{(1 - e\varphi/E_0 - \xi\sin^2\alpha_0)^{1/2}}. \tag{4}$$

The limits of integration are determined by the limits of the region of velocity space in which the distribution function f_0 differs from zero.

We now proceed to find the longitudinal distribution of the potential in the plasma, for which purpose we establish first the connection between the potential and the electron density at each point, making more specific for this purpose the formula (4), which contains such a connection in general form. Assume that in a plasma-filled trap the potential difference between the mirror and the central section is φ_R, with R specified. In accordance with (4), the longitudinal distribution of the electron density $n(\xi)$ is determined by two factors: the electron distribution function in the central plane $f_0(E_0, \alpha_0)$ and the longitudinal distribution of the potential $\varphi(\xi)$. We shall consider the inverse problem—finding the longitudinal distribution of the potential for a specified form of longitudinal density distribution and for a chosen function f_0. Although the arguments presented are rigorously valid only for individual force tubes, we assume that we are referring to the trap as a whole. Such a simplification is justified if the conditions are sufficiently uniform over the section of the trap.

We choose as the electron distribution function in the central plane of the trap a so-called "cut-out Maxwellian distribution," i.e., the usual Maxwellian distribution, from which we remove all the particles which cannot be confined in the trap. The limit of the cut-out region can be found from relation (2) by putting $\xi = R$, $\varphi = \varphi_R$, and $\alpha = \pi/2$. The equation for the boundary surface in velocity space is of the form

$$v_0 = \left(\frac{-2e\varphi_R/m}{1 - R\sin\alpha_0} \right)^{1/2}. \quad (5)$$

Thus, the cut-out region constitutes the internal volume of a two-cavity hyperboloid of revolution with vertices at the points

$$\{v_{\perp 0} = 0, \quad v_{\parallel 0} = \pm(-2e\varphi_R/m)^{1/2}\}$$

and an asymptotic aperture angle equal to the angle of the loss cone in the absence of a potential (see Fig. 5 below, surface 1). In all the remaining velocity space, the electron distribution is isotropic, and the energy part of the distribution function is of the form $\exp(-E_0/T_e)$. A problem formulated in similar fashion was considered earlier by Post[5] and by Ben-Daniel[6]. Since they obtained essentially different expressions, it is useful to pay some attention again to the solution of this problem.

Let us substitute in (4) the chosen distribution function and integrate over the entire volume outside the loss hyperboloid. We have here

$$n(\xi) = \int_0^\infty e^{-E_0/T_e} E_0^{1/2} dE_0 \int_{\alpha_{0\,min}}^{\alpha_{0\,max}} \frac{\sin\alpha_0 \cos\alpha_0}{(1 + e\varphi/E_0 - \xi\sin^2\alpha_0)^{1/2}} d\alpha_0, \quad (6)$$

where

$$\alpha_{0\,min} = \begin{cases} 0, & E_0 < -e\varphi_R \\ \arcsin[R^{-1}(1 + e\varphi_R/E_0)]^{1/2}, & E_0 > -e\varphi_R \end{cases},$$

$$\alpha_{0\,max} = \arcsin[\xi^{-1}(1 + e\varphi/E_0)]^{1/2}.$$

Carrying out the integration, we obtain ultimately

$$n(\xi) = n_0 e^{-\Phi} F(\Phi, \xi) / F_0. \quad (7)$$

Here $\Phi = -e\varphi/T_e$, and the function F is equal to

$$J(\sqrt{2\Delta}) + \left(1 - \frac{\xi}{R}\right)^{1/2} \exp\left(\frac{\Delta}{R/\xi - 1}\right)\left[1 - J\left(\left(\frac{2\Delta}{1 - \xi/R}\right)^{1/2}\right)\right],$$

where

$$J(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2/2} dt$$

is the probability integral; $\Delta = \Phi_R - \Phi$. The quantity F_0 corresponds to the value of F in the central section, i.e., when $\Phi = 0$ and $\xi = 1$ [1)].

