SOVIET PHYSICS JETP VOLUME 21, NUMBER 6 DECEMBER, 1965

SOME PROPERTIES OF A FERROMAGNETIC CRYSTAL CONTAINING A MAGNETIC IMPURITY ATOM

Yu. A. IZYUMOV and M. V. MEDVEDEV

Institute of Metal Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 15, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 1723-1731 (June, 1965)

Certain effects are considered that are due to the presence of local magnetic oscillations in a ferromagnetic crystal containing impurities, at low temperatures. It is shown that if one of the local levels lies close to the bottom of the spin-wave band, an anomaly should be observed in the temperature dependence of the spontaneous magnetization: namely, it should decrease appreciably faster than is required by the $T^{3/2}$ law for an ideal crystal, even when the concentration of impurities does not exceed a few percent. Also considered is ferromagnetic resonance in such a crystal in a uniform radio-frequency field. It is shown that when the g factors of the matrix and impurity atoms are different, the radio-frequency field can excite local oscillations only of s type. When the g factors are equal, only the usual ferromagnetic resonance, with excitation of a uniform precession of the spins, should be observed.

1. LOCAL MAGNETIC OSCILLATIONS

WOLFRAM and Callaway [1] considered the energy spectrum of a ferromagnetic crystal containing a magnetic impurity atom; they showed that in it, along with the continuous spectrum that corresponds to spin-wave excitations of an ideal crystal, there may be separate discrete levels that correspond to so-called local (magnetic) oscillations. For example, in a cubic crystal, in the nearest-neighbors approximation for exchange interaction, there exist in the general case three types of local oscillation: s, p, and d. Of these p is triply degenerate and d doubly degenerate. These oscillations obtained their nomenclature through the fact that the symmetry of the corresponding spin deviations in the neighborhood of an impurity atom coincides with the symmetry of the s, p, and d wave functions in the field of a cubic crystal.

It is necessary to separate two cases: (a) when the distinct local level falls in the continuous-spectrum band, and (b) when it lies outside it. In case (a), in consequence of the interaction of the discrete level with the continuous spectrum, the local level broadens to a certain finite width, and thus it acts as a virtual level. In case (b) the discrete level is infinitely narrow and acts as a discrete level in the full sense. For brevity we shall call these two situations the cases of a virtual and of a local level.

In an earlier paper, [2] the authors developed a rigorous mathematical treatment of the problem; this treatment permitted easy calculation not only of the energy spectrum of a crystal with an impurity, but also of the time-correlation functions that are necessary for a description of various processes in such a crystal. We applied our method to a discussion of inelastic scattering of slow neutrons in a crystal with an impurity, and we showed that in the energy distribution of the scattered neutrons there should be peaks corresponding to excitation or absorption of various local oscillations. In this paper we wish to point out still another possibility for the investigation of local magnetic oscillations, one that has to do with their excitation by an electromagnetic field in ferromagnetic resonance. At the same time we discuss here peculiarities of the temperature dependence of the spontaneous magnetization of a ferromagnetic crystal with an impurity.

2. BASIC FORMULAS

Let S, g and S′, g′ denote the spin and g factor for matrix and impurity atoms, respectively; let J be the exchange integral between nearest neighbors among matrix atoms, and let J′ be the exchange integral between an impurity atom and the nearest matrix atoms. The Hamiltonian of the spin system of a crystal, in an external constant magnetic field H, can be written in the form

$$\mathcal{H} = -J\sum_{n}\sum_{\Delta}\mathbf{S}_n\mathbf{S}_{n+\Delta} + 2J\sum_{\Delta}\mathbf{S}_1\mathbf{S}_{1+\Delta} - 2J'\sum_{\Delta}\mathbf{S}_1'\mathbf{S}_{1+\Delta} - \mu_0 gH\sum_{n}S_n^z + \mu_0 gHS_1^z - \mu_0 g'HS_1'^z. \tag{2.1}$$

Here n denotes the number of any site in the crystal; the impurity is on site 1; $\mathbf{S}_n$ is the spin operator of a matrix atom, and $\mathbf{S}_1'$ is the spin operator of the impurity. The summation with respect to Δ extends over nearest neighbors.

