SOVIET PHYSICS JETP VOLUME 21, NUMBER 6 DECEMBER, 1965

RADIATION ACCOMPANYING TWO-PARTICLE ANNIHILATION OF AN ELECTRON-POSITRON PAIR

V. N. BAĬER and V. A. KHOZE

Novosibirsk State University

Submitted to JETP editor January 11, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **48**, 1708-1716 (June, 1965)

The integral cross section for photon emission in electron-positron pair annihilation into a pair of scalar particles is calculated by a method previously proposed by the authors. An analysis of the photon emission cross section for annihilation of an electron-positron pair into a pair of any two particles is carried out by taking into account the form factors. The cross section for emission by electrons is calculated exactly, whereas in the cross section for emission by the final particles the first two terms of the expansion in powers of ω/E are calculated. The Low method is used to evaluate the second expansion term. The interference term vanishes in all the cross sections considered.

1. INTRODUCTION

In an earlier paper[1] the authors proposed a simple method of calculating the integral cross section for the radiation of a photon with arbitrary energy occurring when an electron-positron pair annihilates into a pair of other particles. The idea of the method consists in integrating the individual parts of the diagrams, making extensive use of the relativistic, gauge, and charge invariance properties. This method was used to calculate the integral cross section for the emission of a photon when a muon pair is produced in an electron-positron collision.

We calculate in the present paper the photon emission cross sections for several annihilation processes. We consider the emission accompanying the production of a pair of scalar particles upon annihilation of an electron-positron pair (Sec. 2). To take into account the influence of the strong interactions on the process of emission by the initial particles, it is sufficiently simple to introduce the form factors of the final particles. If the summation is carried out over the spins of the final particles, then it becomes possible,[2] as a result of relativistic, charge, and gauge invariance properties, to write down a universal formula for the integral cross section of emission by the initial particles, containing two functions that depend on the form factors (Sec. 3).

To take into account the influence of strong interactions on the process of emission by the final particles, it is necessary to know the contributions of the diagrams of the Compton type (two final-particle lines and two photon lines, of which one is virtual). To this end one can either introduce phenomenological form factors of the four-point diagram, or else use dynamic models, or finally, expand the amplitude of the four-point diagram in powers of ω/E. Nothing is known at present concerning the indicated form factors for four-point diagrams, and an analysis, for example, within the framework of the dispersion approach, is a rather complicated independent problem and will not be presented below. At the same time, a rather large amount of information can be obtained by expanding the amplitude of emission of a photon by the final particles in powers of ω/E. It turns out that the zero-order term of the expansion is given by the classical current approximation, for which we can also write out a universal formula. To obtain the next term of the expansion, we used the method of Low,[3] and the expansion itself is carried out as an example for the emission produced when a pair of pions is created (Sec. 4). It can also be carried out in similar fashion for the radiation accompanying the production of any particle pair.

2. CROSS SECTION FOR EMISSION UPON PRODUCTION OF A PAIR OF SCALAR PARTICLES

The emission of a photon when a pair of scalar particles is produced in an electron-positron collision is represented by five diagrams (Fig. 1).

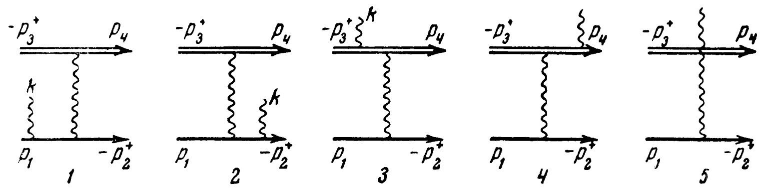

FIG. 1

The matrix element of the process is of the form[1)]

$$M = A[\Delta^{-2}\bar{v}(p_2^+)L_1^\nu u(p_1)P_\nu + \Lambda^{-2}\bar{v}(p_2^+)\gamma^\nu u(p_1)S_\nu], \tag{2.1}$$

where

$$A = \frac{ie^3}{(2\pi)^{7/2}}\frac{m}{(8E_1E_2E_3E_4\omega)^{1/2}}, \tag{2.2}$$

$$P = p_4 - p_3^+, \qquad \Delta = p_4 + p_3^+, \qquad \Lambda = p_1 + p_2^+;$$
$$S_\nu = \left(\frac{ep_4}{\eta'} + \frac{ep_3^+}{\eta}\right)P_\nu + \left(\frac{ep_4}{\eta'} - \frac{ep_3^+}{\eta}\right)k_\nu - 2e_\nu. \tag{2.3}$$