We have obtained a relation between the plasma potential and the electron concentration. We assume, however, that the plasma is sufficiently dense and that the quasineutrality condition is well satisfied. Then the electron and ion densities are nearly equal, and we can take n to mean simply the plasma density.

[1)] The expression obtained coincides with the result of Ben-Daniel[6].

It is convenient to rewrite (7) in the form

$$\Phi(\xi) = \ln\frac{n_0}{n}(1 - \delta), \qquad \delta = \left(1 - \Phi/\ln\frac{F}{F_0}\right)^{-1}. \quad (8)$$

Numerical estimates show that δ is small compared with unity in the entire region where the plasma potential exceeds the potential in the mirror by more than an amount approximately equal to one electron temperature, i.e., where $n \gtrsim n_0 \exp[-(\varphi_R - 1)]$. This means that for a plasma with not too steep a longitudinal distribution density the correction δ can be neglected in the greater part of the trap, with the exception of the regions near the mirrors. By way of illustration Fig. 1 shows the region of the plasma potential φ_R and of the anisotropy exponent of the ionic component $\gamma = T_\perp/T_\parallel$ (see below), in which δ is negligibly small compared with unity, provided the coordinate ξ does not exceed 1.45 for $R = 1.5$ and 1.90 for $R = 2.0$.

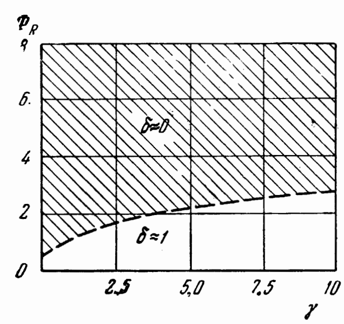

FIG. 1. Region of applicability of Boltzmann's law for longitudinal potential distribution (Φ_R, in T_e units) in the interior of the trap ($\xi < 1.45$ for $R = 1.5$ and $\xi < 1.9$ for $R = 2.0$).

Thus, the potential distribution is well approximated by the formula

$$\Phi(\xi) = \ln(n_0/n), \quad (9)$$

which represents the well known Boltzmann law. The real distribution of the potential is somewhat less steep, but the difference becomes appreciable only near the ends of the trap. With increasing potential Φ_R, the region of applicability of (9) becomes larger. When $\Phi_R \gtrsim 5$, Boltzmann's law can be regarded as unconditionally applicable in the entire interior volume of the trap, with the exception of very narrow sections near the mirrors. We shall therefore use formula (9) in the calculation of the potential without further stipulation.

The calculation is facilitated by specifying some concrete plasma model in which we can calculate the longitudinal density distribution and consequently the potential. We choose as such a model the initial plasma potential, i.e., we neglect the influence of the electric field on the ion motion. At the same time, we assume that the plasma potential exceeds T_e by many times, or at least several times, so that formula (9) can be used.

The ion distribution function in the central section is chosen in the form of the so-called "two-temperature Maxwellian distribution":

$$f_0 = \exp\{-{}^1\!/_2 M(v_{\perp 0}^2 / T_\perp + v_{\| 0}^2 / T_\|)\}. \quad (10)$$

Going over to the chosen coordinates E and α, we can represent this distribution function in the form

$$f_0(E_0, \alpha_0) = \exp\{-E_0 T_\perp^{-1}[1 + (\gamma - 1)\cos^2\alpha_0]\}. \quad (11)$$

We then use again formula (4), putting $\varphi \approx 0$. Integrating over the entire velocity space outside the barrier cones, we obtain

$$n(\xi) = n_0\left(\frac{R-\xi}{R-1}\right)^{1/2}\frac{\xi}{\gamma(\xi-1)+1}. \quad (12)$$

Substituting the obtained expression in Boltzmann's formula (9) we obtain the approximate distribution of the potential in the space between the mirrors for the chosen model.

3. CONFINEMENT OF SLOW IONS

We now consider the case when a certain group of ions (together with an equal number of electrons), having an energy comparable with T_e, is added to the positively charged plasma (with fast ions) described above. We shall call such ions "slow." Let the quantity $\mathscr{E}$ denote their kinetic energy, expressed in units of the electron temperature. We assume that the density of the slow ions is small compared with the density of the fast ones, i.e., the potential distribution is given only by the fast ions.

