COUPLED OSCILLATIONS OF ELECTRONIC AND NUCLEAR SPINS IN ANTIFERROMAGNETS

E. A. TUROV and V. G. KULEEV

Institute of Metal Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 25, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 248-256 (July, 1965)

The spectrum of coupled oscillations of electronic and nuclear spins (the coupling is due to hyperfine interaction) is calculated and studied in an antiferromagnet in which the axis of antiferromagnetism lies in a plane with small magnetic anisotropy. A relaxation mechanism is considered for the oscillations of the nuclear-like branch which appears because of the hyperfine coupling when account is taken of damping in the electronic spin system. The microwave magnetic susceptibility of the whole spin system is calculated, and the amplification coefficient for nuclear magnetic resonance is found. The spatial dispersion of nuclear-like spin waves is studied, and it is pointed out that such waves can be excited by a uniform microwave field ("nuclear spin-wave resonance").

1. In a number of experimental and theoretical papers[1-9] it has been shown that hyperfine interaction (hfi) causes a mixing of the oscillations of electronic and nuclear spins in ferromagnets and antiferromagnets. The mixing leads, in particular, to a shift of the frequencies of electronic and nuclear magnetic resonance (EMR and NMR), as compared with the case in which the hfi is taken into account only as the source of a constant local magnetic field $H_n = AM_0$, acting on the nuclei (A = hfi constant, M_0 = magnetization of the electrons of that magnetic sublattice on whose sites are located the nuclear spins of interest to us).

This shift is particularly large in antiferromagnets ($CsMnF_3$ and $RbMnF_3$) and weak ferromagnets ($MnCO_3$ and $KMnF_3$) in which the antiferromagnetism vector $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$ lies in the plane with small magnetic anisotropy (the "basal plane"). In this case the mixed EMR and NMR frequencies are determined approximately by the formulas[4,5]

$$\omega_1^2 \equiv \omega_e^2 = \omega_{e0}^2(1 + \omega_{EN}^2/\omega_{e0}^2), \quad (1)$$

$$\omega_2^2 \equiv \omega_n^2 = \omega_{n0}^2(1 - \omega_{EN}^2/\omega_e^2). \quad (2)$$

Here ω_{e0} and ω_{n0} are the unmixed EMR and NMR frequencies, and

$$\omega_{EN}^2 = \gamma_e^2 2H_E A m_0, \quad (3)$$

where H_E is the exchange field, describing the interaction between the magnetic sublattices of the antiferromagnet; $m_0 = \chi_n A M_0/2$ is the nuclear magnetization in one of the sublattices; and χ_n is the nuclear magnetic susceptibility, which changes with temperature T as far down as $T \sim 0.01°K$ according to Curie's law ($\chi_n = \text{const}/T$).

The unmixed NMR frequency is

$$\omega_{n0} = \gamma_n H_n, \quad (4)$$

the unmixed EMR frequency of the low-frequency resonance branch, in the case of antiferromagnets with weak ferromagnetism, of the $MnCO_3$ type or of the $KMnF_3$ type, can be expressed in the form

$$\omega_{e0}^2 = \gamma_e^2[H_0(H_0 + H_D) + 2H_E H_\Delta] \quad (5)$$

or

$$\omega_{e0}^2 = \gamma_e^2[H_0(H_0 + 5H_D) + 4H_D^2 + 2H_E H_\Delta] \quad (6)$$

respectively.[1)] In these expressions, H_D is the Dzyaloshinskiĭ field, which is responsible for weak ferromagnetism; H_Δ is a certain effective field, determining an additional energy gap, and connected either with a small magnetic anisotropy in the basal plane[10,11] or with magnetostrictive deformations[12,13]; H_0 is the external constant magnetic field, which lies in the indicated plane and is large enough so that $\mathbf{L}$ may be supposed perpendicular to $\mathbf{H}_0$.