After averaging over the spins of the electrons and summing over the polarizations of the photons, we obtain

$$\bar{S}_i S_f |M|^2 = \frac{|A|^2}{4}\left[\frac{1}{\Delta^4}\frac{M_e^{\nu\nu'}}{m^2}P_\nu P_{\nu'} + \frac{1}{\Lambda^4}\frac{I_e^{\nu\nu'}}{m^2}T_{\nu\nu'} + \frac{2}{\Lambda^2\Delta^2}\overline{\frac{K_1^{\nu\nu'}}{m^2}P_\nu S_{\nu'}}\right]. \tag{2.4}$$

Here

$$T_{\nu\nu'} = -\sum S_\nu S_{\nu'} = 4g_{\nu\nu'} + \xi^2 P_\nu P_{\nu'} + \zeta^2 k_\nu k_{\nu'} + (\xi\zeta)(P_\nu k_{\nu'} + P_{\nu'}k_\nu) - 2(\xi_\nu P_{\nu'} + \xi_{\nu'}P_\nu + \zeta_\nu k_{\nu'} + \zeta_{\nu'}k_\nu), \tag{2.5}$$

$$\xi_\nu = \frac{p_{4\nu}}{\eta'} + \frac{p_{3\nu}^+}{\eta}, \qquad \zeta_\nu = \frac{p_{4\nu}}{\eta'} - \frac{p_{3\nu}^+}{\eta}, \tag{2.6}$$

where the summation is over the photon polarization.

The cross section of the investigated process is conveniently written, as before, in the form

$$d\sigma = d\sigma_e + d\sigma_s + d\sigma_{es}, \tag{2.7}$$

where $d\sigma_e$ is a contribution in which the photon is emitted by the initial particles, $d\sigma_s$—the contribution in which the photon is emitted by the final particles, $d\sigma_{es}$—interference term. Let us consider the cross section

$$d\sigma_e = \frac{\alpha^3}{(2\pi)^2|F|}\int\frac{d^3k}{\omega}M_e^{\nu\nu'}V_{\nu\nu'}, \tag{2.8}$$

$$V_{\nu\nu'} = \frac{1}{4\Lambda^4}\int\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}P_\nu P_{\nu'}\delta(\Delta - p_3^+ - p_4) = -\frac{\pi}{6\Delta^2}\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{3/2}\left(g_{\nu\nu'} - \frac{\Delta_\nu\Delta_{\nu'}}{\Delta^2}\right). \tag{2.9}$$

Owing to gauge invariance of $M_e^{\nu\nu'}$ the contribution to (2.8) is made only by the contraction with the tensor $g_{\nu\nu'}$. Carrying out this contraction and integrating trivially over the azimuthal angle of the photon emission, we obtain the differential cross section with respect to the angle between the directions of the initial electron and the photon in the c.m.s. of the initial particles:

$$\frac{d^2\sigma_e(E,\omega,\vartheta_k)}{d(\cos\vartheta_k)\,d\omega} = \frac{\alpha^3\omega}{24E^2\beta}\frac{1}{\Delta^2}\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{3/2}Z, \tag{2.10}$$

$$Z = m^2(\Delta^2 + 2m^2)\left(\frac{1}{\varkappa^2} + \frac{1}{\varkappa'^2}\right) + 2m^2\left(\frac{1}{\varkappa'} - \frac{1}{\varkappa}\right) + 2\left(\frac{\varkappa}{\varkappa'} + \frac{\varkappa'}{\varkappa}\right) + \frac{4}{\varkappa\varkappa'}[E^2\Delta^2 + m^2(E\omega - m^2)]. \tag{2.11}$$

