DAMPING OF LONGITUDINAL WAVES IN AN INHOMOGENEOUS PLASMA

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Account is taken of the fact that in a weakly inhomogeneous medium, for which the geometrical optics approximation is valid, the number of resonant particles interacting with a longitudinal wave may vary considerably with the distance. As a result, in some cases the wave absorption coefficient may oscillate in space in a certain region of the plasma.

THE damping of a longitudinal wave in a weakly-inhomogeneous plasma was already considered in a number of papers (see, for example,[1-4]). The external field causing the presence of plasma inhomogeneity was calculated in [5,6], where it was found that corrections to the real part of the wave vector $k_0^2(z)$, connected with the account of the external field F, can be neglected in the geometrical-optics approximation, while the correction to the imaginary part of $k_0(z)$ is of the order of κ_0^2 (κ_0 is the geometrical-optics parameter, $\kappa_0 = k_0^{-2} dk_0/dz$). Some decrease in the Landau damping is connected in this case with the fact that the accelerated electrons (moving in the external field F) produce Cerenkov radiation of somewhat lower intensity than the electrons moving with constant velocity.

The cited articles employed the geometrical-optics method, which is valid when the properties of the medium change little over a distance on the order of the wavelength. No attention was paid so far in the calculation of the absorption coefficient of the wave, however, to the fact that the distribution function of the resonant particles $f_0(u(z))$ can change appreciably over distances of the order of the wavelength (u = particle velocity). We show in this article that an account of this circumstance can lead to a radical change in the character of the damping of the longitudinal wave in a weakly inhomogeneous plasma. In particular, it may turn out that in some region of the plasma there will propagate not a damped wave but a wave that grows in space.

To solve this problem we shall follow an analysis close to that used earlier[1]. We consider the one-dimensional case, in which the kinetic equation for the nonequilibrium part of the distribution function ($f = f_0 + \varphi$; $\varphi \ll f_0$) has, neglecting the external field[1)], the form

$$-i\omega\varphi + u\frac{d\varphi}{dz} + \frac{Ee}{m}\frac{\partial f_0}{\partial u} = 0,$$

$$f_0 = N(z)\exp(-u^2/2v_t^2)/\sqrt{2\pi}v_t^2, \tag{1}$$

where u is the velocity of the plasma electrons parallel to the electric field of the normal wave E, $v_t^2 = \kappa T/m$ the average thermal velocity of the plasma electrons, and $N(z)$ the plasma electron concentration. The solution of (1) can be obtained in the elementary fashion. (it is assumed that $-i\omega = -i\omega + \nu$, $\nu \to 0$):

$$\varphi(z) = \delta_0 e^{i\omega z/u}\exp(-u^2/2v_T^2)\int_{-\infty\cdot\mathrm{sign}\,u}^{z} E(z)\,e^{-i\omega z/u}N(z)\,dz;$$

$$\delta_0 = e/\sqrt{2\pi}v_t^3 m. \tag{2}$$

Expanding E and N in Fourier integrals

$$E(z) = \int_{-\infty}^{+\infty} e^{ikz}E_k dk, \quad N(z) = \int_{-\infty}^{+\infty} N_k e^{ikz}\,dk$$

and recognizing that the particle concentration changes little, i.e., $k' \ll k$ in the entire important interval, an assumption which must be made in order to employ later on the approximation of weak spatial dispersion, where $|1 - \omega_0^2/\omega^2| \ll 1$, $\omega_0^2 = 4\pi e^2 N/m$, as well as the geometrical-optics method (see also[5]), we obtain

$$\varphi = \delta_0 N(z)\exp(-u^2/2v_T^2)\int_{-\infty}^{+\infty}\frac{E_k e^{ikz}}{i(k-\omega/u)}dk. \tag{3}$$

[1)]As was already noted, inclusion of the corresponding term in the kinetic equation yields a correction[5] for $\mathrm{Im}\,k_0 \approx \varkappa_0^2$. We neglect corrections of this order in what follows.