Formulas (1) and (2) determine the frequencies of those coupled oscillations of electronic and nuclear spins that are excited by a magnetic micro-

[1)] If we set $H_D = 0$, then these formulas are also applicable to antiferromagnets without weak ferromagnetism, for example $CsMnF_3$. We remark that because of anisotropy in the basal plane, the term containing H_Δ (and also, in general, that containing H_D) can depend on direction in this plane.

wave field $\mathbf{h}$, perpendicular to the constant field $\mathbf{H}_0$. In an antiferromagnet, however, there is also a second pair of EMR and NMR frequencies, excited when $\mathbf{h} \parallel \mathbf{H}_0$[4,5,8]. For this pair of frequencies, formulas (1) and (2) are again correct with this difference only, that for ω_{e0} it is necessary to choose the frequency on the second EMR branch. In the case of uniaxial antiferromagnets (of the type $MnCO_3$ and $CsMnF_3$), for this second branch $\omega_{0e}^2 \approx \gamma_e^2 \cdot 2H_E H_A$, where H_A is the anisotropy field that prevents the vector $\mathbf{L}$ from leaving the basal plane. Since $H_A \gg Am_0$, for the second pair of frequencies the mixed and unmixed frequencies will differ little from each other. Below, we shall be interested only in the first pair of frequencies, for which ω_{e0} has the form (5) or (6).

It is known[1-9] that, for example, for the Mn^{55} nucleus the effective magnetic field Am_0, acting on the electrons and due to the nuclei, is determined in order of magnitude by the expression $Am_0 \sim 10/T$. It is easy to see from (3), on supposing that $2H_E \sim 10^6$ Oe, that at temperatures $T \lesssim 1°K$ the characteristic frequency ω_{EN}, which determines the coupling of the oscillations of electronic and nuclear spins, can be comparable in magnitude with the frequency ω_{e0} of (5) or (6) and may even exceed it. In this case the coupling of the oscillations of electronic and nuclear spins becomes so strong that in order to find the frequency spectrum, it is necessary to carry out a consistent simultaneous solution of the equations of motion for the electronic and nuclear magnetizations, without the simplifying assumptions that were adopted in the cited papers[4,5]. Furthermore, under these conditions there appears a spatial dispersion of the nuclear-like branch of the oscillations. The strong spatial correlation in the motions of nuclear spins (over a distance of the order of 10^2 to 10^3 interatomic distances) leads to the existence of long-wavelength "nuclear spin waves," which evidently can be excited by a uniform microwave field in thin foils, in analogy to the resonance of spin waves in ferromagnetic films[14]. The present paper, also, is basically devoted to this problem.

We have incidentally calculated the microwave susceptibility of an electronic-nuclear spin system and have at the same time found the NMR amplification coefficient. By taking into account the dissipative term in the equation of motion for the electronic magnetization, we have also calculated the NMR relaxation time T_1 for spin-lattice relaxation through the intermediacy of the electron system, and also the nonsecular contribution to the NMR line-width connected with this relaxation mechanism.

2. To find the spectrum of coupled oscillations of electronic and nuclear spins, it is necessary to add to the electronic spin Hamiltonian of an antiferromagnet (which we do not write out; we refer the reader, for example, to [10,11,4,5]) the hfi energy density, of the form

$$A(\mathbf{M}_1\mathbf{m}_1 + \mathbf{M}_2\mathbf{m}_2), \tag{7}$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are the nuclear-spin magnetizations on the sites of the first and the second magnetic sublattices, respectively. We write, in the usual way, the equations of motion for the four magnetizations $\mathbf{M}_j$ and $\mathbf{m}_j$ ($j = 1, 2$) in effective fields defined as the functional derivatives, with sign reversed, of the total energy of the system. We can then linearize the system of equations obtained, with respect to small oscillations of these vectors in the neighborhood of the equilibrium state. The corresponding determinant of eighth order,[2)] whose vanishing is the condition that the system of linear equations be solvable, separates into two determinants of fourth order. Each of the biquadratic equations obtained gives one "electronic-like" oscillation frequency (which reduces to a pure electronic frequency when $A = 0$) and one "nuclear-like" frequency.