Integrating over the angle ϑ_k, we get

$$d\sigma_e = \frac{\alpha^3}{6E^2\beta}\frac{d\omega}{\omega}\frac{1}{\Delta^2}\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{3/2}Y, \tag{2.12}$$

$$Y = (L-1)(\Delta^2 + 2\omega^2) + m^2\left[L\left(\frac{2\omega}{E} - \frac{m^2}{E^2}\right) - 2\right], \tag{2.13}$$

$$L = \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}, \qquad \beta = \frac{(E^2 - m^2)^{1/2}}{E}. \tag{2.14}$$

Integrating over ϑ_k, we get

$$d\sigma_s = \frac{\alpha^3}{4(2\pi)^2|F|}\int\frac{d^3k}{\omega\Lambda^4}I_e^{\nu\nu'}W_{\nu\nu'}, \tag{2.15}$$

$$W_{\nu\nu'} = \int\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}T_{\nu\nu'}\delta(\Lambda - p_3^+ - p_4 - k). \tag{2.16}$$

If we recognize that the tensor $T_{\nu\nu'}$ satisfies the condition

$$\Lambda^\nu T_{\nu\nu'} = \Lambda^{\nu'}T_{\nu\nu'} = 0, \tag{2.17}$$

then the general expression for $W_{\nu\nu'}$ can be represented in the form

$$W_{\nu\nu'} = h_1 g_{\nu\nu'} + \frac{\Lambda^2}{(k\Lambda)^2}(h_1 + \Lambda^2 h_2)k_\nu k_{\nu'} + h_2\Lambda_\nu\Lambda_{\nu'} - \frac{h_1 + \Lambda^2 h_2}{k\Lambda}(k_\nu\Lambda_{\nu'} + k_{\nu'}\Lambda_\nu). \tag{2.18}$$

To calculate the functions h_1 and h_2 it is sufficient to contract the tensor $W_{\nu\nu'}$ with the tensors $g^{\nu\nu'}$ and $k^\nu k^{\nu'}$, and calculate the integrals obtained from (2.16); this is the simplest to do in the c.m.s. of the two final particles. In the c.m.s.

[1)] Here and below we use the same notation as in the previous article[1].

of the initial particles we have

$$2h_1 - \Lambda^2 h_2 = \frac{4\pi}{E^2\omega^2}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}\{(E^2-\mu^2)[(\Delta^2-2\mu^2)L_1 - \Delta^2] + 4E^2\omega^2\}, \tag{2.19}$$

$$h_2 = \frac{\pi}{E^2\omega^2}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}[3\Delta^2 - L_1(\Delta^2+2\mu^2)],$$

$$L_1 = \frac{1}{\beta_0}\ln\frac{1+\beta_0}{1-\beta_0}, \qquad \beta_0 = \left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}. \tag{2.20}$$

Substituting the obtained values in (2.15), we readily get the differential cross section with respect to the angle between the directions of the initial electron and the photon for the contribution of the scalar particles:

$$\frac{d^2\sigma_s(E,\omega,\vartheta_k)}{d(\cos\vartheta_k)\,d\omega} = -\frac{\alpha^3\omega}{64\pi E^4\beta}\left[h_1\left(1+\frac{m^2}{2E^2}\right) + \frac{\varkappa\varkappa'}{2E^2\omega^2}(h_1+\Lambda^2 h_2)\right]. \tag{2.21}$$

Integrating (2.21) over the photon emission angle, we get

$$d\sigma_s = \frac{\alpha^3}{24E^4\beta}\frac{d\omega}{\omega}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}\left(1+\frac{m^2}{2E^2}\right) \times\left\{[(\Delta^2-2\mu^2)L_1-\Delta^2]\left(1-\frac{\mu^2}{E^2}\right)+4\omega^2\right\}. \tag{2.22}$$

In calculating the contribution of the interference term we encounter the integral

$$O_{\nu\nu'} = \int\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}\,\delta(\Lambda - p_3^+ - p_4 - k)S_\nu P_{\nu'}. \tag{2.23}$$

Following the substitution $p_3^+ \longleftrightarrow p_4$, the integrand reverses sign, so that

$$O_{\nu\nu'} = 0. \tag{2.24}$$

We see that the contribution of the interference term in the case of the production of a pair of scalar particles, as in the case of production of a muon pair, [1] vanishes. Thus, the total differential cross section is

$$\frac{d^2\sigma}{d(\cos\vartheta_k)\,d\omega} = \frac{d^2\sigma_e}{d(\cos\vartheta_k)\,d\omega} + \frac{d^2\sigma_s}{d(\cos\vartheta_k)\,d\omega} \tag{2.25}$$

and is given by formulas (2.10) and (2.21). The total integral cross section is

$$d\sigma = d\sigma_e + d\sigma_s \tag{2.26}$$

and is given by formulas (2.12) and (2.22).