In the spin-wave approximation, we transform from spin operators to spin-deviation Bose operators in accordance with the relations

$$\begin{aligned} S_n^+ &= (2S)^{1/2}a_n, & S_1'^+ &= (2S')^{1/2}a_1, \\ S_n^- &= (2S)^{1/2}a_n^+, & S_1'^- &= (2S')^{1/2}a_1^+, \\ S_n^z &= S - a_n^+a_n, & S_1'^z &= S' - a_1^+a_1, \end{aligned} \tag{2.2}$$

and we introduce the two-time Green function, which we define as follows:[3]

$$G_{nm}(t-t') = -i\theta(t-t')\langle[a_n(t), a_m^+(t')]\rangle,$$

$$\theta(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}. \tag{2.3}$$

We shall also consider the Fourier transform $G_{nm}(E)$ (we shall sometimes denote it simply by G_{nm}), such that

$$G_{nm}(t-t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_{nm}(E)e^{-iE(t-t')}dE. \tag{2.4}$$

In[2] it was shown that the Green function G_{nm} for a crystal with an impurity satisfies Dyson's equation

$$G_{nm} = G_{nm}^0 + \sum_{pl} G_{np}^0 V_{pl} G_{lm}. \tag{2.5}$$

Here G_{nm}^0 is the Green function of the ideal crystal,

$$G_{nm}^0 = \frac{1}{N}\sum_{\mathbf{k}} G^0(\mathbf{k})\,e^{i\mathbf{k}(\mathbf{n}-\mathbf{m})} = \frac{1}{N}\sum_{\mathbf{k}}\frac{1}{E-\mathscr{E}_{\mathbf{k}}}e^{i\mathbf{k}(\mathbf{n}-\mathbf{m})}, \tag{2.6}$$

where $\mathscr{E}_{\mathbf{k}}$ is the energy of a spin wave in the ideal crystal; V is the matrix of coefficients of perturbation by the impurity, of size (z + 1) × (z + 1), where z is the number of nearest neighbors in the crystal:

$$V = 2JS\begin{pmatrix} \tilde{\varepsilon} z & -\gamma & -\gamma \ldots & -\gamma \\ -\gamma & \rho & 0 \ldots & 0 \\ -\gamma & 0 & \rho \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ -\gamma & 0 & 0 \ldots & \rho \end{pmatrix}; \tag{2.7}$$

the parameters entering here are:

$$\begin{gathered} \gamma = \sigma^{1/2}(\varepsilon+1) - 1, \quad \rho = \sigma - 1 + \varepsilon\sigma, \quad \varepsilon = (J'-J)/J, \\ \tilde{\varepsilon} = \varepsilon + \lambda\mu_0 gH/2JSz, \quad \sigma = S'/S, \quad \lambda = (g'-g)/g. \end{gathered} \tag{2.8}$$

From the definition (2.8) it is clear that in the absence of an external field, $\tilde{\epsilon} = \epsilon$, and the matrix V coincides with the corresponding matrix from[1,2].

Equation (2.5) can be written in the symbolic form

$$G = G^0 + G^0VG, \tag{2.9}$$

from which a symbolic solution for G can also be found:

$$G = \frac{1}{1-G^0V}G^0 \tag{2.10}$$

The poles of the Green function G, determined by the equation

$$D \equiv \det|1 - G^0V| = 0, \tag{2.11}$$

give the spectrum of the local oscillations of the crystal.

In the case of a cubic crystal we have

$$D = D(s)D^3(p)D^2(d); \tag{2.12}$$

$$\begin{aligned} D(s) &= 1 + 2\gamma + \gamma^2 - \tilde{\varepsilon}\rho - \rho - \rho(\mu_0 gH - E)/2JSz \\ &+ [2JSz(2\gamma + \gamma^2 - \tilde{\varepsilon}\rho - \tilde{\varepsilon} - \rho) + (\mu_0 gH - E) \\ &\times (2\gamma + \gamma^2 - \tilde{\varepsilon}\rho - 2\rho) - \rho(\mu_0 gH - E)^2/2JSz]G_{11}^0, \end{aligned} \tag{2.13}$$

$$D(p) = 1 - 2JS\rho(G_{11}^0 - G_{23}^0), \tag{2.14}$$

$$D(d) = 1 - 2JS\rho(G_{11}^0 + G_{23}^0 - 2G_{24}^0). \tag{2.15}$$

These expressions reduce to the corresponding formulas of[1,2] if we set H = 0.