One of these equations, which mixes oscillations of the nuclear spin system with the low-frequency branch of the spin waves of the antiferromagnet, can be written, to a very good approximation, in the form

$$\omega^4 - \omega^2(\omega_{ek}^2 + \omega_{EN}^2 + \omega_{n0}^2) + \omega_{ek}^2\omega_{n0}^2 = 0, \tag{8}$$

where

$$\omega_{ek} = [\omega_{e0}^2 + \omega_E^2(ak)^2]^{1/2} \tag{9}$$

is the unperturbed spin-wave frequency; the quantities ω_{e0}, ω_{EN}, and ω_{n0} are determined respectively by formulas (5) [or (6)], (3), and (4); the constant ω_E, of the dimensions of a frequency, is still another exchange-interaction parameter, connected with the nonuniform part of the exchange energy of the antiferromagnet; $\mathbf{k}$ and a are, respectively, the wave vector and a constant of the order of the interatomic distance. The second equation has the same form (8), but with this difference, that for ω_{ek} it is necessary to choose the frequency on the second spin-wave branch.

First, before going on to a discussion of the solutions of Eq. (8), we mention that between these

---

[2)] There are in all, of course, twelve equations; but only eight of them are independent (by virtue of the usual conditions $M_j^2 = \text{const}$ and $m_j^2 = \text{const}$).

solutions ω_{1k}^2 and ω_{2k}^2 there exists the following simple relation:

$$\omega_{1k}^2\omega_{2k}^2 = \omega_{ek}^2\omega_{n0}^2. \quad (10)$$

A peculiar "conservation law" is obeyed: the coupling between oscillations of the electronic and nuclear spins changes the characteristic frequencies of these oscillations, but in first approximation their product remains unchanged.

3. We pause first to analyze the frequencies of uniform resonance. For k = 0 we have from (8)

$$\omega_{e,n}^2 = \tfrac{1}{2}(\omega_{e0}^2 + \omega_{EN}^2 + \omega_{n0}^2) \pm \tfrac{1}{2}[(\omega_{e0}^2 + \omega_{EN}^2 + \omega_{n0}^2)^2 - 4\omega_{e0}^2\omega_{n0}^2]^{1/2}. \quad (11)$$

The plus sign refers to the electronic-like frequency, the minus to the nuclear-like. On expanding the root in (11) with respect to the quantity

$$\omega_{n0}^2 / (\omega_{e0}^2 + \omega_{EN}^2),$$

we get in first approximation

$$\omega_e^2 = \omega_{e0}^2 + \omega_{EN}^2\left(1 + \frac{\omega_{n0}^2}{\omega_{e0}^2 + \omega_{EN}^2}\right), \quad (12)$$

$$\omega_n^2 = \omega_{n0}^2\frac{\omega_{e0}^2}{\omega_{e0}^2 + \omega_{EN}^2}, \quad (13)$$

which agrees, to within small terms, with formulas (1) and (2). The latter formulas, however, are sometimes applied to the discussion of experimental data in the range of EMR frequencies $\omega_e \sim \omega_{n0}$ (at sufficiently small fields H_0)[8], where the expansion in question is not valid.

The general case is easier to analyze if we solve Eq. (8) (with k = 0) for $\omega_{e0} = \omega_{e0}(H_0) \equiv \omega_H$:

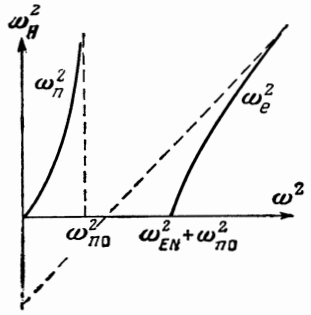
$$\omega_H^2 = \omega^2\left(1 - \frac{\omega_{EN}^2}{\omega^2 - \omega_{n0}^2}\right). \quad (14)$$

This formula actually expresses the dependence of the resonance field H_0 on the frequency of the microwave field, ω, and on the temperature of the nuclear spin system (through ω_{EN}) for both branches of the spectrum (NMR for $\omega^2 \le \omega_{n0}^2$ and EMR for $\omega^2 \ge \omega_{EN}^2 + \omega_{n0}^2$). The dependence of ω_H^2 on ω^2 is shown schematically in the figure.

