MUON POLARIZATION AND THE FORM FACTOR RATIO IN $K^+_{\mu 3}$ DECAY

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Submitted to JETP editor January 26, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 1604-1610 (June, 1965)

Using an emulsion chamber, we have measured the angular distribution of positrons for 887 μ^+ mesons from $K^+_{\mu 3}$ decay and the μ^+-meson longitudinal polarization $P_\mu = +0.68 \pm 0.28$. This result points to a form factor ratio $\xi = 0 \pm 1$.

1. INTRODUCTION

THE study of $K_{\mu 3}$ decay ($K^+_{\mu 3} \to \mu^+ + \pi^0 + \nu$) provides the possibility of verifying the applicability of the universal V–A interaction theory to leptonic decays of strange particles and of clarifying the properties of the form factors characterizing the $K\pi$ interaction in this decay. Specifically, for a two-component neutrino, muons from $\pi \to \mu + \nu$ or $K \to \mu + \nu$ decays must have a polarization identical to that of the emitted neutrino as the result of conservation of angular momentum, since in three-particle $K_{\mu 3}$ decay the muon polarization depends on the spatial correlation of the resulting particles, which is determined by the nature of the interaction.

As Pais and Treiman[1] have shown, the matrix element of the decay depends on two scalar form factors. It can be described[2] in the form

$$M = [f_+(p_k + p_\pi) + f_-(p_k - p_\pi)]_\lambda \bar{u}(p_\nu)\gamma_\lambda(1+\gamma_5)v(p_l), \tag{1}$$

where l indicates electron or muon, f_+ and f_- are the two form factors characterizing the strong interaction between the K meson and the π meson, and p is the particle momentum. If the weak interaction is universal, this matrix element describes both K_{e3} and $K_{\mu 3}$ decay.

One of the possible means of determining the form factors f_+ and f_- is comparison of theoretical and experimental values of the probability ratio for $K_{\mu 3}$ and K_{e3} decay. If we assume that the form factors f are constant, this ratio turns out to be

$$R = W(K_{\mu 3}{}^+) / W(K_{e3}{}^+) = 0.651 + 0.126\xi + 0.019\xi^2, \quad \xi = f_-/f_+. \tag{2}$$

It is thus a quadratic function of the parameter ξ —the ratio of the two form factors.

If we use the value $R = 0.75 \pm 0.07$ given by Giacomelli et al.,[3] then two possible solutions follow from Eq. (2):

$$\xi_1 = +0.7 \pm 0.5 \quad \text{or} \quad \xi_2 = -7.3 \pm 0.5.$$

In order to make a choice between these two solutions, additional experiments are necessary. They can consist, for example, of a measurement of the shape of the muon spectrum or, as Matinyan and Okun',[4] have shown, of a measurement of the sign and magnitude of the longitudinal polarization of muons in $K_{\mu 3}$ decay. These polarization measurements are the purpose of the present work. Preliminary results obtained with poorer statistics were published in 1964.[5]

2. DESCRIPTION OF THE EXPERIMENT AND MEASUREMENTS

To measure the longitudinal polarization of μ^+ mesons from $K^+_{\mu 3}$ decay, an emulsion chamber with a volume of about 1 liter of type R emulsion was irradiated in the proton synchrotron of the Institute of Theoretical and Experimental Physics in an unseparated K^+-meson beam with a momentum of 410 MeV/c. This beam, extracted at an angle of 60° to the primary proton direction, was focused by a pair of quadrupole lenses and a bending magnet onto the center of the emulsion chamber. The total path from the internal target of the synchrotron to the emulsion chamber was nearly 5 m, and the intensity was ~ 30 K^+ mesons per pulse for 10^{10} protons, for a beam diameter of 50 mm.