Using Maxwell's equation

$$-i\omega E+4\pi j=0,\quad j=e\int_{-\infty}^{+\infty}u\varphi\,du,$$

we obtain in the usual manner

$$E=\frac{\omega_0^2(z)}{v_t^2}\int_{-\infty}^{\infty}\left\{-i\sqrt{\pi}\frac{\omega}{\sqrt{2}\,kv_t}W\left(\frac{\omega}{\sqrt{2}\,kv_t}\right)-1\right\}\frac{E_k}{k^2}e^{ikz}dk,\tag{4}$$

where, for example, when $k > 0$ and $\omega > 0$,

$$W(x)=e^{-x^2}+\frac{2i}{\sqrt{\pi}}e^{-x^2}\int_0^x e^{t^2}dt.$$

In the case of weak spatial dispersion, when $\omega/k_m v_t \gg 1$ and $k_m = 2\pi/\lambda_m$ (λ_m is the shortest wavelength of the radiation), Eq. (4) transforms to

$$\frac{d^2E}{dz^2}+k_0^2(z)E=-\frac{\sqrt{\pi}\omega^5 i}{3\sqrt{2}\,v_t^5}\left\{\int_0^{\infty}\frac{E_k}{k^3}\exp\left(-\frac{\omega^2}{2k^2v_t^2}\right)e^{ikz}dk-\int_{-\infty}^{0}\frac{E_k}{k^3}\exp\left(-\frac{\omega^2}{2k^2v_t^2}\right)e^{ikz}dk\right\},$$

$$k_0^2(z)=\frac{\omega^4}{3v_t^2\omega_0^2}\left(1-\frac{\omega_0^2}{\omega^2}\right).\tag{5}$$

It is easy to see that Eq. (5) is similar to the equation for the field of the longitudinal wave of a homogeneous plasma placed in an inhomogeneous dielectric [see formula (1) of [1]].

For the balance of the analysis we must specify the form of the function $k_0^2(z)$. In this paper we shall deal with the case when the properties of the medium vary linearly, i.e.,

$$k_0^2(z)=\alpha_1-\alpha z,\tag{6}$$

where $\alpha > 0$ and $\alpha_1 > 0$ are constants. This case is important also because the analysis presented below, in the geometrical-optics approximation, can be used with some refinement for an arbitrary dependence of k_0^2 on z, provided the function $k_0^2(z)$ has one simple zero. We note also that, as already mentioned, Eq. (5) is valid only for those Fourier components of the field E_k for which $k \ll \omega/v_t$ (the higher Fourier components of the field E attenuate rapidly). Therefore, the integration with respect to dk in (5) must be limited to $k = k_m \ll \omega/v_t$ where necessary. As $\omega/v_t \to \infty$, k_m can also be arbitrarily large.

To explain the singularities that occur here, we solve the following idealized problem: Let an extraneous current $j = j_0\delta(z)e^{-i\omega t}$ be specified in the plane $z = 0$ and let it excite a longitudinal field described by Eq. (5), in the right side of which we must introduce a term corresponding to the specified extraneous current

$$I=-(4\pi i\omega^3/3v_t^2\omega_0^2)j_0\delta(z).$$

In view of the fact that the integral terms on the right side of (5) are small, we make use of the perturbation method, i.e., we seek a solution of (5) with the indicated extraneous current in the form $E = E_0 + E_1$, where E_1 is of the order of the right side of Eq. (5). In the zeroth approximation the field E satisfies the equation

$$\frac{d^2E_0}{dz_1^2}+\frac{k_0^2}{\alpha^2}E_0=B\delta(z),\quad k_0^2=z_1=\alpha_1-\alpha z,$$

$$B=-\frac{4\pi i\omega^3 j_0}{3v_t^2\omega_0^2\alpha^2},\tag{7}$$

and we should have $E_0 \to 0$ as $z_1 \to -\infty$, whereas only an outgoing wave should exist when $z_1 \to +\infty$ (more accurately, as soon as $z_1/\alpha^{2/3} \gg 1$).

Under the foregoing assumptions, we can write down a solution of (7) in elementary fashion, with the aid of Airy functions [7] and in the geometrical optics approximation, i.e., when $\sigma = z_1/\alpha^{2/3} \gg 1$

$$E_0=-B'\sigma^{-1/4}e^{i\gamma}\quad\text{when}\quad z_1>0;$$

$$E_0\approx 0\quad\text{when}\quad z_1<0,$$

$$B'=B\alpha^{5/3}v(0),\qquad \gamma={}^2/_3\sigma^{3/2}+\pi/4.\tag{8}$$

Here $v(0) = 0.63$ is the value of the Airy function $v(t)$ at $t = 0$ [7].

