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ANISOTROPY OF GALVANOMAGNETIC PROPERTIES OF PURE ALUMINUM IN LARGE EFFECTIVE FIELDS

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We investigate the galvanomagnetic properties of pure aluminum ($R_{273}/R_{4.2}$ = 6400—20,000). We find that the anisotropy of the resistance increases in large fields. The resistance of aluminum of such purity increases with increasing magnetic field for all the investigated directions of the magnetic field. It is assumed that the increase in the anisotropy and the growth of the resistance in large fields can be attributed to the existence of a narrow layer of open trajectories.

INVESTIGATIONS of the galvanomagnetic properties of aluminum, carried out on aluminum samples with $R_{273}/R_{4.2} \leq 2000$,[1,2] have shown that the anisotropy of the resistance does not increase in a magnetic field. The dependence of the resistance on the magnetic field for different field directions relative to the crystallographic axes is the same, and resembles a saturation curve. The Hall field is isotropic. These results do not contradict the Harrison model, according to which the Fermi surface of aluminum in the second zone, responsible for the main features of the galvanomagnetic properties, is closed. However, it was noted even earlier[2] that for one of the field directions the resistance increases with increasing magnetic field. Investigations of the dependence of the resistance on the magnetic field in single-crystal samples of purer aluminum ($R_{273}/R_{4.2}$ = 2000—6500), carried out by Balcombe,[3] have shown that the resistance does increase with increasing magnetic field, although only insignificantly. The anisotropy in these samples was equally small,—30—40%. The measurements were made at T = 4.2°K. Balcombe notes that such a behavior of the resistance in a magnetic field can be explained by assuming that the Fermi surface in the second zone touches the corners of the Brillouin zone, and this leads to the appearance of open trajectories.

In this paper we present the results of the investigation of the galvanomagnetic properties of much purer aluminum than in earlier work ($R_{273}/R_{4.2}$ = 6500—20,000) at T = 4.2°K. The results of measurements made with such aluminum at T = 20.4°K, which have shown an anomalous behavior as compared with more highly contaminated aluminum (large increase in the resistance and violation of the Köhler rule), were reported earlier.[4]

MEASUREMENT RESULTS

The samples were prepared from type AV-0000 aluminum, which has been subjected to zone purification[1)]. The samples cut from the ingot were deep-etched to remove the damaged surface layer, after which they were annealed at T = 500°C for 8—10 hours. Some samples were electrically polished to improve their surface. The orientation of the samples was determined optically with a goniometer. For some samples, for control purposes, x-ray pictures were taken, which confirmed the results of the optical measurements.

We investigated several dozen samples, the characteristics of some of which are listed in the Table. The resistance below T = 4.2°K remained practically unchanged, obviously because of the high Debye temperature of aluminum (θ = 395°K). We could therefore regard $R_{4.2}$ as the residual resistance, characterizing the purity of the sample. During the time of all the measurements, the magnetic field was perpendicular to the current flowing along the sample axis. In plotting the rotation diagrams, the angle φ between the magnetic field and some fixed direction in a plane perpendicular to the current was varied. The measurements were made in fields up to 35 kOe, using an ordinary potentiometer circuit, the output of which

[1)]The authors take the opportunity to thank B. N. Aleksandrov for graciously furnishing the high-purity aluminum.

Characteristics of the investigated samples

No. of sample	Size, mm	$\frac{R_{273}}{R_{4.2}}\cdot 10^{-3}$	$\frac{R_{273}}{R_{20.4}}\cdot 10^{-3}$	Orientation of sample axes
Al — 10	36×1.92×0.6	18.0	—	$\theta = 39°,\ \psi = 3°$
Al — 12	33×2.1×0.6	12.0	2.64	$\theta = 57°,\ \psi = 44°$
Al — 13	∅ = 2	15.5	2.97	$\theta = 42°,\ \psi = 40°$
Al — 13′	2×0.15	8.3	2.3	$\theta = 42°,\ \psi = 40°$
Al — 17	50×1.5×1.5	6.4	2.0	$\theta = 35°,\ \psi = 3°$
Al — 18	50×1.5×1	11.8	2.83	$\theta = 35°,\ \psi = 4°$
Al — 19	45×2×1.5	18.0	—	$\theta = 38°,\ \psi = 41°$
Al — 20	55×2×1.8	22.0	3.33	$\theta = 50°,\ \psi = 17°$
Al — 21	220×1.7×1.7	15.6	2.78	large-grain polycrystal
Al — 22	∅ = 1.35	12.7	2.78	[100] ±5°
Al — 23	80×1.8×1.8	18.5	3.16	[110] ±5°
Al — 28	20×1.8	14.3	2.8	$\theta = 42°,\ \psi = 40°$