We introduce also the expression for the density of states,

$$g(E) = \pi^{-1}\,\mathrm{Im}\,\mathrm{Sp}\,G(E - i0^+), \tag{2.16}$$

obtained in[1,2] (formula (3.15) in[2]):

$$g(E) = g_0(E) + \frac{1}{\pi}\,\mathrm{Im}\,\frac{D_E'(E - i\mathscr{E})}{D(E - i\mathscr{E})}, \tag{2.17}$$

where $D_E' \equiv dD/dE$. Here $g_0(E)$ is the density of states of the ideal crystal. By virtue of the relation (2.12) we have

$$\frac{D_E'}{D} = \frac{D_E'(s)}{D(s)} + 3\frac{D_E'(p)}{D(p)} + 2\frac{D_E'(d)}{D(d)}. \tag{2.18}$$

This means that the various branches (s, p, and d) of the local oscillations make additive contributions to the density of states, with weights corresponding to the degree of degeneracy of the local level.

3. ANOMALIES IN THE TEMPERATURE DEPENDENCE OF THE SPONTANEOUS MAGNETIZATION

For simplicity we restrict ourselves to the case in which g = g′. The spontaneous magnetic moment in the spin-wave approximation is

$$M = (N-1)\mu_0 gS + \mu_0 gS' - \mu_0 g\sum_n \langle a_n^+a_n\rangle, \tag{3.1}$$

where $\langle\ \rangle$ denotes the statistical mean value over a Gibbs ensemble with Hamiltonian (2.1). In accordance with the known spectral relation[3], connecting the Green function with the correlation function, we have

$$\langle a_n^+ a_n\rangle = \int_{-\infty}^{\infty} \frac{1}{e^{E/T}-1}\,\frac{1}{\pi}\operatorname{Im} G_{nn}(E-i\mathscr{E})\,dE. \qquad (3.2)$$

On summing both members of this expression over n, we get in accordance with (2.16)

$$\sum_n \langle a_n^+ a_n\rangle = \int_{-\infty}^{\infty} \frac{g(E)\,dE}{e^{E/T}-1}, \qquad (3.3)$$

where g(E) is the density of excitation states of the crystal; it is given by formula (2.17).

In view of the relations (3.1)—(3.3) and (2.18), we write the expression for the magnetization of the crystal in the following general form:

$$M = (N-1)\,\mu_0 g S + \mu_0 g S' - \mu_0 g \sum_{\mathbf{k}} [e^{\mathscr{E}_{\mathbf{k}}/T} - 1]^{-1}$$

$$-\mu_0 g \int_{-\infty}^{\infty} \frac{dE}{e^{E/T}-1}\,\frac{1}{\pi}$$

$$\times \operatorname{Im}\left\{\frac{D_E'(s,E-i\mathscr{E})}{D(s,E-i\mathscr{E})} + 3\,\frac{D_E'(p,E-i\mathscr{E})}{D(p,E-i\mathscr{E})}\right.$$

$$\left. + 2\,\frac{D_E'(d,E-i\mathscr{E})}{D(d,E-i\mathscr{E})}\right\}. \qquad (3.4)$$

The last expression takes account of the contributions of all the local oscillations.

We consider in more detail, for example, oscillations of s type. Let the point E = E_s correspond to the virtual level defined by the condition

$$\operatorname{Re} D(s, E_s) = 0. \qquad (3.5)$$

We expand Re D(s, E) as a series in the vicinity of the point E_s:

$$\operatorname{Re} D(s,E) = \operatorname{Re} D(s,E_s) + (E-E_s)\operatorname{Re} D'_{E=E_s}(s,E) + \ldots$$