Using (8) we can readily obtain, for example by the saddle point method with $\alpha \to 0$, also an expression for the Fourier component of the field E_{0k}

$$E_{0k}=-\frac{B'i}{\sqrt{\pi}\,\alpha^{1/3}}e^{ik^3/3\alpha},\quad k<0.\tag{9}$$

We then have for the determination of E_1 the equation

$$\frac{d^2E_1}{dz_1^2}+\frac{k_0^2E_1}{\alpha^2}=-2\mu B'J^*,$$

$$J=\frac{1}{2\sqrt{\pi}\alpha^{1/3}}\int_0^{\infty}\frac{1}{k^3}\exp\left[-\omega^2/2v_t^2k^2+\frac{i}{\alpha}\left(-z_1k+\frac{k^3}{3}\right)\right]dk;$$

$$\mu=2\sqrt{\pi}\omega^5/3\sqrt{2}v_t^5\alpha^2,\tag{10}$$

where $E_1 \to 0$ when $z_1 \ll -1$ and $E = E_0 + E_1 \to A_0(e^{i\gamma} - pe^{i\gamma}) \approx A_0\exp(i\gamma - p)$ when $z_1 \gg 1$; $A_0 = -B'/\sigma^{1/4}$, and $p(z_1)$ is some real function of $p \ll 1$.

As already noted, in the case of a linear dependence of the wave vector on the coordinate (6), geometrical optics is valid provided

$$\varkappa_0^{-1} \approx \varkappa = z_1^{3/2} / \alpha \gg 1. \quad (11)$$

Using this circumstance, we can calculate the integral (10) by one of the asymptotic methods. We must exercise certain caution here, since the argument of the exponent in the integrand of (10) contains a second large parameter $\eta = \omega^2/v_t^2 v_1$, corresponding to the approximation of small spatial dispersion.

We write the integral of (10) in the form ($k = \sqrt{z_1}\ \xi$)

$$J = \frac{1}{2\sqrt{\pi}\,\alpha^{1/3} z_1} \int_0^\infty \exp\left[\frac{z_1^{3/2}}{\alpha} f(\xi)\right]\frac{d\xi}{\xi^3},$$

$$f(\xi) = -\varepsilon / 2\xi^2 + i(-\xi + \xi^3/3), \quad (12)$$

where $\epsilon = (\omega^2/v_t^2 z_1)\alpha/z_1^{3/2}$ is a dimensionless parameter equal to the ratio of the two large parameters indicated above, $\epsilon = \eta/k$. Thus, J depends essentially on the value of ϵ. We shall calculate the integral J by the saddle point method. The presence of an essential singularity of the integrand of (12) at $\xi = 0$ is of no significance, for as $\zeta \to 0$ we have

$$\int_0^\zeta \exp\left\{\frac{z_1^{3/2}}{\alpha} f(\xi)\right\}\frac{d\xi}{\xi^3} \approx \int_0^\zeta \frac{e^{-\eta/2\xi^2}}{\xi^3}\, d\xi = -\frac{e^{-\eta/2\zeta^2}}{\eta} \to 0.$$

Therefore the integration in (12) can be carried out not from zero, but from some quantity[2)] $\zeta \ll \alpha/Z_1^{3/2}$. As already indicated, the quantity $k = \xi\sqrt{z_1}$ is bounded by the condition $\eta \gg 1$, i.e., $k \ll \omega/v_t$. On the other hand, the presence of infinity in the upper limit of (12) is insignificant, for at large values of k the integrand function oscillates rapidly.

To determine the saddle points, we use the equation

$$df / d\xi = \varepsilon + i\xi^3(-1 + \xi^2) = 0. \quad (13)$$

This is a fifth-degree equation, which cannot be solved for an arbitrary value of the parameter ϵ. However, we can easily find the roots of (13) if $\epsilon \gg 1$ or $\epsilon \ll 1$.