In order not to fill the scanned region of the chamber with K-meson and pion tracks, the irradiation was carried out perpendicular to the plane of the emulsion. A drawing of the experimental arrangement is shown in Fig. 1. The protons in the beam are slowed down in a carbon absorber, and the K^+ mesons stop in the center of the emulsion chamber. For pions with the same momentum the comparatively thin chamber is practically transparent. In order to preserve the muon polarization, the chamber was placed in a magnetic field of strength 6 kG perpendicular to

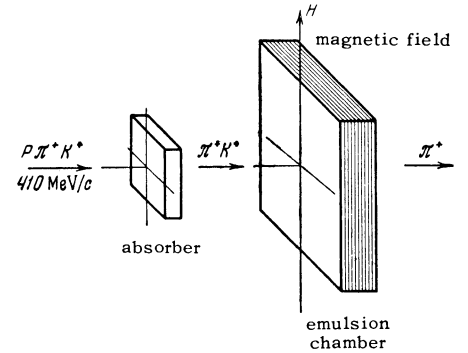

FIG. 1. Setup for irradiation of the emulsion chamber by K^+ mesons.

the beam and directed vertically along the plane of the emulsion. The muon polarization was measured from the asymmetry of the spatial distribution of positrons in $\mu^+ \to e^+$ decay. The search for $\mu^+ \to e^+$ decays was carried out by area scanning. On finding such a decay, the scanner followed the muon track. If this procedure terminated after $\sim 600\,\mu$ in a π^+-meson stopping, we were dealing with a $\pi^+ \to \mu^+ \to e^+$ decay. If the muon track was longer than $600\,\mu$ and turned out to be directed to the central spot of the chamber where the K^+-meson stoppings occurred, we assumed that the observed event occurred as the result of $K^+_{\mu 3}$ decay.

We scanned only the relatively small part of the emulsion where the greatest number of μ^+-meson stoppings from $K^+_{\mu 3}$ decay should occur (μ^+-meson kinetic energy $T_\mu \cong 40$–100 MeV). π^+ mesons arising in $K^+_{\pi 2}$ and $K^+_{\pi 3}$ decays also stop in this emulsion region. We used the decays of these π^+ mesons to monitor the correctness of the asymmetry measurements. In addition, in order to exclude the effect of cosmic-ray μ mesons on the polarization measurements, we used only the part of the emulsion in which muons from $K^+_{\mu 3}$ decay were directed upward, along the direction of the magnetic field (Fig. 2).

To verify the correctness of the method which we used for selection of μ^+ mesons from $K^+_{\mu 3}$ decay, we carried out a control experiment in which we measured the ranges and the sign of the polarization of μ^+ mesons from $K^+_{\mu 2}$ decay. It is well known that the range of these mesons is close to 20 cm of emulsion, which is 3 cm greater than the maximum range of μ^+ mesons from $K^+_{\mu 3}$ decay.

The arrangement for irradiation of the special emulsion chamber intended for this experiment was the same as in the experiment shown in Fig. 1, with the single exception that the K^+-meson stoppings occurred in a graphite target, and between the emulsion chamber and the target was placed an absorber. This absorber was chosen so that all the μ^+ mesons from $K^+_{\mu 2}$ decay and part of the mesons from $K^+_{\mu 3}$ decay stopped in the emulsion layers. Figure 3 shows the distribution of the distance between the point of stopping of μ^+ mesons from $K^+_{\mu 2}$ decay and the edge of the emulsion chamber. We see that this selection method actually permits separation of the range group of μ^+ mesons from $K^+_{\mu 2}$ decay. The spatial asymmetry coefficient for emission of positrons from these μ^+ mesons is $a = -0.40 \pm 0.30$, which agrees with the expected value $a = -0.25$.[6]

In the decay of μ^+ mesons in a magnetic field the decay probability depends on the cosines of the angle ϑ_μ between the magnetic field and the μ^+-meson direction and the angle ϑ_e between the magnetic field and the positron direction, and has the form

$$dN \sim (1 + a\cos\vartheta_\mu\cos\vartheta_e)\,d\vartheta_\mu d\vartheta_e,$$

where a is the asymmetry coefficient. By measuring the forward-backward asymmetry (along the direction of the magnetic field and against the field), we can determine the sign and absolute value of the μ^+-meson polarization.