was fed to a photo-electro-optical amplifier with sensitivity 4×10^{-9} V/mm.

We investigated the anisotropy of the resistance in the magnetic field, the anisotropy of the Hall field, and the dependence of the resistance on the magnetic field. Figure 1 shows the results of measurements of the anisotropy of the resistance in a magnetic field for single crystals of different purity (Al-10, Al-17, and Al-18), with sample axes close to the [110] direction.

For the less pure sample Al-17 (Fig. 1, curves 1 and 2), the anisotropy of the resistance at 20.4° and 4.2° was low. The deviation of $(\Delta R/R)_{max}$ from $(\Delta R/R)_{min}$ was 30—35%. The symmetry of the rotation diagrams corresponds to a two-fold symmetry axis. For the purer aluminum Al-10, at T = 4.2°K (curve 5 of Fig. 1) the anisotropy increased $(\Delta R_{max}/\Delta R_{min}) = 1.8$. Additional maxima, connected with the [111] direction, appear then on the rotation diagram. A similar behavior was observed at T = 4.2°K also in the resistance of Al-18, which was close to Al-10 in purity and in orientation. Less symmetrical compared with Al-10 was the diagram obtained for Al-18, owing to the fact that the axis of the Al-18 sample was inclined further away from the [110] direction.

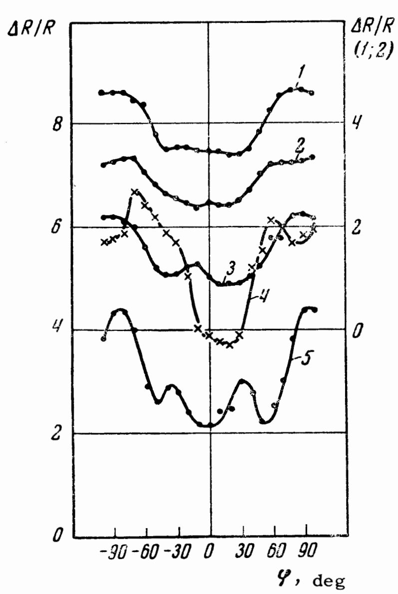

FIG. 1. Anisotropy of the resistance of single-crystal aluminum in a magnetic field. Al-17: curve 1 – T = 20.4°K, curve 2 – T = 4.2°K, H = 35,000 Oe; Al-18: curve 3 – T = 20.4°K, curve 4 – T = 4.2°K, H = 35,000 Oe; Al-10: curve 5 – T = 4.2°K. H = 24,600 Oe.

Figure 2 shows the rotation diagram for the sample Al-12, the axis of which was practically parallel to the [111] direction. As can be seen from the figure, at T = 20.4°K (curve 2), the symmetry of the rotation diagram corresponds approximately to three-fold symmetry. The maxima on the dia-

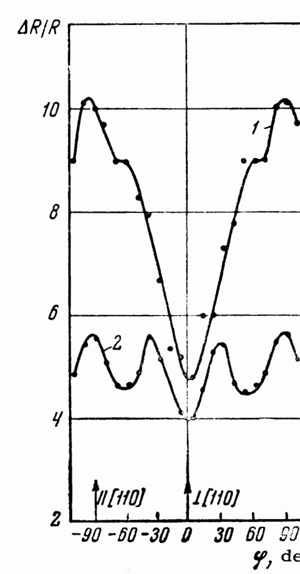

FIG. 2. Anisotropy of resistance of Al-12 in a magnetic field: curve 1 – 4.2°K, curve 2 – 20.4°K, H = 24,600 Oe.