There exists still another simple method of obtaining (2.22), in which the integration is carried out simultaneously over all the momenta of the final particles (with fixed ω). This method is presented in the Appendix.

The expression (2.26) obtained for $d\sigma$ is exact. We now consider the behavior of the cross sections in various limiting cases. Near the threshold we have $\mu^2/E^2 \sim 1$ and $\omega/E \ll 1$; then, expanding up to the first-order terms in ω/E, we get

$$d\sigma_e^{th} = \frac{\alpha^3}{3E^2}\frac{d\omega}{\omega}\beta_0^3\left[\ln\left(\frac{2E}{m}\right)-\frac{1}{2}\right], \tag{2.27}$$

$$d\sigma_s^{th} = \frac{2\alpha^3}{9E^2}\frac{d\omega}{\omega}\beta_0^3\left(1-\frac{\omega}{E}\right)\left(1-\frac{\mu^2}{E^2}\right); \tag{2.28}$$

near the threshold $\beta_0 \ll 1$; we see that the cross section for emission by heavy particles has near the threshold an additional smallness, proportional to the square of the velocity of these particles, as should be the case, since the emission of heavy particles near threshold is of the dipole type.

Far from the threshold $\mu^2/E^2 \ll 1$; then, accurate to terms of first order in μ^2/E^2 and ω/E, we have

$$d\sigma_e = \frac{\alpha^3}{3E^2}\left(1-\frac{3\mu^2}{2E^2}\right)\left[\ln\left(\frac{2E}{m}\right)-\frac{1}{2}\right]\frac{d\omega}{\omega}, \tag{2.29}$$

$$d\sigma_s = \frac{\alpha^3}{3E^2}\left(1-\frac{3}{2}\frac{\mu^2}{E^2}\right)\left(1-\frac{\omega}{E}\right)\left[\ln\left(\frac{2E}{\mu}\right)-\frac{1}{2}\right]\frac{d\omega}{\omega}. \tag{2.30}$$

We see that near the threshold the contribution of emission from a heavy particle is very small. With increasing energy of the initial particles over the production threshold, the contribution of the emission by the heavy particles increases, and when $E \sim 2\mu$ we have $d\sigma_S \sim 0.1\, d\sigma_e$. This situation is analogous to that occurring when a muon pair is produced. [1] As to the hard part of the photon spectrum, when $\omega \to \omega_{max}$ we have

$$d\sigma_e \approx (\omega_{max}-\omega)^{3/2}, \quad d\sigma_s \approx (\omega_{max}-\omega)^{1/2}.$$

3. CROSS SECTION FOR EMISSION UPON CREATION OF A PAIR OF ARBITRARY PARTICLES

The method proposed in [1] can be used to calculate the cross section of any process in which an electron-positron pair is converted into a particle-antiparticle pair and a photon. It is clear that the contributions of the diagrams in which the initial electrons radiate can be calculated in the same manner as in [1] (see also Sec. 2 of the present paper). To describe the vertex of the created particles we introduce the matrix element for the transition current $\langle p_4, p_3^+ | J | 0 \rangle$. It can be shown [2] that it follows from relativistic and gauge invariance requirements and from the law of current conservation that for particles with arbitrary spin the sum over the polarization of the final particles can be written in the form

$$X_{\mu\nu} = \sum \langle p_4, p_3^+|J_\mu|0\rangle \langle p_4, p_3^+|J_\nu|0\rangle^*$$
$$= \frac{1}{4E_3E_4}\left[4D_1(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu}{\Delta^2} - g_{\mu\nu}\right) - 2D_2(\Delta^2)P_\mu P_\nu\right], \qquad (3.1)$$

where D_1 and D_2 are functions of the form factors of the final particles. Thus, for example, for pions

$$D_1 = 0, \quad D_2 = -|F(\Delta^2)|^2/2. \qquad (3.2)$$

For nucleons

$$D_1 = {}^1/_2\Delta^2|\mathcal{F}_1 + g\mathcal{F}_2|^2, \quad D_2 = |\mathcal{F}_1|^2 - \Delta^2 g^2|\mathcal{F}_2|^2/4\mu^2. \qquad (3.3)$$

Here $F(\Delta^2)$, $\mathcal{F}_1(\Delta^2)$, and $\mathcal{F}_2(\Delta^2)$ are respectively the electromagnetic form factors of the pion and of the nucleons in the time-like region of momentum transfer. The corresponding expressions for vectons are given in [2].