On separating the real and imaginary parts in the expression for D(s, E − i$\mathscr{E}$), we get on the basis of the expansion just written

$$\frac{1}{\pi}\operatorname{Im}\frac{D_E'(s,E-i\mathscr{E})}{D(s,E-i\mathscr{E})} = \frac{1}{\pi}\,\frac{\Gamma_s(E) - (E-E_s)\,d\Gamma_s(E)/dE}{(E-E_s)^2 + \Gamma_s^2(E)},$$

where

$$\Gamma_s(E) = -\operatorname{Im} D(s,E)\,/\operatorname{Re} D'_{E=E_s}(s,E) \qquad (3.6)$$

is the amount of damping of the virtual level. If we neglect the dependence of $\Gamma_s(E)$ on E, we shall have a Lorentzian form for the density of states of the virtual level:

$$\frac{1}{\pi}\operatorname{Im}\frac{D_E'(s,E-i\mathscr{E})}{D(s,E-i\mathscr{E})} = \frac{1}{\pi}\,\frac{\Gamma_s}{(E-E_s)^2+\Gamma_s^2}. \qquad (3.7)$$

In the case of a local level, $\Gamma_s = 0$, and instead of the right member of formula (3.7) we get $\delta(E - E_s)$.

A similar treatment can be carried out for the other types of local oscillation. The problem of calculating the contribution to the magnetization from the virtual oscillations thus reduces to the calculation of the integral

$$\int_{-\infty}^{\infty} \frac{dE}{e^{E/T}-1}\,\frac{1}{\pi}\,\frac{\Gamma_s}{(E-E_s)^2+\Gamma_s^2}. \qquad (3.8)$$

This integral can be evaluated by use of the theorem of residues, if we close the path of integration with a semicircle of infinite radius. Since the integrand has an infinite discrete series of poles on the imaginary axis, we get the result in the form of a series:

$$\frac{1}{2}\left\{\frac{2TE_s}{E_s^2+\Gamma_s^2} + {\sum_{n=1}^{\infty}}' \frac{4TE_s}{E_s^2 + (2\pi nT + \Gamma_s)^2}\right\} - \frac{1}{2}. \qquad (3.9)$$

For $\Gamma_s \to 0$, we get from (3.8) the well known expansion of the hyperbolic cotangent:

$$\coth x = \frac{1}{x} + {\sum_{n=1}^{\infty}}' \frac{2x}{x^2+\pi^2 n^2}, \qquad (3.10)$$

which explains the structure of the expression (3.9).

In the case of a high-lying virtual level, when $E_s/T > 1$, it is easy to see directly from (3.8) that this integral gives an exponentially small contribution $\sim e^{-E_s/T}$. The case of a local level in general is not of interest to us, because for it always $E_s/T \gg 1$ and the integral (3.8) becomes negligibly small. Thus the physically interesting case is determined by the condition $E_s/T < 1$, when the local level falls in the range of the temperature interval being considered. Since our treatment is restricted to low temperatures (because of the spin-wave approximation), the condition $E_s/T < 1$ means that E_s falls close to the bottom of the band. However, it is known[1] that near the bottom of the band (and also near its top) the damping of the virtual level is small, so that we may suppose that $\Gamma_s \ll E_s$ and expand the expression (3.9) as a series in powers of Γ_s.

Thus for the integral in (3.8) we get

$$[e^{E_s/T} - 1]^{-1} - \Gamma_s \sum_{n=1}^{\infty} \frac{8T^2 E_s \pi n}{[E_s^2 + (2\pi nT)^2]^2}$$

$$-\frac{1}{2}\Gamma_s^2\left[\frac{1}{e^{E/T}-1}\right]''_{E=E_s} + \ldots . \qquad (3.11)$$

The first and third terms of this expansion agree with the corresponding terms of the expansion of integrals of the type (3.8) according to the theorem of Tikhonov and Samarskiĭ[4]; the term containing the first power of Γ_s does not reduce to a simple function, since in their method it is expressed as a complicated integral, to be taken in the sense of a principal value. We can give a more convenient representation of the series that occurs in the expression (3.11):

$$\Gamma_s \sum_{n=1}^{\infty} \frac{8T^2 E_s \pi n}{[E_s^2 + (2\pi n T)^2]^2} = \frac{\Gamma_s}{T} \sum_{n=1}^{\infty} (-1)^{n-1} n \frac{\zeta(2n+1)}{\pi^{2n+1}} \left(\frac{E_s}{2T}\right)^{2n-1}, \qquad (3.12)$$

where $\zeta(l)$ is the Riemann function.