Still another useful formula can be obtained from the relation (10) (with k = 0) by rewriting it in the form

$$\omega_H = \omega_e\omega_n / \omega_{n0}. \quad (15)$$

It expresses the general relation between the resonance field H_0 for EMR (at fixed frequency ω_e) and the NMR frequency ω_n. This relation is observed in experiments on double electronic-nuclear resonance[8].



Schematic curves determining the frequency dependence of the resonance field H_0 for the electronic (ω_e) and nuclear (ω_n) branches of the resonance, excited by a microwave field $h \perp H_0$ (formula 14)). The field H_0 enters through the unmixed NMR frequency $\omega_H \equiv \omega_{e0}$, so that, for example, in the case of $MnCO_3$ the conversion from ω_H to H_0 can be carried out according to formula (5).

We emphasize that the quantity ω_{EN}^2 is determined by the temperature T_n of the nuclear spin system; this is in general not equal to the lattice temperature T because of the presence of a saturation effect in NMR. The latter leads to a possibility of observing a number of nonlinear effects: for example, the double EMR–NMR resonance already mentioned. These effects depend significantly on the characteristic times of longitudinal and transverse nuclear relaxation (T_1 and T_2) and on the size of the NMR amplification coefficient η in the antiferromagnet[5].

4. One of the possible mechanisms of spin-lattice relaxation (and, at the same time, of nonsecular broadening of the NMR line-width $\Delta\omega_n = 1/2T_1$) is connected with damping in the electronic system; this, because of hfi, leads to damping of oscillations for the nuclear-like branch also. If we solve the equations of motion for $\mathbf{M_j}$ and $\mathbf{m_j}$ with inclusion of a dissipative term in the equations for $\mathbf{M_j}$ (for example, in accordance with Landau and Lifshitz), it is not difficult to obtain the following value of the indicated NMR relaxation parameter:

$$\frac{1}{2T_1} = \Delta\omega_n \approx \frac{\omega_{n0}^2 - \omega_n^2}{\omega_e^2}\Delta\omega_e. \quad (16)$$

Here $\Delta\omega_e \approx \lambda\gamma_e H_E$ is the EMR line-width (λ is the dimensionless parameter in the dissipative term in the equations of motion of $\mathbf{M_j}$[10,11]).

With (13) and (15) we get instead of (16)

$$\frac{1}{2T_1} = \Delta\omega_n \approx \frac{\omega_{n0}^2\omega_{EN}^2}{(\omega_{EN}^2 + \omega_{e0}^2)^2}\Delta\omega_e. \quad (17)$$

On assuming, for example, $\Delta\omega_e/\omega_e \sim 10^{-2}$, $\omega_{e0} \sim 2\pi \times 10^{10}$, $\omega_{n0} \sim 2\pi \times 6 \times 10^8$, $H_E \sim 10^6$ Oe, and $Am_0 \sim 10/T$, we get at temperature $T \sim 1°K$

$$\Delta\omega_n \sim 10^5 - 10^6 \text{ sec}^{-1}.$$

This is generally comparable in order of magnitude with the NMR line-width in $KMnF_3$ and $RbMnF_3$.

It is characteristic that this relaxation mechanism should give a maximum for $\Delta\omega_n$ in its dependence on temperature. If we suppose that $\Delta\omega_e$ varies only slightly with temperature, then this maximum should occur at a temperature determined by the condition $\omega_{EN}^2 = \omega_{e0}^2$. It is interesting that in $KMnF_3$ there is observed an increase of NMR line-width with decrease of temperature (in the interval 4.2 to 2.1°K), the increase slowing down at a temperature of about 2.5°K. It would be very desirable to extend the measurements into the region of lower temperatures.