Let us consider in greater detail the case $\epsilon \ll 1$. We see readily that the expression for the root of Eq. (13) with $\operatorname{Re}\xi > 0$ can be sought in the form

$$\xi = 1 + \sum_{n=1}^\infty a_n \varepsilon^n, \qquad \varepsilon \ll 1. \quad (14)$$

Substituting (14) in (13), we obtain recurrence relations for all the coefficients a_n. If we confine ourselves to terms of order ϵ^2, then

$$\xi = \xi_n = 1 + {}^1/_2 i\varepsilon + {}^7/_8\varepsilon^2, \quad (15)$$

$$J = \frac{\alpha^{1/6}}{2z_1^{7/4}} \exp\left\{-\frac{\omega^2}{2z_1 v_t^2} + i\left[-\frac{2}{3}\frac{z_1^{3/2}}{\alpha} + \frac{\pi}{4}\right] + i\delta\right\}. \quad (16)$$

When $\epsilon\delta \ll 1$, the value of δ is equal to $\delta_1 = \omega^4\alpha/4v_t^4 z^{7/2}$. If the condition $\epsilon\delta_1 \ll 1$ is not satisfied, then we have for δ a somewhat more complicated expression, in which the largest term, as before, is equal to δ_1. If $\delta_1 \ll 1$, then

$$J = \frac{\alpha^{1/6}}{2z^{7/4}} \exp\left\{i\left[-\frac{2}{3}\frac{z_1^{3/2}}{\alpha} + \frac{\pi}{4}\right]\right\}\left(1 + i\delta_1 - \frac{\delta_1^2}{2}\right) e^{-\eta/2},$$

$$\varepsilon \ll 1. \quad (17)$$

If we disregard small terms of order δ, then formula (17) corresponds to the results obtained by Silin and Rukhadze[3,4]. On the other hand, if the condition $\delta_1 \ll 1$ is not satisfied, then a factor $e^{i\delta}$ appears in (16), and its inclusion can lead to a radical change in the character of the damping of the wave in the plasma (see below). Let us discuss now the second limiting case, when the number of resonating particles $N \approx e^{-\eta/2}$ changes appreciably over a distance on the order of the wavelength $\lambda = 2\pi/\sqrt{z_1}$, i.e., $\epsilon \gg 1$. In this case it is convenient to introduce in (12) a new variable $\xi = \xi_1 i$. Then the roots of (13) can be sought in the form

$$\xi_1 = \varepsilon^{1/5} \sum_{n=0}^\infty a_n \varepsilon^{-2n/5}, \qquad \varepsilon \gg 1.$$

Using the obtained recurrence relations for the coefficients a_n, we have

$$\xi_1 = \varepsilon^{1/5}\left[1 + \sum_{n=0}^\infty \frac{(-1)^n 3^n}{5^{n+1}} \varepsilon^{-2(n+1)/5}\right], \quad (18)$$

where, of course, $\epsilon^{1/5}$ has five values. It is easy to show that the saddle point ζ_{1n} corresponds to the value $\arg \epsilon^{1/5} = 2\pi/5$. Since the integral (12) is a rapidly oscillation function in this case, we can confine ourselves only to its most rapidly oscillating part (see below). We therefore have from (18)

$$\xi_{1n} \approx \varepsilon^{1/5} e^{2\pi i/5} - {}^1/_5 \varepsilon^{-1/5} e^{-2\pi i/5}, \qquad \varepsilon \gg 1, \quad (19)$$

and we put here and below $\epsilon^{1/5} \equiv |\epsilon^{1/5}|$, and

$$J \approx -\frac{\alpha^{1/6}}{\sqrt{10}\,\varepsilon^{7/10} z_1^{7/4}} \exp\left\{\frac{z_1^{3/2}}{\alpha}\left(\frac{5}{6}\varepsilon^{3/5} e^{-4\pi i/5} + \varepsilon^{1/5} e^{2\pi i/5}\right)\right\}$$

$$= -\frac{\alpha^{1/6}}{\sqrt{10}\,\varepsilon^{7/10} z_1^{7/4}} \exp\left\{\frac{5}{6} e^{-4\pi i/5}\left(\frac{\omega^6}{v_t^6\alpha^2}\right)^{1/5}\right.$$