The expansion (3.12) is valid under the condition

$$E_s / 2T < \pi. \qquad (3.13)$$

For $E_S/T < 1$ we can expand the first term in (3.11) as a series and neglect the other terms, which are proportional to various powers of $\Gamma_S/T \ll 1$. Thus the integral in (3.8) is approximately equal to T/E_S. At low temperatures ($T \ll T_C$, the Curie point), this expression in the formula (3.4) for the magnetization should be compared with the expression

$$\sum_{\mathbf{k}} [e^{\mathscr{E}_{\mathbf{k}}/T} - 1]^{-1} \sim (T/T_c)^{3/2}, \qquad (3.14)$$

which describes the number of spin deviations in an ideal crystal.

Thus we see that of the two terms determining the temperature variation of the magnetization, $(T/T_C)^{3/2}$ and T/E_S, at low temperatures the principal one is T/E_S, connected with the impurity, since $E_S \ll T_C$ by hypothesis; and this term contains the smaller power of T. It should be noticed that both terms have the same sign.

And thus we arrive at the following conclusion: if any virtual level of the local oscillation lies near the bottom of the spin-wave band, then the spontaneous magnetization of a ferromagnetic crystal containing impurities should decrease anomalously fast with temperature, much faster than according to Bloch's $T^{3/2}$ law, at temperatures for which $E_S < T$. Physically this is a very understandable result, since under this condition the local state of the crystal with energy E_S should be occupied, and this is the state with a deviated spin on the impurity atom itself and near it. Thus under the condition stated, when there is still very weak excitation in the whole crystal, the spins can be already strongly deviated on the impurity atoms and near them, and this leads to a rapid decrease of magnetization. Broadening of the virtual level Γ_S diminishes this effect, since some of the local states then enter the interval T; but since, under the conditions considered (local level near bottom of band), Γ_S is very small, the effect of broadening can be neglected. From the anomalous temperature variation of the magnetization in a crystal with impurities, it should be possible to determine the quantity of virtual local levels.

4. EXCITATION OF LOCAL LEVELS IN FERROMAGNETIC RESONANCE

We shall calculate the dynamic susceptibility $\chi_{\alpha\beta}(\omega)$ of a crystal in a uniform field of frequency ω, by using the known relation from the theory of kinetic coefficients[5]:

$$\chi_{\alpha\beta}(\omega) = i \int_{-\infty}^{\infty} \theta(\tau) \langle [M_\alpha(\tau), M_\beta] \rangle e^{i\omega\tau - \mathscr{E}\tau} d\tau, \qquad (4.1)$$

where $\mathscr{E} = 0^+$, and where M_α is the magnetic-moment operator of the whole crystal. We are interested in one component of the tensor susceptibility $\chi^+(\omega) \equiv \frac{1}{2}\chi_{+-}(\omega)$, which is obtained by setting $\alpha = +$ and $\beta = -$ in (4.1); then

$$M_+ = \mu_0 \sum_n g(n) S^+(n) = \mu_0 \sum_n g(n) [2S(n)]^{1/2} a_n.$$

Thus (4.1) can be put into the form

$$\chi^+(\omega) = -\mu_0^2 \sum_{mn} g(m) g(n) [S(m) S(n)]^{1/2} G_{nm}(\omega + i\mathscr{E}). \qquad (4.2)$$

In describing the sum over m and n, it is important here to take account of the different g factors for the matrix and impurity atoms. On introducing the parameters defined in (2.8), we can conveniently write (4.2) in the form

$$\chi^+(\omega) = -\sigma(\lambda+1)^2 \mu_0^2 g^2 S G_{11} - 2\sigma^{1/2}(\lambda+1) \times \mu_0^2 g^2 S \left(\sum_{m=1}^{N} G_{1m} - G_{11}\right) - \mu_0^2 g^2 S \left(\sum_{mn=1}^{N} G_{mn} - 2\sum_{m=1}^{N} G_{1m} + G_{11}\right). \qquad (4.3)$$

