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THE MAGNETOELECTRIC EFFECT

V. G. SHAVROV

Institute of Metal Physics, Academy of Sciences, USSR

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A study is made of the magnetoelectric effect in antiferromagnets and ferrimagnets with two sublattices. Conclusions are drawn about the possibility of the existence of this effect in all possible antiferromagnetic structures. Formulas are derived for the energy spectrum of the spin waves in rhombohedral antiferromagnets of the Cr_2O_3 and Ti_2O_3 type; account is taken of the magnetoelectric energy, and in the latter case also of magnetoelastic energy. A discussion is given of the influence of the electric field on the antiferromagnetic resonance frequency in the "magnetoelectrics" under consideration.

1. INTRODUCTION

We consider an antiferromagnet with a collinear or slightly noncollinear ground-state structure, which can be described by two magnetic sublattices with local magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$. We introduce the total magnetic-moment density vector $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$ and the antiferromagnetism vector $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$; these are connected through the conditions $\mathbf{ML} = 0$ and $M^2 + L^2 = L_0^2$, where $L_0 = 2M_0$ is the maximum value of the modulus of $\mathbf{L}$. Using the concept of the parity of an antiferromagnetic structure (a. s.), introduced by Turov[1], we introduce the following classification of antiferromagnets.[1] First we separate them into two groups on the basis of whether the a. s. is even or odd with respect to all crystallochemical translations T of the lattice. In an a. s. of the first group, there can exist various effects (static and kinetic) caused by the presence of a preferred axis of antiferromagnetism. Hereafter we shall consider only such a. s. We subdivide these, in turn, into two subgroups: into structures even or odd with respect to a center of inversion I. We shall enumerate effects, related to antiferromagnetic ordering, which can exist in an a. s. of one or another of the indicated subgroups.

A. For a. s. even with respect to a center of inversion:

1) Weak ferromagnetism[2,1], determined by the relation

$$M_\alpha = D_{\alpha\beta} L_\beta. \tag{1.1}$$

2) Piezomagnetism[3,4],

$$M_\alpha = \Pi_{\alpha\beta\gamma\delta} L_\beta \sigma_{\gamma\delta} \tag{1.2}$$

where σ is the elastic-stress tensor.

3) The even magnetoelectric effect[4],

$$M_\alpha = B_{\alpha\beta\gamma\delta} L_\beta E_\gamma E_\delta \tag{1.3}$$

where $\mathbf{E}$ is the electric field.

4) Certain kinetic (galvano- and thermomagnetic) effects, which can be derived from the general relation between fluxes $\mathbf{j}$ and forces $\mathbf{F}$, namely

$$F_\alpha = \rho_{\alpha\beta}(\mathbf{H}, \mathbf{M}, \mathbf{L}) j_\beta, \tag{1.4}$$

by separating out the terms linear in $\mathbf{L}$ in the expansion of ρ in components of the vectors $\mathbf{H}$, $\mathbf{M}$, and $\mathbf{L}$.[5]

B. For a. s. odd with respect to a center of inversion:

1) The magnetoelectric effect[6-11],

$$M_\alpha = G_{\alpha\beta\gamma} L_\beta E_\gamma. \tag{1.5}$$

2) The piezomagnetoelectric effect[10],

$$M_\alpha = \pi_{\alpha\beta\gamma\delta\varepsilon} L_\beta \sigma_{\gamma\delta} E_\varepsilon. \tag{1.6}$$

3) The gyroelectric effect (rotation of the specimen with angular velocity Ω in an external electric field[10]),

$$\Omega_\alpha = C_{\alpha\beta\gamma} L_\beta E_\gamma. \tag{1.7}$$

In the presence of a center of symmetry in the crystal, effects of types A and B are mutually exclusive. But if there is no center of symmetry, then in the given a. s. effects of both types may exist simultaneously.