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2)The integral (12) converges near $\xi = 0$ for values $|\arg \xi| \leqslant \pi/4$.

$$+ e^{2\pi i/5}\left(\frac{\omega^2}{v_t^2\alpha^4}\right)^{1/5} z_1\Big\}, \qquad \varepsilon \gg 1. \tag{20}$$

It follows from (19) that in the case $\epsilon \gg 1$, considered here the root of Eq. (13), determined by relation (19), corresponds approximately to a plane wave with a wave number

$$k = \operatorname{Re} k + i \operatorname{Im} k,$$

$$\operatorname{Re} k = -\left(\frac{\omega^2}{v_t^2}\alpha\right)^{1/5}\sin\frac{2\pi}{5}, \qquad \operatorname{Im} k = \left(\frac{\omega^2}{v_t^2}\alpha\right)^{1/5}\cos\frac{2\pi}{5}.$$

It is important to note that J oscillates in space much more frequently than the normal wave, whose length is

$$\lambda = \frac{2\pi}{\sqrt{z_1}}, \qquad \lambda \gg \left[\frac{v_t^2}{\omega^2\alpha}\right]^{1/5} = \lambda_1/2\pi, \qquad \lambda/\lambda_1 = \varepsilon^{1/5} \gg 1.$$

The solution of Eq. (10) under the boundary conditions indicated above is written in the form

$$E_1 = 2\alpha^{2/3}\mu B'\Big[i\Phi(\sigma)A + \Phi_1(\sigma) \times \int_{-\infty}^{z_1}\Phi(\sigma)J^* dz_1 - \Phi(\sigma)\int_{\infty}^{z_1}\Phi_1(\sigma)J^* dz_1\Big],$$

$$A = \lim_{z_1\to\infty}\int_{-\infty}^{z_1}\Phi(\sigma)J^* dz_1, \tag{21}$$

where $\Phi(\sigma)$ and $\Phi_1(\sigma)$ are connected with the Airy functions v and u [7] by the formulas $\Phi(\sigma) \equiv v(-\sigma)$ and $\Phi_1(\sigma) \equiv u(-\sigma)$. In particular, as $z_1 \to -\infty$ we certainly have $E_1 \ll E_0$, for it can be readily shown, by using arguments similar to that leading to formula (16), that J is exponentially small as $z_1 \to -\infty$, and

$$\Phi(\sigma) = {}^1/_2|\sigma|^{-1/4}\exp(-{}^2/_3|\sigma|^{3/2}),$$

$$\Phi_1(\sigma) = |\sigma|^{-1/4}\exp({}^2/_3|\sigma|^{3/2})$$

as $\sigma \to -\infty$.

When $z_1 \to \infty$, using the asymptotic formulas for the Airy functions, we get

$$E_1 \to 2\mu B'\alpha^{2/3}A\sigma^{-1/4}e^{i\gamma}, \qquad z_1 \to \infty. \tag{22}$$

The total field can be represented in the form ($z_1 \to \infty$)

$$E = E_0 + E_1 = -\frac{B'}{\sigma^{1/4}}e^{i\gamma}\left(1 - \frac{2\mu A}{\alpha^{4/3}}\right) = -\frac{B'}{\sigma^{1/4}}e^{i\gamma - p},$$

$$p = \frac{2\mu A}{\alpha^{4/3}} \ll 1, \tag{23}$$

$$p = \frac{4\sqrt{\pi}\,\omega^5}{3\sqrt{2}\,v_t^5\alpha^{4/3}}\int_{-\infty}^{z_1}\Phi(\sigma)J^* dz_1 \approx \frac{4\sqrt{\pi}\,\omega^5}{3\sqrt{2}\,v_t^5\alpha^{4/3}}\int_0^{z_1}\Phi(\sigma)J^* dz_1 \equiv \int_0^{z_1}k_1(z)\,dz_1. \tag{24}$$

The lower limit of integration, equal to $-\infty$, has been replaced in (24) by zero because the function $\Phi(\sigma)$ is exponentially small when $z_1 < 0$.