gram correspond to a magnetic field direction H ∥ [110], while the minima correspond to H ⊥ [110]. The lack of perfect symmetry in the diagram is connected with the small deviation of the sample axis from the [111] direction. On going to T = 4.2°K, one minimum remains on the diagram, corresponding to H ⊥ [110], and one maximum corresponding to H ∥ [110]; only weak traces are seen at the locations of the other minima. A similar change in the form of the rotation diagram was observed also for Al-13 and Al-28. However, owing to a large inclination of the sample axis to the [111] direction, the form of the diagrams for these samples changed more noticeably at T = 4.2°K. The vanishing of some minima and maxima and the appearance or the deepening of others, at T = 4.2°K, were observed also for samples with intermediate orientation (for example, Al-19, Al-20 and others).

The anisotropy which we observed at T = 4.2°K is larger than that observed by Balcombe,[3] obviously because of the higher purity of the samples (by an approximate factor of 2—3). The directions of the fields at the minima and maxima of the rotation diagrams, observed by us at T = 20.4°K, coincide with the observations of Balcombe[3] on samples of similar orientation. He did not carry out measurements at 20.4°K; however, in samples whose purity was similar to that of the samples used in [3], the changes which we observed in the form of the diagrams will obviously not occur. Such a conclusion can be drawn by comparing the results for Al-10 and Al-17 (Fig. 1); Al-17 is close in purity to the samples from [3]. The change in the form of the rotation diagram is observed in the purer aluminum.

It seems surprising at first glance that, for example, on the rotation diagram of a sample along which a three-fold crystallographic [111] symmetry axis is directed (Fig. 2), one observes two-fold symmetry. However, a sharp change in the form of the rotation diagram of the resistance in large effective fields has already been reported in the literature for a metal with large anisotropy (Zn). When the sample axis is inclined away from the six-fold axis by merely 5°, the diagram hardly shows six-fold symmetry, [5] which appears only with Zn samples whose axes are inclined less than 2° to the six-fold axis (the measurements were made by us). It is quite possible that for aluminum, in which a large anisotropy of resistance is observed at T = 4.2°K, even a small tilting leads to an asymmetrical picture in which some directions predominate, as is indeed observed by us in very pure samples.

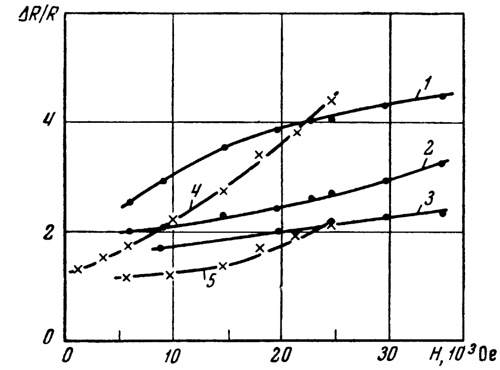

FIG. 3. Dependence of the resistance on the magnetic field. Al-17: curve 1 – T = 20.4°K, φ = 90°, curve 2 – T = 4.2°K, φ = 90°, curve 3 – T = 4.2°K, φ = 0°; Al-10: curve 4 – φ = 90°, curve 5 – φ = 0°, T = 4.2°K.

We measured the dependence of the resistance on the magnetic field in the directions of the minima and maxima of the rotation diagram in all the investigated samples. At T = 20.4°K, in all the samples, the resistance vs field curves tend to saturation. However, no saturation is reached even at maximum fields (see curve 1 of Fig. 3), in spite of the fact that for samples of such purity, large effective fields, $H/H_0 = 40$, were attained even at T = 20.4°K (H_0 is the field in which the mean free path l is equal to the Larmor radius r[6]).