All the sums that occur here, over Green functions that describe a crystal with an impurity, were calculated by us[2] for a simple cubic lattice. Thus according to formula (4.9) of[2], we have

$$\sum_{mn} G_{nm} = \sum_{n} G^{(n)}(\mathbf{k}) \,|_{k=0} = \left[N G^0(\mathbf{k}) + \frac{Y(\mathbf{k}, s)}{D(s)} + 3\frac{Y(\mathbf{k}, p)}{D(p)} + 2\frac{Y(\mathbf{k}, d)}{D(d)} \right]_{k=0}, \qquad (4.4)$$

where $G^0(\mathbf{k}) \equiv (E - \mathscr{E}_{\mathbf{k}})^{-1}$ are the Fourier components of the Green function G^0_{nm} of an ideal crystal.

By virtue of the expressions (4.13) and (4.14) in[2], we have

$$Y(0, p) = 0, \qquad Y(0, d) = 0, \qquad (4.5)$$

the expression for Y(k, s), (4.12) of[2], can (in the presence of a constant field H) be transformed to the form

$$
\begin{aligned}
Y(\mathbf{k}, s) = {} & \frac{\rho}{2JSz} - (\tilde{\varepsilon}\rho - \gamma^2) G_{11}{}^0 \\
& + 2\left[\frac{\rho}{2JSz}(\mu_0 gH - E) - \gamma - \gamma^2 + \tilde{\varepsilon}\rho + \rho\right] G^0(\mathbf{k}) \\
& - \Big[2JSz(2\gamma + \gamma^2 - \tilde{\varepsilon}\rho - \tilde{\varepsilon} - \rho) \\
& + (\mu_0 gH - E)(2\gamma - \gamma^2 - \tilde{\varepsilon}\rho - 2\rho) \\
& - \frac{\rho}{2JSz}(\mu_0 gH - E^2)\Big] [G^0(\mathbf{k})]^2. \qquad (4.6)
\end{aligned}
$$

Thus $\Sigma_{mn} G_{nm}$ is calculated. As regards the other sum $\Sigma_m G_{1m}$, with the help of relation (2.10) it can be put into the form

$$
\begin{aligned}
\sum_m G_{1m} &= \sum_m \sum_{p=1}^{z+1} \left(\frac{1}{1 - G^0 V}\right)_{1p} G_{pm}{}^0 \\
&= G^0(0) \sum_{p=1}^{z+1} \left(\frac{1}{1 - G^0 V}\right)_{1p}. \qquad (4.7)
\end{aligned}
$$

On calculating the last sum, we get as our result

$$
\begin{aligned}
\sum_m G_{1m} = {} & \frac{1 + \gamma}{D(s)} G^0(0) \\
& + \frac{\rho + [\rho(2JSz + \mu_0 gH - E) - 2JSz\gamma] G_{11}{}^0}{2JSzD(s)}. \qquad (4.8)
\end{aligned}
$$

An expression for G_{11}, (5.10), was obtained in [2]:

$$
G_{11} = \frac{[2JSz + \rho(2JSz + \mu_0 gH - E)] G_{11}{}^0 + \rho}{2JSz\, D(s)}. \qquad (4.9)
$$

Now, on summing the results (4.3) to (4.9), we present the desired quantity in the form

$$
\begin{aligned}
\chi^+(\omega) = {} & -N\mu_0{}^2 g^2 S \frac{1}{\omega - \omega_0} \\
& - \mu_0{}^2 g^2 S(\sigma - 1) \frac{[D(s, \omega)]_{\lambda=0}}{D(s, \omega)} \frac{1}{\omega - \omega_0} \\
& - \mu_0{}^2 g^2 S\lambda \Big\{ (\rho + 1)\omega_0 \left[\frac{1}{\omega - \omega_0}\right]^2 \\
& + \frac{\rho\omega_0 + 2(\rho + 1)2JSz}{2JSz} \frac{1}{\omega - \omega_0} \\
& + \frac{2(\sigma - 1)(\rho + 1)\cdot 2JSz + (2\sigma - 1)\rho\omega_0 - 2\sigma\rho\omega}{2JSz} G_{11}{}^0(\omega) \\
& + \frac{2\sigma\rho}{2JSz} \Big\} \frac{1}{D(s, \omega)} - \mu_0{}^2 g^2 S\lambda^2 \sigma \Big\{ \frac{2JSz(\rho + 1) + \rho(\omega_0 - \omega)}{2JSz} \\
& \times G_{11}{}^0(\omega) + \frac{\rho}{2JSz} \Big\} \frac{1}{D(s, \omega)}. \qquad (4.10)
\end{aligned}
$$