On moving from the center to some point ξ, the longitudinal energy of the ion decreases because of the action of the increasing magnetic field, by an amount $\mathscr{E}_{\perp 0}(\xi - 1)$. At the same time, the ion acquires in the electric field an energy equal to Φ. The total change in the longitudinal energy can be described by the function $\Phi_\|(\xi) = \mathscr{E}_{\perp 0}(\xi - 1)$, which we shall arbitrarily call the "longitudinal potential." Figure 2 shows the variation of the longitudinal potential in the model in question at different transverse energies of the slow ions. We see that, for ions with $\mathscr{E}_{\perp 0}$ above a certain value, a potential well is produced in the center of the trap in which the ions can be confined. The larger the transverse energy of the ion, the higher the well and the closer its sides are to the mirrors.

Let us find the minimum total energy, starting

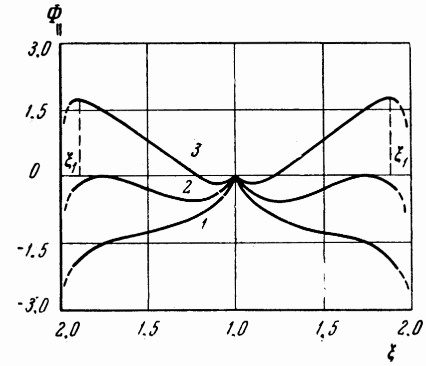

FIG. 2. Variation of the longitudinal potential $\Phi_\| = \mathscr{E}_{\perp 0}(\xi - 1) - \Phi$ (in T_e units) for ions with different transverse energies: 1 – $\mathscr{E}_{\perp 0} = T_e$, 2 – $\mathscr{E}_{\perp 0} = 3T_e$, 3 – $\mathscr{E}_{\perp 0} = 5T_e$; R = 2.0, $\gamma = 10$ ($\xi = H(z)/H_0$).

with which the slow ions can be confined in the center of a positively charged plasma, and also the value of the effective mirror ratio R_{eff} for ions whose energy exceeds the minimum value. By R_{eff} we mean, as usual, $\sin^{-2}\alpha_{0\,min}$, where $\alpha_{0\,min}$ is the minimum angle (between the velocity and the magnetic field in the central plane) at which the ion still does not leave the trap.

Obviously, the boundary of the confinement region—the "effective mirror"—is determined by the conditions

$$d\Phi_\| / d\xi = 0, \quad (13)$$

$$\Phi_\| > 0. \quad (14)$$

The second condition is necessary in order to exclude from consideration a possible minimum of the potential curve, and also the cases when the ion energy is lower than the minimum value. In addition, it is necessary to add to relations (13) and (14) the condition for the turning of the ion at the point ξ, in the form

$$\mathscr{E}_0 + \Phi = \xi\mathscr{E}_{\perp 0}. \quad (15)$$

This condition makes it possible to express the transverse energy $\mathscr{E}_{\perp 0}$, contained in the quantity $\Phi_\|$, in terms of the total energy $\mathscr{E}_0$. Making such a substitution, we solve (13) and pick out only those solutions which satisfy (14). As a result we obtain the location ξ_1 of the effective mirror (see Fig. 2) for a specified total ion energy $\mathscr{E}_0$. Of all the ions with energy $\mathscr{E}_0$, only those will be confined for which, in accordance with (15),

$$\sin\alpha_0 \geqslant [\xi_1^{-1}(1 + \Phi(\xi_1) / \mathscr{E}_0)]^{1/2}.$$

Thus, the effective mirror ratio is

$$R_{eff} = \frac{\xi_1}{1 + \Phi(\xi_1)/\mathscr{E}_0}. \quad (16)$$

The minimum energy will be that for which R_{eff}

= 1[2)]. At this energy relation (14) turns into an equality.