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[1] An a. s. is called even or odd with respect to a symmetry element n according as the operator n connects sites with parallel or antiparallel spins. The parity is recorded by indices plus or minus, respectively, on n (n^+ or n^-).

The present work treats the magnetoelectric effect (m. e.) in antiferromagnetic crystals with two magnetic sublattices. It investigates the possibility of existence of the effect in various types of a. s. For the case of the rhombohedral antiferromagnets Cr_2O_3 and Ti_2O_3, the spin-wave spectrum is calculated; account is taken of the magnetoelectric energy, and the influence of an electric field on the antiferromagnetic resonance frequency is considered.

2. SYMMETRY OF THE MAGNETOELECTRIC COEFFICIENTS

The magnetoelectric coefficients are determined either by formula (1.5) or by the following expression for the thermodynamic potential responsible for the m. e.:

$$\Phi = -\gamma_{\alpha\beta\gamma} M_\alpha L_\beta P_\gamma, \tag{2.1}$$

where **P** is the electric polarization vector. On the basis of the transformation properties of the parameters that occur in these formulas, it is easy to derive the explicit form of the coefficients γ (or G) for all possible a. s. We shall not give the corresponding table of all magnetoelectric tensors (it will be published later). We shall merely carry out certain deductions from the results thus obtained.

If a center of inversion is present in the crystal, the m. e. can exist in antiferromagnets of an arbitrary crystallographic class, without exception, if their structure is odd with respect to this center of inversion (these include a. s. of the triclinic syngony and crystals of the cubic classes T and T_h, in which effects of type A in general cannot occur; see [1,4,5]).

In case a center of symmetry is absent, the m. e. can be observed both in pure antiferromagnets and also in ferrimagnets, that is in substances with crystallographically nonequivalent magnetic sublattices. In the latter, the effect will manifest itself in a change of the already present spontaneous magnetic moment.[2] In the presence of a compensation point in the ferrimagnet, it will be possible to observe a shift of this point in one or the other direction under the influence of an electric field. In pure antiferromagnets, in this case, the m. e. can be observed against the background of the weak ferromagnetic moment (1.1). Such a phenomenon was recently observed experimentally by Rado[12] in an orthorhombic compound without a center of symmetry, $Ga_{2-x}Fe_xO_3$ (with x close to unity). The symmetry of this crystal is evidently described by the space group C_{2v}^9; or according to the terminology of Turov[1,13], here the structure $\sigma_x^+ 2_y^- \sigma_z^-$ occurs, with $\mathbf{L} \parallel \mathbf{y}$.[3]

The m. e. was first observed by Astrov[7] in the antiferromagnet Cr_2O_3, which has the structure $3_z^+ 2_x^- I^-$ and is in the state with $\mathbf{L} \parallel \mathbf{z}$. Then this effect was observed by Al'shin and Astrov[11] in titanium oxide, Ti_2O_3. Crystallographically this compound is isomorphic with chromic oxide. From this it follows that Ti_2O_3 has either the structure (a) $3_z^+ 2_x^- I^-$ or the structure (b) $3_z^+ 2_x^+ I^-$. In [11], on the basis of measurements of the m. e., the conclusion is drawn that the vector **L** more probably lies in the basal plane than along the trigonal axis. The paper of Al'shin and Astrov [11] was followed soon by the work of Abrahams[14], in which, on the basis of neutron diffraction studies, the conclusion is drawn that the a. s. of titanium oxide is like that of $\alpha\,Fe_2O_3$ in the temperature range 250° K < T < 950° K; that is, the structure present is $3_z^+ 2_x^- I^+$, $\mathbf{L} \perp \mathbf{z}$. This is in direct contradiction with the results of [11], since in such an a. s. (even with respect to I) the m. e. cannot exist. In this connection, in our judgment, there is a need for verification of the results of both researches.