Formula (17) also predicts a very rapid decrease of $\Delta\omega_n$ with increase of field H_0 (when $\omega_{EN} < \omega_{e0}$). A decrease of $\Delta\omega_n$ in its dependence on H_0 has been observed in $RbMnF_3$ [7] ($\Delta\omega_n$ decreased from 500 to 60 kcps on change of field from 6 to 13 kOe). Furthermore, in this compound $\Delta\omega_n$ also increased with lowering of temperature at low fields and became independent of temperature at high fields. Such a behavior of $\Delta\omega_n$ can be explained in principle by supposing that in $RbMnF_3$ two broadening mechanisms occur together: the one described above and the Suhl-Nakamura mechanism (interaction of nuclear spins through spin waves).

5. Now about the amplification coefficient η. It can be found by calculating, from the equations of motion, the whole microwave susceptibility of the electronic-nuclear spin system. Such a calculation (made by us for the case of $MnCO_3$) gives the following approximate expressions for the susceptibility in two mutually perpendicular directions, χ_{yy} ($\mathbf{h} \parallel \mathbf{L} \parallel \mathbf{Y}$) and χ_{zz} ($\mathbf{h}$ perpendicular to the $(\mathbf{L}, \mathbf{H}_0)$ plane):

$$\chi_{yy} \approx \chi_{0\perp} \frac{(\omega_{e0}^2 - \omega_\Delta^2)(\omega_{n0}^2 - \omega^2)}{(\omega_e^2 - \omega^2)(\omega_n^2 - \omega^2)}, \qquad (18)$$

$$\chi_{zz} \approx \chi_{0z} \frac{\omega_e^2\omega_n^2 - \omega^2(\omega_{e0}^2 + \omega_{EN}^2)}{(\omega_e^2 - \omega^2)(\omega_n^2 - \omega^2)}, \qquad (19)$$

where $\omega_\Delta^2 = \gamma_e^2 2H_E H_\Delta$, and $\chi_{0\perp}$ and χ_{0z} are the static transverse electronic susceptibilities of the antiferromagnet in directions perpendicular and parallel, respectively, to the z axis (they are defined as quotients of the static magnetization in the given directions divided by the field that excites it). For a weak ferromagnet,

$$\chi_{0\perp} = (M_0/H_E)(H_0 + H_D)/H_0, \quad \chi_{0z} = M_0/H_E.$$

Near the NMR frequency, $\omega \sim \omega_n$, we get from these formulas

$$\chi_{yy} \approx \eta_y^2 \chi_n \omega_n^2/(\omega_n^2 - \omega^2), \qquad (20)$$

$$\chi_{zz} \approx \eta_z^2 \chi_n \omega_n^2/(\omega_n^2 - \omega^2), \qquad (21)$$

$$\eta_y^2 = \left(\frac{\chi_{0\perp}}{\chi_n}\right)\frac{\omega_{EN}^2(\omega_{e0}^2 - \omega_\Delta^2)}{\omega_e^2\omega_{e0}^2} = \left(\frac{H_n}{H_0}\right)^2 \frac{(\omega_{e0}^2 - \omega_\Delta^2)^2}{\omega_e^2\omega_{e0}^2}, \qquad (22)$$

$$\eta_z^2 = \left(\frac{\chi_{0z}}{\chi_n}\right)\frac{\omega_{EN}^2\omega_{n0}^2}{\omega_e^4} = \left(\frac{\gamma_e H_n}{\omega_e}\right)^2\left(\frac{\omega_{n0}}{\omega_e}\right)^2. \qquad (23)$$

The quantities η are by definition the NMR amplification coefficients.[3)]

As was to be expected, $\eta_y \gg \eta_z$; and as in the case of ferromagnets, $\eta_y \sim H_n/H_0$ (for $\omega_{EN}^2 \ll \omega_{e0}^2$). The latter fact is also clear from simple physical considerations: the effective microwave field that acts on the nuclear spins through hfi is defined in the given case (small anisotropy in the basal plane) as

$$h_y^* \sim AM_0\theta \sim AM_0 h_y/H_0 = \eta_y h_y \qquad (24)$$

(θ is the angle through which the vector $\mathbf{L}$ rotates under the influence of the field h_y). We remark that for the low-temperature limiting case ($\omega_{EN}^2 \gg \omega_{e0}^2$), the square of the amplification coefficient is simply equal to the ratio of the electronic and nuclear static susceptibilities (for $\omega_{e0}^2 \gg \omega_\Delta^2$):