In the scanning it turned out that the plates have a background of μ^+ mesons whose direction of motion coincides with the plane of the accelerator orbit and is practically perpendicular to the magnetic field (Fig. 2). For these μ^+ mesons the asymmetry of $\mu^+ \to e^+$ decay along and against the magnetic field must be close to zero (the right-left asymmetry). We used these decays also for control of the measurement of muon polarization in $K^+_{\mu 3}$ decay. The experimental results

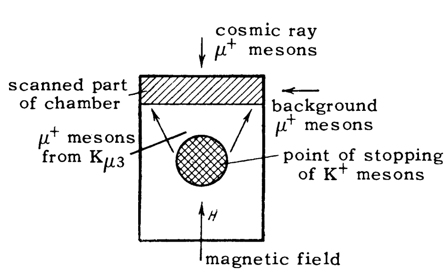

FIG. 2. Location of the K^+-meson stoppings and the scanned region of the chamber in the emulsion layers.

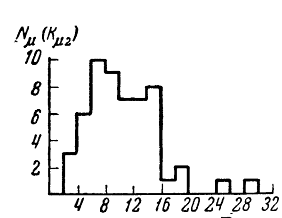

FIG. 3. Range group of μ^+ mesons from $K^+_{\mu 2}$ decay.

Measured values of spatial asymmetry coefficient for positrons in $\mu^+ \to e^+$ decay

No.	Type of decay	Asymmetry	Expected value	Measured value $a \pm \Delta a$	Number of events
1	$\pi^+ \to \mu^+ \to e^+$	Forward-backward Right-left	—0.25 0	—0.235±0.035 0.03±0.035	6971
2	$\mu^+ \to e^+ (K^+_{\mu 2})$	Forward-backward	—0.25	—0.40±0.30	51
3	$\mu^+ \to e^+$ (cosmic rays)	Forward-backward	—0.10	—0.15±0.16	160
4	$\mu^+ \to e^+$ (background)	Right-left	0	0.04±0.08	692
5	$\mu^+ \to e^+ (K^+_{\mu 3})$	Forward-backward Right-left	+0.12 (for $\xi = +1$) or —0,10 (for $\xi = -8$) 0	+0.17±0.07 0.02+0.07	887

for the spatial asymmetry coefficient are given in the table. The asymmetry coefficient was determined from the formula

$$a = k(N_f - N_b) / (N_f + N_b),$$

where $k = 1.94$ is a coefficient determined by the geometry of the measurements; N_f and N_b are the number of decays in which the positron is directed forward or backward (left or right); the error in the measurement is

$$\Delta a \approx 2 / (N_f + N_b)^{1/2}.$$

In the first four lines of the table are shown the results of the control measurements of the asymmetry. The measurement of the $\pi^+ \to \mu^+ \to e^+$ decay asymmetry (first line) gave a value for the asymmetry coefficient $a = -0.235 \pm 0.035$, which is in excellent agreement with the value $a = -0.25 \pm 0.02$ obtained by one of us[7] for a field of 6 kG. In the second line is listed the result mentioned above on measurement of the asymmetry for μ mesons from $K^+_{\mu 2}$ decay. In the third line is listed the value of a for cosmic-ray μ mesons. We observed a total of 160 such mesons traveling upward. The asymmetry $a = -0.15 \pm 0.16$ observed for them agrees with the expected value.[8] In the fourth line of the table is listed the value of asymmetry for background μ mesons emitted in the plane of the accelerator orbit. For them the directions "along the field" and "against the field" are the directions "right" and "left" and the result means that asymmetry must be absent. The measurements actually give $a = 0.04 \pm 0.08$ for these μ^+ mesons. A similar result was also obtained in measurements of the "right-left" asymmetry in other cases (see the table). This series of control measurements verifies the absence of any appreciable systematic shifts in the asymmetry measurements in our chamber.

The fifth line of the table lists the value of the asymmetry parameter for μ^+ mesons from $K^+_{\mu 3}$ decay. Measured from 887 cases of $K^+_{\mu 3}$ decay, this value is

$$a = +0.17 \pm 0.07.$$

3. DISCUSSION OF RESULTS

Thus we see that the observed spatial asymmetry coefficients for μ^+ mesons arising from π^+ and $K^+_{\mu 2}$ decays on the one hand and $K^+_{\mu 3}$ decays on the other hand have opposite signs. Figure 4a shows the angular distribution measured by us for positrons from $K^+_{\mu 3} \to \mu^+$ decay. The abscissa of this graph represents the quantity $\cos\vartheta_\mu \cos\vartheta_e$. The dashed line represents the straight line corresponding to the obtained value $a = +0.17 \pm 0.07$. An evaluation of the agreement of the angular distribution with the dashed line according to a χ^2 test gives a value $\chi^2 = 7.4$ compared to an expected value $\chi^2 = 5$.