If we again represent the cross section in the form

$$d\sigma = d\sigma_e + d\sigma_f + d\sigma_{ef} \qquad (3.4)$$

[cf. (2.7)], then to calculate $d\sigma_e$ it is sufficient to replace the product $P_\nu P_{\nu'}$ in (2.9) by $4E_3E_4X_{\nu\nu'}$ [formula (3.1)]; we denote the obtained integral by $U_{\nu\nu'}$. Then one must substitute in (2.8) in lieu of $V_{\nu\nu'}$ the expression

$$U_{\nu\nu'} = \frac{1}{\Delta^4}\int d^3p_3\, d^3p_4\, X_{\nu\nu'}\delta(\Delta - p_3^+ - p_4)$$
$$= -\frac{2\pi}{\Delta^4}\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{1/2}\left[D_1 - \frac{D_2}{6}(\Delta^2 - 4\mu^2)\right]$$
$$\times\left(g_{\nu\nu'} - \frac{\Delta_\nu\Delta_{\nu'}}{\Delta^2}\right). \qquad (3.5)$$

If we now calculate the integral in (2.8), we obtain the differential cross section with respect to the angle of emission of the photon ϑ_k:

$$\frac{d^2\sigma_e(E, \omega, \vartheta_k)}{d(\cos\vartheta_k)\,d\omega} = \frac{\alpha^3\omega}{2E^2\beta}\frac{1}{\Delta^4}$$
$$\times\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{1/2}\left[D_1 - \frac{D_2}{6}(\Delta^2 - 4\mu^2)\right]Z, \qquad (3.6)$$

where Z is given by formula (2.11), and the integral cross section is of the form

$$d\sigma_e = \frac{2\alpha^3}{\beta E^2}\frac{d\omega}{\omega}\frac{1}{\Delta^4}\left(\frac{\Delta^2 - 4\mu^2}{\Delta^2}\right)^{1/2}\left[D_1 - \frac{D_2}{6}(\Delta^2 - 4\mu^2)\right]Y, \qquad (3.7)$$

where Y is given by (2.13).

In calculating the contribution of the radiation by the produced particles we should also take into account their structure. It is necessary here to calculate the contribution of diagrams of the Compton type. As a result we obtain an expression containing a certain (spin-dependent) number of functions of invariant kinematic parameters. If the produced particles are pions, then there are three such functions, if they are nucleons, we already have 12, etc. At the present time nothing is known concerning these functions. We can therefore express in terms of these functions only the differential cross section, and this approach cannot be used to calculate the integral cross sections.

On the other hand, we can use the expansion of the indicated amplitudes in powers of ω/E. This expansion can be used in general in a rather wide range above threshold, since in this region, owing to the jump in the masses, $\omega/E \ll 1$. Then the integral cross section of the process can be represented in the form (see, for example, [3])

$$d\sigma_f = \sigma_{f0}d\omega/\omega + \sigma_{f1}d\omega + \sigma_{f2}\omega d\omega + \ldots \qquad (3.8)$$

It is easy to see here that the term σ_{f0} is given exactly by the classical current approximation in the case of annihilation processes. Indeed, by definition, the classical currents contain all the terms which do not have ω in the numerator; on the other hand, in annihilation processes, when integrating over the angles of emission of the final particles, no additional powers of ω arise [unlike the case of emission upon scattering, where the limits of integration with respect to the angles depend on ω; thus, $\Delta^2_{\min} = (\omega m^2/E^2)^2$, and the exact expression for σ_0 cannot be obtained from the expression with the classical currents; see, for example, [4]].