Figure 3 shows the relations obtained in this manner between R_{eff} and the ion energy for mirror-ratio values 1.5 and 2.0. The calculation was made for a plasma with different degrees of fast-ion anisotropy.

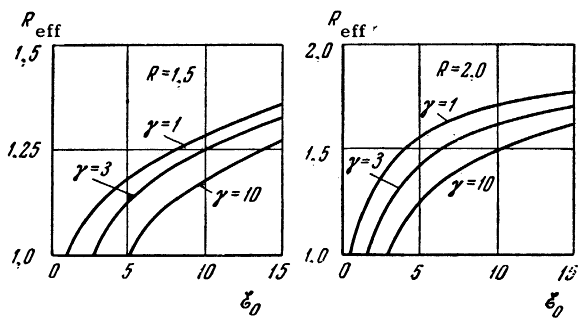

FIG. 3. Effective mirror ratio as a function of the ion energy for two values of R ($\mathscr{E}$ is in T_e units).

An important feature of the results is that R_{eff} (as well as $\mathscr{E}_{0\,min}$) is independent of the total plasma potential Φ_R. It is obvious that in this case this is due to our assumption that the ionic component is made up of fast particles, and therefore the presence of the potential does not influence the longitudinal density distribution. It will be shown later that the independence of the slow-ion confinement conditions of the total potential has a more general character and can occur also when the fast particles constitute only a relatively small fraction of the ionic component.

From the quantitative point of view it must be noted that the minimum energy is of the order of several times T_e, and that R_{eff} has at $\mathscr{E}_0 \approx 10$ a value not much smaller than the mirror ratio which would exist at zero plasma potential.

4. MAXIMUM DENSITY OF SLOW IONS

It is obvious that with increasing number of confined slow ions, the depth of the potential well will decrease, since the density of the slow ions is added to the density of the fast ions in the center, and the ratio of the total densities $n_0/n(\xi_1)$ increases, so that consequently the potential difference between the center and the point ξ_1 also increases. At some sufficiently high slow-ion density the longitudinal potential $\Phi_{\|}$ at the point ξ_1 reaches zero, and any further increase in the number of slow ions at the center becomes impossible.

Let us determine the maximum attainable density of the slow ions. We note that only the slow ions whose longitudinal energy is equal to zero can be confined in a well filled to the limit. Therefore $\mathscr{E}_{\perp 0} = \mathscr{E}_0$. We note also that since the point ξ_1 is the limit of motion of the slow ions, the total density at this point and everywhere beyond it, closer to the mirrors, is determined by the initial density of the fast ions, the longitudinal distribution of which does not depend on the presence of slow ions. Thus, when $\xi \geq \xi_1$ we have $n(\xi) = n_f(\xi)$ (the subscripts f and s pertain to the fast and slow ions).

For the limiting points we have, as before, the condition $\Phi_{\|}' = 0$. Taking into account the two remarks made above, this condition takes the form

$$\mathscr{E}_0 + n_f'(\xi)\,/\,n_f(\xi) = 0. \tag{17}$$

Solving this equation, we obtain the coordinate ξ_1 of the limiting point of motion of the slow ions (at low values of $\mathscr{E}_0$ Eq. (17) can have two roots, of which the larger one should be chosen).

To determine the density ratio of the fast and slow ions at the center of the trap, we use the condition for filling the well to the limit, $\Phi_{\|}(\xi_1) = 0$. We then obtain

$$\mathscr{E}_0(\xi_1 - 1) = \ln\frac{n_s(0) + n_f(0)}{n_f(\xi_1)} \tag{18}$$

hence

$$\frac{n_s(0)}{n_f(0)} = \frac{n_f(\xi_1)}{n_f(0)}\exp[\mathscr{E}_0(\xi_1 - 1)] - 1. \tag{19}$$

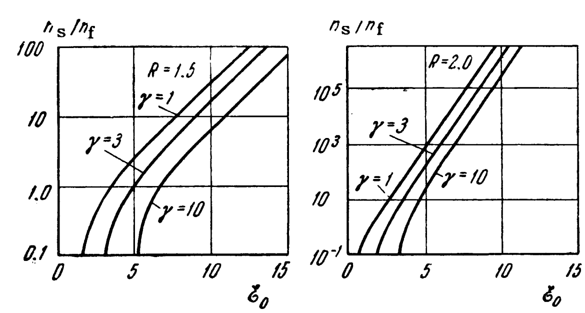

FIG. 4. Maximum possible relative concentration of the slow ions in the central section of the trap as a function of their energy for two values of R.