For comparison we have shown in Fig. 4b the angular distribution of positrons from $\pi^+ \to \mu^+$

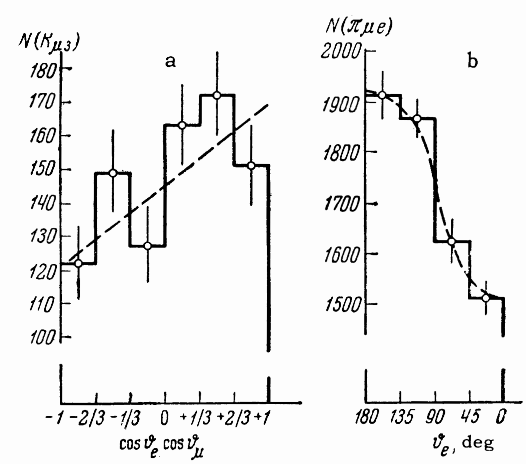

FIG. 4. Angular distributions of positrons: a – $K^+_{\mu 3} \to \mu^+$ decay ($N = 887$, $a = +0.17 \pm 0.07$, $P = +0.68 \pm 0.28$); b – $\pi^+ \to \mu^+$ decay ($N = 6971$, $a = -0.235 \pm 0.035$, $P = -0.94 \pm 0.14$).

decay, averaged over $\cos\vartheta_\mu$:

$$dN \sim 1 + a^* \cos\vartheta_e,$$

where

$$a^* = a\overline{\cos\vartheta_\mu}.$$

This distribution, as we indicated above, is in excellent agreement with the value $a = -0.25 \pm 0.02$ obtained by us[7] for $\pi^+ \to \mu^+ \to e^+$ decays in a field of 6 kG.

It is known that the polarization of muons in $\pi^+ \to \mu^+ \to e^+$ decay is close to 100%. Therefore the following polarization values follow from our data:

1) for muons from π^+ decay

$$P = -0.235/0.25 = -0.94 \pm 0.14;$$

2) for muons from $K^+_{\mu 3}$ decay

$$P = 0.17/0.25 = +0.68 \pm 0.28.$$

Measurements of muon polarization in $K^+_{\mu 3}$ decay have also been carried out by Gidal et al.[9] using a 30-inch Freon bubble chamber in a magnetic field. These workers obtained the following value of muon polarization for an energy interval close to ours:

$$P = +0.74 \pm 0.16.$$

The agreement of the results permits us to combine the two values of polarization. This gives a value

$$P = +0.72 \pm 0.14,$$

which we will use in the subsequent discussion.

In Fig. 5 the parabola shows the theoretical dependence of the quantity R on the parameter ξ (see Eq. (2)). The second curve in this figure gives the dependence of the polarization on the parameter ξ for our muon energy interval.[4]

We see that the polarization value

$$P = +0.72 \pm 0.14$$

corresponds to two possible ranges of the parameter ξ:

$$\xi = 0 \pm 0.6, \quad \xi = -3.8 \pm 0.6.$$

From the same graph it follows that the experimental value $R = 0.75 \pm 0.07$[3] also corresponds to two possible ranges:

$$\xi = +0.7 \pm 0.5, \qquad \xi = -7.3 \pm 0.5.$$

As we have shown above, the shape of the muon spectrum in $K^+_{\mu 3}$ decay also depends on the parameter ξ. This spectrum has been measured by the Turin group,[10] who used two bubble chambers. In one of these, filled with hydrogen, they measured the range of muons with energies less than 35 MeV, and the other, a Freon chamber, served for measurement of the spectrum in the high-energy region. The result obtained by the Turin group is that the shape of the muon spectrum agrees both with small negative values of ξ lying in the region from 0 to -2 and with large positive values of the order of $+10$. Giacomelli et al.[3] have measured the muon spectrum from $K^+_{\mu 3}$ decay in the low-energy region (up to 28 MeV), where the shape of the spectrum is most sensitive to the value of the parameter ξ. Giacomelli et al. determined the muon energy from the range in nuclear emulsion. The measured spectrum is in good agreement with a small positive value $\xi = +0.7$ and contradicts the large negative value $\xi = -7.3$. Thus, measurements of the spectrum shape, considered together with measurements of polarization and of the quantity

$$R = W(K_{\mu 3}) / W(K_{e3})$$

argue in favor of a value of ξ close to zero.