$$\eta_y^2 \approx \chi_{0\perp}/\chi_n.$$

It should be noticed that for the second NMR branch, excited when $\mathbf{h} \parallel \mathbf{H}_0 \parallel \mathbf{x}$, amplification is absent. (This is clear from the chain of relations of the form (24), in which, in the given case, the angle $\theta \sim h_x/H_E$, and consequently $\eta_x \sim H_n/H_E$). Therefore observation of the second NMR branch is very difficult. We must bear in mind also that in the absence of strict parallelism of $\mathbf{h}$ and $\mathbf{H}_0$, the first NMR branch can be excited; it possesses a large amplification coefficient, and this can cause additional difficulties in the observation of the second NMR.

It is interesting that formula (18), along with NMR amplification, leads also to the inference that there is some weakening of the EMR intensity. In fact, near the frequency $\omega \sim \omega_e$ we have

$$\chi_{yy} \approx \chi_{0\perp}\frac{\omega_{e0}^2}{\omega_e^2 - \omega^2} = \xi^2\chi_{0\perp}\frac{\omega_e^2}{\omega_e^2 - \omega^2} \qquad (25)$$

[3)] It is easy to understand (by introducing the substitution $\omega \to \omega - i\Delta\omega_n/2$ in (20) or (21) and separating out the imaginary part of the susceptibility $\chi'' = \eta^2\chi_n''$) that the multiplier η^2 is also preserved in the absorption curve. Since the absorptive power $P \sim \chi''h^2 = \chi_n''(\eta h)^2$, η can actually be regarded as the amplification coefficient of the microwave field that acts on the nuclear spins.

(for simplicity we have again assumed $\omega_{e0}^2 \gg \omega_{n0}^2, \omega_\Delta^2$), where the quantity

$$\zeta = \omega_{e0}/\omega_e \approx (1 - \omega_{EN}^2/\omega_e^2)^{1/2} \quad (26)$$

can be called the EMR weakening coefficient. It is easy to see that it can give a decrease of intensity of EMR at low temperatures at given frequency ω_e. In fact, by the substitution $\omega \to \omega - i\Delta\omega_e/2$ we find from formula (25) at $\omega = \omega_e$

$$(\chi_{yy}'')_{max} = \zeta^2\chi_{0\perp}\omega_e/\Delta\omega_e. \quad (27)$$

Thus in the known relation for the relative height of the resonance peak[10,14],

$$\chi_{max}''/\chi_0 = \omega_{res}/\Delta\omega$$

(where χ_0 is the static susceptibility), in the given case there appears on the right side a multiplier $\zeta^2 < 1$. It is possible that the decrease of intensity of the EMR line with lowering of the temperature, observed down to very low temperatures in $CsMnF_3$[4] and $MnCO_3$[8], is related to this effect.

6. We return now to the general formula (8) for the spectrum of coupled oscillations, and we consider the "nuclear-like" branch of the oscillations for $k \neq 0$. Its frequency, determined by the relation

$$\omega_{2k}^2 = {}^1/_2(\omega_{ek}^2 + \omega_{EN}^2 + \omega_{n0}^2) - {}^1/_2[(\omega_{ek}^2 + \omega_{EN}^2 + \omega_{n0}^2)^2 - 4\omega_{n0}^2\omega_{ek}^2]^{1/2} \approx \omega_{n0}^2\frac{\omega_{e0}^2 + \omega_E^2(ak)^2}{\omega_{e0}^2 + \omega_{EN}^2 + \omega_E^2(ak)^2}, \quad (28)$$

represents the frequency of collective oscillations of the nuclear spins, whose motion becomes correlated because of their indirect interaction through electronic spin waves. With increase of the wave vector k, the dispersion of these "nuclear spin waves" very rapidly becomes inconsequential, and their frequency ω_{2k} approaches the unmixed value of the NMR frequency.