As follows from (24), the character of the damping of the wave, determined by the behavior of the function $J(z_1)$, depends essentially on the value of ϵ. If $\epsilon \ll 1$, we can readily separate by using formula (16) the slowly varying part of k_1 from the rapidly oscillating one. As a result we obtain for the slowly varying part of k_1 the following expression ($k_1(z_1) = \alpha k_1(z)$)

$$k_{1s}(z) = \frac{\sqrt{\pi}\,\omega^5}{6\sqrt{2}\,v_t^5}\,\frac{\exp(-\omega^2/2v_t^2 z_1)}{z_1^2}\cos\delta, \qquad \varepsilon \ll 1. \tag{25}$$

When $\epsilon\delta \ll 1$ we have $\delta = \delta_1 = \omega^4\alpha/4v_t^4 z_1^{7/2}$.

On the other hand, if the indicated condition is not satisfied, then δ has a much more complicated form, and, as already noted, the largest term in δ is still equal to δ_1. When $\delta_1 \ll 1$, we obtain a formula which coincides with the corresponding formula given in the articles of Silin and Rukhadze [3,4].

It follows from (25) that the wave damping, determined by the integral

$$\int_0^{z_1} k_{1s}\,dz,$$

is smaller than that obtained by an analysis that does not take into account the circumstance noted above. Moreover, it follows from (25) that in some region, where $\cos\delta < 0$, the wave grows in space.

In the case when $\epsilon \gg 1$, the expression for J is determined by formula (20) and, as already noted, J oscillates more rapidly than the normal wave, i.e., J executes many oscillations over a distance of the order of the wavelength λ. Because of this, the quantity k_1 determined by (24), oscillates more rapidly than the normal wave ($\zeta_2 \gg \sqrt{z_1}/\alpha$), since

$$k_1(z) \approx \frac{\sqrt{\pi}\,\omega^5}{3\sqrt{5}\,v_T^5\varepsilon^{7/10}z_1^2}e^{\varkappa_1+\varkappa_2 z_1}\cos(\zeta_1+\zeta_2 z_1)\cos\gamma, \qquad \varepsilon \gg 1,$$

$$\varkappa_1 = \frac{5}{6}\left(\frac{\omega^6}{v_t^6\alpha^2}\right)^{1/5}\cos\frac{4\pi}{5}, \qquad \varkappa_2 = \left(\frac{\omega^2}{v_t^2\alpha^4}\right)^{1/5}\cos\frac{2\pi}{5},$$

$$\zeta_1 = \varkappa_1\tan\frac{4\pi}{5}, \qquad \zeta_2 = \varkappa_2\tan\frac{2\pi}{5}. \tag{26}$$

We note that in (26) we retained only the most rapidly oscillating terms [see (20)].

It follows from (24) that although there exist regions where the amplitude of the wave grows in space, the wave loses energy on the whole propagation path, since

$$p|_{z_1=\infty} \approx \frac{\sqrt{\pi}\,\omega^3}{3\sqrt{2}\,v_t^3\alpha}\exp\left(-\frac{\omega^2}{2v_t^2k_s^2}\right).$$

To clarify the meaning of the foregoing results, let us determine directly the interaction between Maxwell-distributed electrons of the inhomogeneous plasma with the field of a normal longitudinal wave of such a plasma. We assume as before that the properties of the plasma change in accordance with the linear law (6). We consider the one-dimensional case, when the Fourier component E_ω of the longitudinal field excited by the current $j = euN^{2/3}\delta(z-ut)$ is determined by the equation[1]:

$$\frac{d^2E_\omega}{dz^2}+k_0^2E_\omega=\mu_1 e^{i\omega z/u},$$

$$\mu_1=-ieN^{2/3}\omega^3/3\omega_0^2v_t^2,\qquad k_0^2=z_1=\alpha_1+\alpha z, \tag{27}$$

and that the dependence of μ_1 on z can be neglected ($|1-\omega_0^2/\omega^2|\ll 1$, see above).

If we assume that the field sources are located in a finite region of space, then the boundary conditions of the problem are as follows: E_ω is finite everywhere, and as $z_1\to+\infty$ the field $E_\omega\to e^{i\gamma}$, i.e., E_ω should contain only an outgoing wave as $z_1\to+\infty$.