By definition, for the emission of a single photon,

$$d\sigma_{cl} = \sigma_{el}\frac{\alpha}{4\pi^2}\int \omega\, d\omega\, d\Omega_k$$
$$\times\left[\frac{p_1}{(p_1k)} - \frac{p_2^+}{(p_2^+k)} + \frac{p_3^+}{(p_3^+k)} - \frac{p_4}{(p_4k)}\right]^2. \qquad (3.9)$$

Carrying out integration over the emission angles of the photon and of the final particles, we obtain

$$d\sigma_{cl}(E, \omega) = \sigma_{f0}\frac{d\omega}{\omega} = \frac{\alpha^3}{E^4\beta}\left(\frac{E^2 - \mu^2}{E^2}\right)^{1/2}\left(1 + \frac{m^2}{2E^2}\right)$$
$$\times\left\{D_1^0 - \frac{2}{3}(E^2 - \mu^2)D_2^0\right\}\left[\frac{2E^2 - \mu^2}{4E^2}L_1^0 - \frac{1}{2}\right]\frac{d\omega}{\omega}. \qquad (3.10)$$

The previously defined functions D and L_1 [see (2.20), (3.2), and (3.3)] are taken in this formula at the point $\omega = 0$ (when $\Delta^2 = 4E^2$).

It must also be noted that the interference term $d\sigma_{ef}$ (3.4) vanishes, as in all the preceding cases, if the amplitude for the emission by the electron is taken in exact form, and the amplitude for the emission by the final particles is taken in the classical-current approximation.

4. INVESTIGATION OF THE RADIATION BY THE FINAL PARTICLES WITH THE AID OF THE LOW METHOD

There exists a general method of calculating the first two terms of expansion (3.8) with account of strong interaction in all orders.[3] We shall consider the calculation of the cross section σ_{f1} using as an example the radiation produced upon creation of a pair of pions. The matrix element for the emission of the photon by the final particles can be obtained directly from the second term of (2.1), in which we replace S_ν by $T_{\nu\mu}e_\mu$. The latter quantity we break up into two parts:

$$T_{\nu\mu} = T_{\nu\mu}{}^A + T_{\nu\mu}{}^B. \qquad (4.1)$$

By definition, $T^A_{\nu\mu}$ consists of the sum of the contributions of all the diagrams, on which the vertex with emission of a real photon is connected with the remaining part of the diagram by a single pion line (Fig. 2); $T^B_{\nu\mu}$ is the contribution of the remaining diagrams. We note that as $\omega \to 0$ only the quantity $T^A_{\nu\mu}$ diverges, while $T^B_{\nu\mu}$ remains finite.[3]

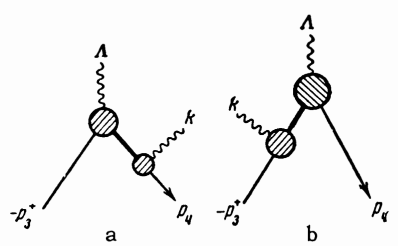

FIG. 2

For $T^A_{\nu\mu}$ we have the following expression:

$$T_{\nu\mu}{}^A = \Gamma_\nu(\Lambda^2, p_3^{+2}, (p_4+k)^2)\Delta(p_4+k)\Gamma_\mu(0, (p_4+k)^2, p_4^2)$$
$$+ \Gamma_\mu(0, p_3^{+2}, (p_3^+ + k)^2)\Delta(p_3^+ + k)$$
$$\times \Gamma_\nu(\Lambda^2, (p_3^+ + k)^2, p_4^2), \qquad (4.2)$$

where Δ and Γ_ν are exact renormalized propagation functions and the electromagnetic vertex operator of the pion.