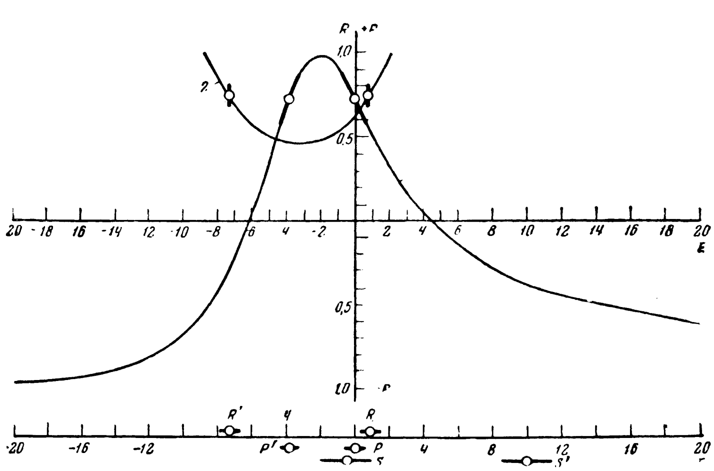

FIG. 5. Dependence of the polarization of μ^+ mesons from $K^+_{\mu 3}$ decay (curve 1) and the ratio of the decay probabilities $K = W(K_{\mu 3})/W(K_{e3})$ (curve 2), as a function of the parameter ξ.

All of these regions of possible values are shown in the lower part of Fig. 5 (R represents measurements of the ratio $W(K_{\mu 3})/W(K_{e3})$, P —the measured polarization, S —the spectrum), and thus we see that only small values of the parameter ξ, close to zero, agree with the data on the polarization, the spectrum, and the decay probability.

We note that if the value of ξ were exactly zero, this would mean that in the matrix element for process (1) the term with f_- is absent and the matrix element has the same form for the decay $\pi \rightarrow \pi^0 + e + \nu$ as for $K_{\mu 3}$ decay. This result would indicate the validity of applying the unitary symmetry scheme for strong interactions to the decay of mesons.[11]

The authors take pleasure in thanking A. I. Alikhanov for his interest in their work, L. B. Okun', P. Yu. Kobzarev, and N. I. Kostanashvili for discussion of the problem, D. M. Samoĭlovich, in whose laboratory the emulsion chambers were developed, and our co-workers of the scanning group.

[1] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957).

[2] Brene, Egardt, and Qvist, Nucl. Phys. **22**, 553 (1961).

[3] Giacomelli, Monti, Quareni, Quareni-Vignudelli, Püschel, and Tietge, Nuovo cimento **34**, 1134 (1964).

[4] S. G. Matinyan and L. B. Okun', JETP **36**, 1317 (1959), Soviet Phys. JETP **9**, 933 (1959).

[5] V. A. Smirnitski and A. O. Weissenberg, Phys. Rev. Lett. **12**, 233 (1964).

[6] Coombes, Cork, Galbraith, Lambertson, and Wenzel, Phys. Rev. **108**, 1348 (1957).

[7] A. O. Vaĭsenberg, Myu-mezon (The Mu Meson), Nauka, 1964, p. 346.

[8] A. O. Vaĭsenberg, ibid. p. 334.

[9] Gidal, Powell, March, and Natali, Phys. Rev. Lett. **13**, 95 (1964).

[10] V. Bisi et al., Phys. Rev. Lett. **12**, 490 (1964).

[11] I. Yu. Kobzarev and L. B. Okun', JETP **42**, 1400 (1962), Soviet Phys. JETP **15**, 970 (1962).

Translated by C. S. Robinson
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