As a critical value k_0 of the wave vector k, above which the dispersion disappears, we may conditionally take its value in the center of the dispersion band; this is determined by the following equation:

$$\omega_{2k} = {}^1/_2(\omega_n + \omega_{n0}), \quad (29)$$

From this, with use of (28), we find approximately

$$(ak_0)^2 = \left(\frac{\omega_{e0}}{\omega_E}\right)^2 \frac{(\omega_e/\omega_{e0} + 1)^2 - 4}{4 - (\omega_{e0}/\omega_e + 1)^2}$$

In the limiting cases of high and low temperatures we have

$$ak_0 = \frac{\omega_{e0}}{\omega_E}\left(1 + \frac{3}{8}\frac{\omega_{EN}^2}{\omega_{e0}^2}\right) \text{ for } \omega_{EN}^2 \ll \omega_{e0}^2, \quad (30)$$

$$ak_0 = \frac{1}{\sqrt{3}}\frac{\omega_{EN}}{\omega_E}\left(1 + \frac{4}{3}\frac{\omega_{e0}}{\omega_{EN}}\right) \text{ for } \omega_{EN}^2 \gg \omega_{e0}^2. \quad (31)$$

Consequently, at arbitrary temperatures, in order of magnitude,

$$k_0 \sim \omega_e/a\omega_E, \quad (32)$$

where ω_e is the electronic resonance frequency. The inverse quantity

$$b = k_0^{-1} \sim a\omega_E/\omega_e, \quad (33)$$

as also in the case of a ferromagnet, gives the effective linear dimensions of a region within which there is correlation in the motion of the nuclear spins. We remark that the correlation radius b in our case greatly exceeds the analogous quantity in ferromagnets (for which the formula of the form (33) contains, instead of the ratio ω_E/ω_e, the square root of this quantity) and amounts to hundreds or thousands of interatomic distances.

Finally, to speak of a "nuclear spin wave," as of any elementary excitation with a definite wave vector **k**, makes sense only in the case in which the width of the level corresponding to such a state, $\Delta\omega_{nk}$, is small in comparison with the total width of the whole dispersion band $\delta\omega_n = \omega_{n0} - \omega_n$.[4)] In the case of antiferromagnets of the type considered, this condition is already well satisfied in the range of helium temperatures; for the width of the band $\delta\omega_n$ here becomes comparable in order of magnitude with the unmixed frequency ω_{n0}, and for the width $\Delta\omega_{nk}$ of the nuclear spin-wave level it is possible, for crude estimates, to take the NMR line-width. Consequently

$$\Delta\omega_{nk}/\delta\omega_n \sim 10^{-3}\text{—}10^{-4} \ll 1.$$

It is possible to estimate the speed $v = \partial\omega_{2k}/\partial k$ and the length of the free path $l = v/\Delta\omega_{nk}$ of a nuclear spin wave. According to (28), we have

$$v = \frac{\omega_{EN}^2\omega_{n0}^2\omega_E^2a^2k}{\omega_{2k}(\omega_{e0}^2 + \omega_{EN}^2 + \omega_E^2a^2k^2)^2} \quad (34)$$

In particular, for $k \sim k_0$ in the region of high temperatures, where, according to (30), $ak_0 \sim \omega_{e0}/\omega_E$, we get from (34) with the aid of (29)

$$v \sim a\omega_{EN}^2\omega_E\omega_{n0}/4\omega_{e0}^3. \quad (35)$$

On taking for an estimate $a \sim 3 \times 10^{-8}$ cm, $\omega_E \sim 10^{13}$, $\omega_{e0} \sim 3 \times 10^{10}$, $\omega_{n0} \sim 3 \times 10^9$, $\omega_{EN} \sim 10^{10}$,

---

4)In analogy to the case of ferromagnets[5], fulfillment of this requirement is also, exactly, a necessary condition for justification of the procedure, carried out above, of linearization of the equations of motion for $\mathbf{M}_j$ and $\mathbf{m}_j$.