Since we are interested in the emission of soft photons, we shall expand the quantities in (4.2) in powers of k and retain only the first two terms. Using the generalized Ward identity, we can easily show[3] that

$$T_{\nu\mu}{}^A = \Gamma_\nu(\Lambda^2, p_3^{+2}, (p_4+k)^2)\frac{p_{4\mu}}{(p_4k)}$$
$$- \Gamma_\nu(\Lambda^2, (p_3^+ + k)^2, p_4^2)\frac{p_{3\mu}^+}{(p_3^+k)}. \qquad (4.3)$$

The operator Γ_ν can obviously be represented in the form

$$\Gamma_\nu(\Lambda^2, p_3^{+2}, (p_4+k)^2) = (p_3^+ + p_4 + k)_\nu\varphi_1$$
$$+ (p_4 + k - p_3^+)_\nu\varphi_2,$$
$$\Gamma_\nu(\Lambda^2, (p_3^+ + k)^2, p_4^2) = (p_3^+ + p_4 + k)_\nu\widetilde{\varphi}_1$$
$$+ (p_4 - k - p_3^+)_\nu\widetilde{\varphi}_2, \qquad (4.4)$$

where φ_k and $\widetilde{\varphi}_k$ are scalar functions of the same arguments. From the requirement of gauge invariance

$$\Lambda_\nu\Gamma^\nu(\Lambda^2, p_3^{+2}, p_4^2) = 0 \qquad (4.5)$$

we have here

$$\varphi_1(\Lambda^2, p_3^{+2}, p_4^2) = \widetilde{\varphi}_1(\Lambda^2, p_3^{+2}, p_4^2) = 0. \qquad (4.6)$$

Expanding the functions φ_2 and $\widetilde{\varphi}_2$ in powers of k and recognizing that

$$\varphi_2(\Lambda^2, p_3^{+2}, p_4^2) = \widetilde{\varphi}_2(\Lambda^2, p_3^{+2}, p_4^2) = F(\Lambda^2) \qquad (4.7)$$

is the electromagnetic form factor of the pion, we obtain for $T^A_{\nu\mu}$ from (4.3)

$$T_{\nu\mu}{}^A = \left[(p_4 + k - p_3^+)_\nu\frac{p_{4\mu}}{(p_4k)} - (p_4 - p_3^+ - k)_\nu\frac{p_{3\mu}^+}{(p_3^+k)}\right]$$
$$\times F(\Lambda^2) + 2(p_4 + p_3^+)_\nu[p_{4\mu}(\varphi_1)_3 - p_{3\mu}^+(\widetilde{\varphi}_1)_2]$$
$$+ 2p_{4\mu}(p_4 - p_3^+)_\nu(F)_3 - 2p_{3\mu}^+(p_4 - p_3^+)_\nu(F)_2. \qquad (4.8)$$

Here $(F)_3$ and $(F)_2$ are the derivatives of the form factor F with respect to the corresponding argument, taken at $k = 0$.

In addition, the current conservation law must be satisfied

$$k_\mu T_{\nu\mu} = k_\mu T_{\nu\mu}{}^A + k_\mu T_{\nu\mu}{}^B = 0. \qquad (4.9)$$

Hence, recognizing that $T^B_{\nu\mu}$ does not contain infrared divergences, we obtain

$$T_{\nu\mu}{}^B = -2g_{\mu\nu}F(\Lambda^2) - 2(p_4 + p_3^+)_\nu[p_{4\mu}(\varphi_1)_3 - p_{3\mu}^+(\widetilde{\varphi}_1)_2]$$
$$-2p_{4\mu}(p_4 - p_3^+)_\nu(F)_3 + 2p_{3\mu}^+(p_4 - p_3^+)_\nu(F)_2. \qquad (4.10)$$

Consequently, the complete expression for $T_{\nu\mu}$ (we have retained only the first two terms of the expansion) is

$$T_{\nu\mu} = \left[(p_4 + k - p_3^+)_\nu\frac{p_{4\mu}}{(kp_4)} - (p_4 - p_3^+ - k)_\nu\frac{p_{3\mu}^+}{(kp_3^+)}\right.$$
$$\left. - 2g_{\mu\nu}\right]F(\Lambda^2). \qquad (4.11)$$

We see that the derivatives with respect to the masses have cancelled out, as is always the case.[3,5] The expression obtained is the matrix element of the radiation from a point-like particle, multiplied by the form factor. This corresponds to the well-known statement[5-7] that the first two terms of the expansion of the amplitude in the frequencies of the photons are determined by the total charge of the system and, consequently, depend only on the form factor. It is clear that the calculation of the integral cross section is carried out in the same manner as for point-like particles. The cross section for the emission of a photon by pions [see (2.22)], accurate to terms of first order in ω/E and in the ultrarelativistic limit with respect to the electrons, is of the form

$$d\sigma_\pi = \frac{\alpha^3}{24E^4}\frac{d\omega}{\omega}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}\left(1-\frac{\mu^2}{E^2}\right)[(\Delta^2-2\mu^2)L_1 - \Delta^2]F(\Lambda^2). \qquad (4.12)$$