Here $\omega_0 = \mu_0 gH$ is the Larmor frequency for uniform resonance (ħ has here been set equal to 1); D(s, ω) is what we earlier denoted simply by D(s) and is given by the expression (2.13).

Despite a certain cumbersomeness of the expression (4.10), it permits a clear physical interpretation. We notice first of all that the parameter $\lambda = (g' - g)/g$ is a measure of the difference of the g factors of the matrix and impurity atoms. For equal g factors, $\lambda = 0$, and the last two terms vanish. Since for $\lambda = 0$

$$
[D(s, \omega)]_{\lambda=0} / D(s, \omega) = 1,
$$

we get instead of (4.10) the elementary formula

$$
\chi^+(\omega) = -\mu_0{}^2 g^2 [(N - 1)S + S'] \frac{1}{\omega - \omega_0}. \qquad (4.11)
$$

This expression coincides in form with the susceptibility of an ideal ferromagnetic crystal. The result (4.11) means that when the g factors are equal, neither type of local oscillation is excited in ferromagnetic resonance. This is completely analogous to the case in which ferromagnetic resonance is studied in an ideal crystal with two sublattices. As is well known, in such a crystal, if the g factors of the atoms of the different sublattices are equal, then in uniform resonance the spin-wave branches that correspond to optical magnetic oscillations are not excited. The local magnetic oscillations in a crystal containing impurities are in a certain sense analogous to the optical oscillations of an ideal crystal composed of two sublattices. A local oscillation of an impurity atom is a peculiar nucleus of an optical band of oscillations, which originates if we replace half the original atoms of the crystal and form a second sublattice from the magnetic atoms brought in.

The last two terms in (4.10) when $\lambda \neq 0$ describe the contribution to the susceptibility from the local oscillation of s type. Oscillations of the p and d types in general make no contribution to the susceptibility of the crystal in a uniform field, because of conditions (4.5). These relations are essentially selection rules, connected with the symmetry of these oscillations in a lattice of the type considered. Thus it turns out that a uniform field can excite only the most symmetrical oscillation, the s type.

We restrict ourselves to this qualitative investigation. It would be possible to separate the real and imaginary parts in (4.10), bearing in mind that $\omega = \omega + i0^+$; but the expressions obtained would be very cumbersome. We point out only that the imaginary part of $\chi^+(\omega)$, which is connected with the absorption of electromagnetic radiation, contains a contribution proportional to the quantity

$$
\operatorname{Im} \frac{1}{D(s, \omega)} \sim \frac{\operatorname{Im} D(s, \omega)}{[\operatorname{Re} D(s, \omega)]^2 + [\operatorname{Im} D(s, \omega)]^2}, \qquad (4.12)
$$

which in the vicinity of the local oscillation $\omega = \omega_s$

has approximately the Lorentzian form (3.7). It is not difficult to see, however, that (4.12) has an asymmetric form with respect to the resonance point $\omega = \omega_s$.

---

[1] T. Wolfram and J. Callaway, Phys. Rev. **130**, 2207 (1963).

[2] Yu. A. Izyumov and M. V. Medvedev, JETP **48**, 574 (1965), Soviet Phys. JETP **21**, 381 (1965).

[3] N. N. Bogolyubov and S. V. Tyablikov, DAN SSSR **126**, 53 (1959), Soviet Phys. Doklady **4**, 589 (1959).

[4] A. N. Tikhonov and A. A. Samarskiĭ, DAN SSSR **126**, 26 (1959).

[5] R. Kubo, J. Phys. Soc. Japan **12**, 570 (1957).

Translated by W. F. Brown, Jr.
244