and $\Delta\omega_{nk} \sim 10$, we find

$$v \sim 10^3 \text{ cm/sec}, \qquad l \sim 10^{-3} \text{ cm}.$$

Similarly we get for the region of low temperatures (with $ak_0 \sim \omega_{EN}/\omega_E$ and $\omega_{EN} \sim 10^{11}$)

$$v \sim a\omega_E\omega_{n0}/\omega_{EN} \sim 10^4 \text{ cm/sec}, \quad l \sim 10^{-2} \text{ cm}.$$

It is possible that in thin antiferromagnetic plates at low temperatures one could observe resonance of nuclear spin waves (excitation of standing nuclear spin waves by a uniform microwave field), similar to the resonance of electronic spin waves observed in ferromagnets. If we assume that the thickness d of the plate contains an integral number of half-waves (for example, in case the boundary conditions are such that on the surface of the plate the nuclear spins are pinned), we have

$$k = k_z = \pi p/d, \tag{36}$$

where p is an integer; under the indicated boundary conditions, only harmonics with odd p can be excited (we are not counting the uniform resonance, which corresponds to p = 0). Thus according to (28) and (36), there should be observed a discrete set of resonance peaks, separated from the frequency of uniform resonance ω_n by the relative amount (for small numbers p)

$$\delta_p = \frac{\omega_{np} - \omega_n}{\omega_n} \approx \frac{1}{2}\frac{\omega_E^2\omega_{EN}^2}{\omega_{e0}^2(\omega_{e0}^2 + \omega_{EN}^2)}\left(\frac{\pi pa}{d}\right)^2. \tag{37}$$

In the region of low temperatures, a fully resolvable value of δ_p is obtained. For example, with $\omega_E/\omega_{e0} \sim 10^3$, p = 3, $a = 3 \times 10^{-8}$ cm, and $d = 3 \times 10^{-3}$ cm, we have $\delta_p \sim 10^{-2}$, whereas the relative NMR line-width amounts to $\Delta\omega_n/\omega_n \sim 10^{-3}$ to 10^{-4}. At higher temperatures, where the different peaks will be superposed on one another, this effect may lead to an apparent broadening of the NMR line.

[1] Heeger, Portis, Teaney, and Witt, Phys. Rev. Letters **7**, 307 (1961).

[2] Teaney, Freiser, and Stevenson, Phys. Rev. Letters **9**, 212 (1962).

[3] Borovik-Romanov, Kreĭnes, and Prozorova, JETP **45**, 64 (1963), Soviet Phys. JETP **18**, 46 (1964).

[4] Lee, Portis, and Witt, Phys. Rev. **132**, 144 (1963).

[5] de Gennes, Pincus, Hartmann-Boutron, and Winter, Phys. Rev. **129**, 1105 (1963).

[6] Nakamura, Minkiewicz, and Portis, J. Appl. Phys. **35**, 842 (1964).

[7] A. J. Heeger and D. T. Teaney, J. Appl. Phys. **35**, 846 (1964).

[8] H. Fink and D. Shaltiel, Phys. Rev. **136**, A218 (1964).

[9] A. S. Borovik-Romanov and V. A. Tulin, report at the 11th All-Union Conference on the Physics of Low Temperatures, Minsk, 1964; JETP Letters to the Editor **1**, no. 5, 18 (1965), Soviet Phys. JETP Letters **1**, 134 (1965).

[10] E. A. Turov, Fizicheskie svoĭstva magnitouporyadochennykh kristallov (Physical Properties of Magnetically Ordered Crystals), Izd. AN SSSR, 1963.

[11] E. A. Turov and N. G. Guseĭnov, JETP **38**, 1326 (1960), Soviet Phys. JETP **11**, 955 (1960).

[12] A. S. Borovik-Romanov and E. G. Rudashevskiĭ, JETP **47**, 2095 (1964), Soviet Phys. JETP **20**, 1407 (1965).

[13] E. A. Turov and V. G. Shavrov, FTT **7**, 217 (1965), Soviet Phys. Solid State

[14] Ferromagnitnyĭ rezonans (Ferromagnetic Resonance), ed. S. V. Vonsovskiĭ, Fizmatgiz, 1961.

Translated by W. F. Brown, Jr.
32