Using the foregoing boundary conditions, we obtain the solution of (27) in a manner similar to that used for the preceding case. As a result, going over in the resultant expression for the field to the limit as $z_1\to\infty$, we obtain the radiation field

$$\underset{z_1\to\infty}{E_\omega}=E_{\text{rad }\omega}=\frac{\mu_1\sqrt{\pi}i}{\alpha^{1/2}z_1^{1/4}}\exp\left[i\left(\frac{\omega\alpha_1}{\alpha u}-\frac{\omega^3}{3\alpha u^3}\right)\right]e^{i\gamma}.$$

Calculating the work performed by the charge per unit time on the radiation field at the point z, we obtain

$$\frac{dw}{dt}=-euE_\omega e^{-i\omega t}\big|_{t=(\alpha_1-z_1)/\alpha u}=-\frac{ie\mu_1\sqrt{\pi}\,\alpha^{5/6}u}{z_1^{1/4}}$$

$$\times\exp\left[i\left(\frac{\omega}{\alpha u}z_1-\frac{\omega^3}{3\alpha u^3}+\gamma\right)\right]+\text{ c.c.},\quad z_1\to\infty, \tag{28}$$

If we now average (28) over the Maxwellian distribution

$$f_0=\frac{N^{1/3}}{(2\pi)^{1/2}v_t}\exp\left(-\frac{u^2}{2v_t^2}\right)$$

and introduce a new variable $k=\omega/e$, then

$$\frac{d\overline{w}}{dt}=\frac{\omega^5 m}{3\sqrt{2}\pi\alpha^{1/2}v_t^3z_1^{1/4}}\cos\left[\frac{2}{3}\frac{z_1^{3/2}}{\alpha}+\frac{\pi}{4}\right]\operatorname{Im}\int_0^\infty\exp\left[-\frac{\omega^2}{2v_t^2k^2}\right.$$

$$\left.+\frac{i}{\alpha}\left(-kz_1+\frac{k^3}{3}\right)\right]\frac{dk}{k^3}. \tag{29}$$

It is evident that for the slowly varying part, which can be readily separated when $\epsilon\ll 1$, formulas (25) and (29) satisfy a relation of the type of the Kirchoff theorem [see (16)]. When $\epsilon\gg 1$ the quantity $\overline{dw(z_1)}/dt$ is, like k_1, a rapidly oscillating function ($\zeta_2\gg\sqrt{z_1}/\alpha$)

As can be seen from the foregoing, the presence of the effects considered above is connected with the circumstance that the number of particles which interact with the normal wave in the given section of the plasma can change appreciably over a distance on the order of the wavelength. Such a situation takes place, for example, for a Maxwellian distribution in the case of weak spatial dispersion ($u^2=\omega^2/z_1\gg v_t^2$). It follows from (28) that the work performed by one charge on the normal wave behaves, as a function of z, in the following manner: dw/dt changes little in the region where $u\approx\omega/z_1^{1/2}(z_0)$, and oscillates outside this region (when z is close to z_0, the oscillation is slow, and when z is greatly different from z_0, dw/dt oscillates rapidly). If the number of resonant particles changes appreciably with the distance, then the character of the interaction of the wave with the charges at the given point of the plasma is influenced much more strongly not by the charges for which $u\approx\omega/\sqrt{z_1}$ for this point of space $z=z_0$, but by the charges for which the velocity u is of the order of the phase velocity of the wave ($u\approx\omega/z_1^{1/2}(z)$) in neighboring sections ($z\neq z_0$), since there are many more such charges. However, in accordance with the statements made above with respect to formula (28), these charges give an oscillating radiation reaction dw/dt. If the number of the resonating particles N_r does not change strongly over one wavelength ($\epsilon\ll 1$), then these oscillations are slower ($\overline{dw}/dt\sim\cos\delta$). On the other hand, if N_r changes appreciably over one wavelength ($\epsilon\gg 1$), then $\overline{dw}/dt$ oscillates rapidly [see (20)]. This effect is absent only when $\delta\ll 1$.

In conclusion we point out that the effects under consideration should take place also in the case of a weakly inhomogeneous magnetoactive plasma.

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