Measurements of the dependence of the resistance on the magnetic field at T = 4.2°K have shown that the resistance increases with increasing magnetic field in almost all directions. In the samples Al-10 and Al-17, (Fig. 3, curves 2—5) the resistance increases both in the maximum at H ∥ [100] and in the minimum H ⊥ [100], up to the maximum fields at which the measurements were made. In the sample Al-12 (Fig. 4), the resistance

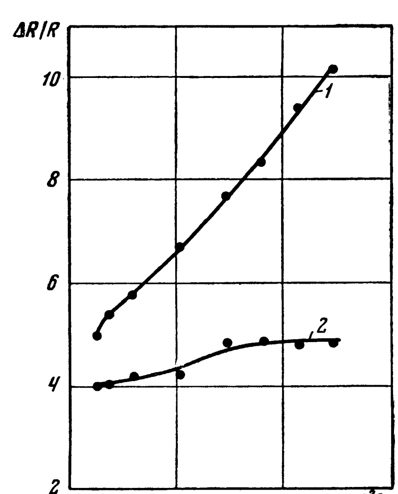

FIG. 4. Dependence of the resistance on the magnetic field in Al-12: curve 1 – φ = 90°, curve 2 – φ = 0°, T = 4.2°K.

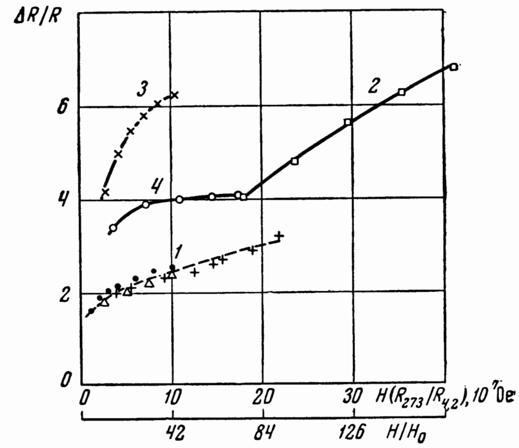

FIG. 5. Dependence of the resistance on the magnetic field. Al-18: curve 2 – T = 4.2°K, curve 3 – T = 20.4°K; Al-23: curve 4 – T = 14°K; curve 1 shows the results of [3] at T = 4.2°K; △ – sample 3, ● – sample 5, + – Al-17; H_0 – field in which l = r. The curves are shown for the field direction H || [100].

at the maximum for H ∥ [110] increases much more rapidly than in the direction of the minimum, corresponding to H ⊥ [110]. A similar behavior is typical also of the other samples with similar orientations (Al-18 with the sample axis close to [110], samples Al-13 and Al-28 with axes close to [111]), and also for samples with intermediate orientations.

It is of interest to compare the results obtained by us for the samples Al-17 and Al-18 with the results for sample No. 5 of [3]. These samples are close in orientation (sample 5: $\psi = 0°$ and $\theta = 44°$; Al-17: $\psi = 3°$ and $\theta = 35°$; Al-18: $\psi = 4°$ and $\theta = 35°$). The sample Al-17 is close in its purity to sample 5 of [3]; Al-18 is approximately twice as pure as Al-17 or sample 5. As already mentioned, in the purer sample Al-18 a larger anisotropy is observed at T = 4.2°K, and the resistance rotation diagram is more complicated than the diagram at T = 20.4°K. This is barely noticeable in the more contaminated Al-17.

Figure 5 shows the dependence of the resistance on the magnetic field in Köhler coordinates [7] for these samples. It is seen from the figure that for the samples of like purity Al-17 and sample 5 (curve 1), the Köhler rule is fairly well satisfied. This agrees with the observations of Balcombe [3] and our observations made on samples of similar and lower purity ($R_{273}/R_{4.2} \le 6500$). For the purer sample Al-18 (Fig. 5, curve) with the same orientation, Köhler's rule is not satisfied. This offers evidence of the fact that the observed anomalies (increasing resistance, larger anisotropy) are directly connected with the purity of the samples.

Investigations of the Hall effect were made on samples Al-12, Al-22, and Al-23. The measurements have shown the absence of Hall-field anisotropy, with a measurement accuracy ±2%. The dependence of the Hall field on the magnetic field is linear. The value of the Hall constant R_∞ (R_∞ —in large effective fields) at temperatures 20.4, 14, and 4.2°K is the same, namely $R_\infty = 10.5 \pm 0.5 \times 10^{-4}$ cgs emu, and coincides within the limits of experimental accuracy with the value $R_\infty = 10.9 \pm 0.2 \times 10^{-4}$ cgs emu, obtained earlier.[2] An interesting feature is the isotropy of the properties of the Hall field even in those pure samples, in which large resistance anisotropy was observed at T = 4.2°K.