Figure 4 shows plots of the ratio $n_s(0)/n_f(0)$ against the ion energy $\mathscr{E}_0$ for the case when the distribution of the fast ions is described by a two-temperature Maxwellian distribution. We see that the limiting slow-ion density increases very rapidly

[2)]If the potential curve $\Phi_{\|}(\xi)$ has minima off the central section (similar to curves 2 and 3 on Fig. 2), then ions with lower energy than determined above can be confined, in the region of these minima. However, these ions do not enter the regions of the central section of the trap.

with increasing energy, and when $\mathscr{E}_0 \approx 10$ it can exceed by several orders of magnitude the density of the fast ions. An increase in the mirror ratio increases very strongly the limiting density, while an increase in the anisotropy reduces the limiting density somewhat. The given values of the maximum density correspond, obviously, to the case $R_{eff}(E_0) = 1$. At densities below maximum, R_{eff} increases, tending, if the number of slow ions is small, to the values represented by the plots in Fig. 3

5. CONCLUSION

The results obtained are important in two respects.

First, they are of interest for experimental applications, since they answer the question raised at the beginning of the article, whether it is possible to confine relatively slow ions ($E_0 \lesssim e\varphi_0$) in a plasma with positive potential. It has been established that such a possibility exists when the electron temperature is lower than the plasma potential and when the larger part of the ions, or at least some group of ions, has an energy much higher than the indicated potential. The conditions for confinement of the slow ions in the central region of the trap do not depend in this case on how large the plasma potential is, and are determined primarily by the value of the electron temperature, and also by the character of the longitudinal distribution of the fast ionic component and the number of slow ions. The minimum energy of the confined ions constitutes several electron temperatures, and the limiting density can exceed by many times the density of the fast component. If the density is far from the limit then the effective mirror ratio for slow ions does not differ strongly from the initial value.

A second more general point of interest attached to the obtained results is the problem of ambipolar loss. The plasma contained in a trap, even in the absence of any instabilities, should be positively charged, since the Coulomb escape of the electrons to the loss cone is more rapid than the escape of the ions (provided only $T_e/T_i < (M/m)^{1/3}$). The value of the positive potential can amount to $\gtrsim 5\,T_e$. On the other hand, if the confinement time is appreciable, the electron temperature is raised as a result of heating by the ions, and reaches a value comparable with the ion temperature. Therefore the potential to which the plasma is charged also turns out to be of the order of the ion temperature. This accelerates the ion loss due to the decrease in the effective mirror ratio. If the degree of the decrease is close to that given by formula (1), then for traps with small mirror ratio ($R \lesssim 2$) this possible effect would lead to an increase in the rate of ion loss by a factor of many times. However, the question of the actual change in the mirror ratio under real conditions has not been exhaustively investigated, in view of its great complexity. Ben-Daniel, in the paper cited above, considered the case when all the ions have the same energy, comparable with the plasma potential, and showed that the effective mirror ratio is determined by expression (1). This is connected with the fact that the longitudinal drop in potential under these conditions is quite steep in the entire volume of the trap including the central region (in practice the potential drop is linear in the coordinate ξ). In this case the longitudinal ion density distribution, which is established under the influence of the electric field, also is exceedingly steep, and only a negligible fraction of the ion moves in the regions away from the central section.