The cross section for the emission of the photon by electrons is

$$d\sigma_e = \frac{\alpha^3}{6E^2}\frac{d\omega}{\omega}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{3/2}(L-1) \times\left[F(\Lambda^2)-\frac{\omega}{E}\Lambda^2\frac{dF(\Lambda^2)}{d\Lambda^2}\right], \qquad (4.13)$$

and the interference term, as shown at the end of Sec. 2, vanishes.

The total emission cross section for production of a pair of pions in an electron-positron annihilation, accurate to terms of first order in ω/E, is[2)]

$$d\sigma = d\sigma_e + d\sigma_\pi. \qquad (4.14)$$

As expected, it is expressed in terms of the electromagnetic form factor of the pion and its derivative with respect to the momentum transfer. In exactly the same manner we can obtain the integral cross section for the emission in the case of proton-antiproton pair production.

The authors are grateful to V. M. Galitskiĭ for numerous discussions.

[2)]We note that in a broad energy interval $d\sigma_\pi \approx 0.1 d\sigma_e$, so that the main contribution is made by the cross section $d\sigma_e$, which we calculate exactly. In addition, if we expand $d\sigma_s$ in (2.22) in powers of ω/E and retain the first two terms of the expansion, then this provides a rather high accuracy, up to $\omega/E \sim 1/2$, as follows directly from the comparison of the expansion with the exact result.

APPENDIX

The cross section for emission from point-like final particles can be calculated with the aid of the following simple procedure. We now present $d\sigma_s$ (2.15) in the form

$$d\sigma_s = \frac{\alpha^3}{4(2\pi)^2|F|}\frac{1}{\Lambda^4}I_e^{\nu\nu'}R_{\nu\nu'}, \qquad (A.1)$$

where

$$R_{\nu\nu'} = \frac{d\omega}{\omega}\int\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}\omega^2\, d\Omega_k\,\delta(\Lambda - p_4 - p_3{}^+ - k)T_{\nu\nu'} \qquad (A.2)$$

can depend only on the four-vector that fixes the reference frame in which the photon energy ω is chosen. Such a vector is $n_\mu(1, 0, 0, 0) = \Lambda_\mu/\sqrt{\Lambda^2}$. Taking gauge invariance into account, we get

$$R_{\nu\nu'} = \frac{d\omega}{\omega}\left[g_{\nu\nu'} - \frac{\Lambda_\nu\Lambda_{\nu'}}{\Lambda^2}\right]f. \qquad (A.3)$$

The quantity f is calculated, as usual, with the aid of contraction with the tensor $g^{\nu\nu'}$ and is equal to

$$f = \frac{16\pi^2}{3}\left(\frac{\Delta^2-4\mu^2}{\Delta^2}\right)^{1/2}E^2\left\{[(\Delta^2-2\mu^2)L_1-\Delta^2]\left(1-\frac{\mu^2}{E^2}\right) + 4\omega^2\right\}. \qquad (A.4)$$

Substituting (A.3) in (A.1), we obtain (2.22).

[1]V. N. Baĭer and V. A. Khoze, JETP **48**, 946 (1965), Soviet Phys. JETP **21**, 629 (1965).

[2]V. N. Baĭer and V. S. Fadin, DAN SSSR **161**, 74 (1965), Soviet Phys. Doklady **10**, 204 (1965).

[3]F. Low, Phys. Rev. **110**, 974 (1958).

[4]V. N. Bayer and V. Galitsky, Phys. Lett. **13**, 355 (1964).

[5]S. M. Bilen'kiĭ and R. M. Ryndin, JETP **40**, 819 (1961), Soviet Phys. JETP **13**, 575 (1961).

[6]F. Low, Phys. Rev. **96**, 1428 (1954).

[7]M. Gell-Mann and M. Goldberger, Phys. Rev. **96**, 1433 (1954).

Translated by J. G. Adashko
242