INFLUENCE OF SIDE EFFECTS ON THE MEASUREMENT RESULTS

One might assume that the increase in the resistance and the large anisotropy of pure aluminum are not true properties of the metal, but are connected with some of the following side effects: 1) bending of the current lines due to the finite dimensions of the sample, similar to the effect observed by Alekseevskiĭ and co-workers[8]; 2) the fact that the dimensions of the samples are comparable with the mean free path at T = 4.2°K and that the symmetry of the shape effect is superimposed on the crystallographic symmetry[9]; 3) the appearance of the static skin effect predicted by Azbel'.[10]

To check on the influence of these side effects, the following experiments were performed. To exclude the influence of the bending of the curve lines, measurements were made of the resistance anisotropy and the dependence of the resistance on the magnetic field in sample Al-21, the length of which was 5–6 times larger than the lengths of the usually employed samples. Two pairs of current leads were soldered to the sample—one pair on the ends and the other on the side, at a distance of one quarter of the length of the sample from its ends. The current was made to flow alternately through each pair of leads. The values of the resistance obtained in both cases were the same. The measurements in a magnetic field have shown that this sample exhibits an increase in resistance with a field at T = 4.2°K, in spite of all precautions.

To ascertain the influence of the shape effect, measurements were made with a single-crystal sample in the form of a thick rod going over into a plate, similar to the procedure used by one of the authors and Lazarev.[9] The thick cylindrical part of the sample had a diameter of 2 mm, the plate cross section was 2×0.15 mm, i.e., approximately 5—10 times thinner than all the samples used by us. On the thick part of the sample $R_{273}/R_{4.2} = 15{,}500$, and on the thin part $R_{273}/R_{4.2} = 8300$. Such a difference in the values of the residual resistance can be attributed to the fact that the smallest dimension of the plate was smaller than the mean free path, and this, as is well known, leads to an increase in the resistance as compared with a bulk sample. Judging from the value of the residual resistance of the cylindrical part of the sample, the mean free path at $T = 4.2°K$ is $l = 0.3—0.4$ mm. In calculating the mean free path we used data of Aleksandrov[11] and Fossheim and Olsen[12]. If we use the calculations of Dingle,[13] then for a plate with this mean free path $\rho_\infty/\rho \sim 0.6$, which agrees with the values of the resistances of the thin and thick parts of the sample, which were obtained by us (ρ_∞ —specific resistance of the bulk sample).

Measurements of the anisotropy of the resistance at $T = 20.4°K$ yielded the same rotation diagram for both parts of the sample. At $T = 4.2°K$, the anisotropy of the resistance of the plate does not increase, while that of the thick part of the sample increases with increasing magnetic field, and the diagram becomes more complicated compared with the diagram obtained at $T = 20.4°K$. Thus, the shape effect, which should be most clearly pronounced in the thin part of the sample, cannot be the cause of the large anisotropy of the resistance, observed at $T = 4.2°K$ in pure samples. The dimensions are more likely to cause a decrease of the effect.

Relatively recently Azbel' and Peschanskiĭ[10] have shown that an account of the boundaries of the sample, and consequently the state of its surface, can complicate the $\rho(H)$ dependence. In particular, when $r \ll l^2/d$, where d is of the order of the wire thickness, the resistance depends linearly on the magnetic field. In the case of a rough surface, we have observed on some samples an influence of the arrangement of the contacts used to measure the potential difference on the value of the resistance. In samples with good surface, this effect was small. This may be connected with the manifestation of the Azbel' effect,[10] but we did not investigate this phenomenon in detail. We used for the measurement samples in which a change in the arrangement of the contacts caused only a small change (several percent) in the value of the resistance in a magnetic field.

Thus, the large anisotropy observed in very pure aluminum and the increase of the resistance with increasing magnetic field cannot be attributed to any of the foregoing side effects.