Our analysis shows that the position can change noticeably if there is some group of very fast ions, whose velocities are sufficiently strongly inclined to the magnetic field. Such a group may be, for example, the hot tail of a partially maxwellianized distribution. In this case the longitudinal potential distribution in the central part becomes gently sloping, and the main drop is concentrated near the mirrors. Physically this is connected with the fact that the fast ions do not change their distribution under the influence of the electric field and ensure, away from the central section, such a density that, according to Boltzmann's formula, the potential drop relative to the center will amount to only several electron temperatures. Therefore in the center there may be retained the slower ions, forming the main part of the ionic component, and the effective mirror ratio for them will be more favorable than (1). In the direct vicinity of the mirrors, in the region of steep descent of the potential, there will be no slow ions; the fast ions absorb, as it were, the external field and screen the central part of the plasma.

It can be seen from the foregoing calculations that if the potential inside the plasma is sufficiently gradual, then the actual effective mirror ratio may not differ strongly from the initial value, even for relatively slow ions. The resultant reduction in the ambipolar loss can be intuitively represented in the following form. The problem of the escape of the ions through the magnetic mirror reduces to the problem of diffusion escape of the particles in velocity space inside the loss region. In the absence of a potential difference between the center of the trap and the mirror, the loss region is bounded by a conic surface. In the case of a decelerating potential, this boundary represents a two-cavity hyperboloid, as we have seen with electrons as an example. If a drawing potential is produced, the loss region is made up of the internal volumes of a single-cavity hyperboloid, conjugate to that considered in Sec. 2 (see Fig. 5).

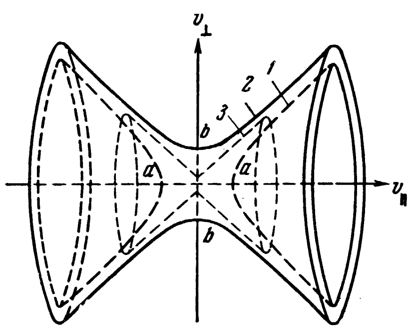

FIG. 5. Limits of forbidden region of velocity space for particles in the central section of the trap for a retarding (1) and drawing (2) plasma potentials. The surface 3 shows the deformation of the surface 2 when the potential distribution is gradual ($R = 2$, $n_f \gg n_s$, $\gamma = 1$, $T_e = 0.1\ \varphi_0$). Here $a = (2e\varphi_0/M)^{1/2}$ and $b = [(2e\varphi_0/M(R-1)]^{1/2}$.

It is obvious that in this case the diffusion of the particles inside the loss volume will be more intense, especially if the average distance between the particles and the origin of coordinates is comparable with the real semiaxis of the hyperboloid, i.e., if φ_0 is comparable with $E_{av}(R-1)/e$. This is the effect of a drawing potential in velocity space. However, a boundary surface in the form of a regular single-cavity hyperboloid occurs only in the case of sufficiently sharp drop of potential inside the trap, when relation (1) is satisfied for ions of all energies. A gradual distribution of the potential deforms the hyperboloid, making it closer to a conic surface, which corresponds not only to zero potential, but also to the limiting case of a drawing potential of arbitrary magnitude, provided the entire potential drop is concentrated on the trap boundary. An intermediate longitudinal potential distribution leads to an intermediate form of the boundary surface. Figure 5 shows such a surface for the case considered in the calculations, when the ionic component consists of fast particles with a cut-out Maxwellian distribution function. The ion diffusion into such a near-conical hyperboloid is smaller than into the undeformed hyperboloid, i.e., the ambipolar loss is smaller.

Fowler and Rankin[7] calculated the ambipolar loss under some particular conditions, for a trap with small R. Unlike Ben-Daniel[6], an extensive energy spectrum was used in the calculations. However, the change in mirror ratio due to the presence of the plasma potential was chosen for each energy group of ions in accordance with formula (1), i.e., the longitudinal distribution of the potential was in fact disregarded. We can therefore expect the actual increase in loss to be smaller than that obtained by Fowler and Rankin.

In view of the incomplete and approximate nature of our calculations, we cannot indicate the degree to which the presence of fast ions can influence quantitatively the value of the ambipolar loss. Qualitatively, however, the existence of such an effect is apparently quite reliably established.

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