CONCLUSION

The main new fact observed in this investigation is the presence in pure aluminum of large resistance anisotropy in a magnetic field, and the absence of saturation of resistance for most directions of the magnetic field. For some orientations, we even observed a difference in the behavior of the resistance at the maximum and at the minimum of the diagram (see Fig. 4). As already mentioned, such a behavior of the resistance, but in weaker form (slight increase of the resistance in the field, small anisotropy) were observed earlier.[2,3] It was pointed out in [3] that the growth of the resistance could be explained by assuming that the Fermi surface of aluminum in the second zone touches the corners of the Brillouin zone. This leads to the appearance of open and strongly elongated trajectories. If such is the case, then the fact that a similar growth is observed only in very pure aluminum can be explained by assuming that these bridges are very narrow. With such an assumption, the dependence of the resistance on the magnetic field in large effective fields will have in the direction of the open trajectories the form

$$\rho_{xx} = b_{xx}'(1 + \nu H^2/H_0^2),$$

where $\nu \ll 1$,[14] i.e., in fields $H_0 < H < H_0/\nu^{1/2}$ the openness of the trajectories will not come into play and $\rho(H)$ will behave as in the case of a closed surface. This is observed for aluminum with $R_{273}/R_{4.2} \leq 2000$ at $H \sim 30 H_0$. The existence of a narrow layer of open trajectories is manifest only in fields $H \gg H_0/\nu^{1/2}$. The small anisotropy of the Hall constant can be explained by the fact that in the large fields a narrow layer of open trajectories makes a different contribution to the dependence of the resistance and the Hall constant. In the expression for the Hall constant, the value of ν is not multiplied by the large factor $(H/H_0)^2$. And although the resistance is influenced by the existence of narrow bridges on the Fermi surface, this cannot affect the Hall field.

The fact that the Hall constant is isotropic in the pure aluminum investigated by us makes the use of its value for the determination of the carrier density valid. We have obtained a value $n/N_a = (0.96 \pm 2)\%$ (n —the difference in the densities of the holes and the electrons; N_a —number of atoms per unit volume). Chambers and Jones[15] obtained $n/N_a = (0.95 \pm 0.5)\%$. According to the Harrison model, the Fermi surface of aluminum in the second band is of the hole type, which is confirmed by the positive sign of the Hall constant, while in the third band it is of the electron type. The first band is completely filled. If the surface is of the electron type in the third band, then there should be more than one hole in the second band, and the difference in the densities n should equal unity. From our measurements and the measurements of Chambers and Jones[15] it follows that the difference in the concentrations of the holes and of the electrons is smaller than unity, and we cannot attribute this to experimental error.

We can thus state that the results obtained by us in the investigation of pure aluminum at $T = 4.2°K$ do not contradict the hypothesis that narrow bridges exist on the Fermi surface in the second band. Such an assumption is likewise not in disagreement with the results of Bezuglyĭ et al. and the results of Kamm and Bohm,[16] since they do not give exact data on the shape of the surface near the corners, owing to the low measurement accuracy in these directions, and the bulk of the surface remains the same as before under such assumptions. We note that similar effects can result also from magnetic breakdown,[17] but our experiments offer no choice between these two possibilities.

We emphasize once more that we are unable to explain the fact that the increase of the resistance in the magnetic field and the increase of its anisotropy are observed only at $T = 4.2°K$. In the same effective fields, but at values of T equal to 20.4 and 14°K, the anisotropy is small, and the dependence of the resistance on the field resembles a curve that tends to saturation. By way of illustration, Fig. 5 shows the dependence of the resistance of the magnetic field, in Köhler coordinates, for the samples Al-17, Al-18, and Al-23 as well as samples 3 and 5 from Balcombe's paper.[3] To illustrate the magnitude of the effective magnetic field, we show the values of H/H_0. What remains unexplained within the framework of the Harrison model is the fact that the difference in the hole and electron densities is smaller than unity. Thus, for a complete clarification of the properties of the electrons in the main band of aluminum it becomes necessary to carry out further research.

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223