We shall give the formulas that determine the m. e. for substances in which structure (a) or (b) occurs and which are in the state with $\mathbf{L} \perp \mathbf{z}$.[4]

Structure $3_z^+ 2_x^- I^-$, state $\mathbf{E} \perp \mathbf{z}$. The longitudinal and transverse components of the magnetic moment with respect to the direction of **E** (the longitudinal and transverse effects) have here the following form:

$$M_\parallel = G_1 LE \sin(3\varphi_E + \varphi_{EL}),$$
$$M_\perp^{(xy)} = G_1 LE \cos(3\varphi_E + \varphi_{EL}), \quad M_\perp^{(z)} = G_2 LE \cos\varphi_{EL}. \tag{2.2}$$

Structure $3_z^+ 2_x^- I^-$, state $\mathbf{E} \parallel \mathbf{z}$:

$$M_\parallel = 0, \qquad M_\perp^{(xy)} = G_3 LE \cos\varphi_{ML}. \tag{2.3}$$

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[2] We remark that the m. e. can be detected also simply in a ferromagnet in which there is no center of symmetry. The change of magnetization ΔM under the influence of an electric field will in this case be determined by

$$\Delta M_\alpha = G'_{\alpha\beta\gamma} M_\beta E_\gamma.$$

[3] Whereas in the consideration of effects of type A the symbol $2^\pm$ denotes both an axis of second order and a plane of symmetry $\sigma^\pm$, in the case of effects of type B the symbol $\sigma^\pm$ corresponds to the symbol $2^\mp$.

[4] The structures (a) and (b) considered correspond to the structures (c) and (b) considered by Abrahams (see[14], Fig. 3).

Structure $3_z^+2_x^+I^-$, state $\mathbf{E} \perp \mathbf{z}$:

$$M_{\|} = -G_1'LE\cos(3\varphi_E + \varphi_{EL}),$$
$$M_{\perp}^{(xy)} = G_1'LE\sin(3\varphi_E + \varphi_{EL}), \quad M_{\perp}^{(z)} = G_3'LE\sin\varphi_{EL}. \tag{2.4}$$

Structure $3_z^+2_x^+I^-$, state $\mathbf{E} \parallel \mathbf{z}$:

$$M_{\|} = 0, \qquad M_{\perp}^{(xy)} = G_2'LE\sin\varphi_{ML}. \tag{2.5}$$

Here φ_E determines the direction of **E** with respect to **x** in the basal plane, and φ_{mn} is the angle between the vectors **m** and **n**. The index (xy) denotes the plane in which the given vector lies.

If in Cr_2O_3, independently of the orientation of the electric field, only the longitudinal effect with respect to the direction of **E** is observable ($M_\perp = 0$) (see also[7]), here the picture is different. On the contrary, when $\mathbf{E} \parallel \mathbf{z}$, $M_\perp \neq 0$ but $M_{\|} = 0$; when $\mathbf{E} \perp \mathbf{z}$, both the longitudinal and the transverse effects can occur. Formulas (2.2), (2.3) and (2.4), (2.5) involve different angular dependences of the effect at corresponding directions. This, in turn, gives a possibility of determining from the m. e. (if one already knows the direction of the twofold axis) which of the two indicated structures, (a) or (b), is realized in the given rhombohedral antiferromagnet; that is, of determining the parity of the a. s. with respect to the second-order axis.

3. SPIN-WAVE SPECTRUM OF "MAGNETOELECTRICS." RESONANCE FREQUENCIES.

We consider in more detail the m. e. in rhombohedral antiferromagnets with the structure $3_z^+2_x^-I^-$. The energy density in this case can be expressed as

$$\mathcal{H} = \mathcal{H}_{\mathrm{m}} + \mathcal{H}_{\mathrm{e}} + \mathcal{H}_{\mathrm{me}} \tag{3.1}$$

The magnetic energy is

$$\mathcal{H}_{\mathrm{m}} = \frac{1}{2}A\mathbf{M}^2 + \frac{1}{2}bL_z^2 - \mathbf{MH} + \frac{1}{2}A_{\beta\gamma}\frac{\partial M_\alpha}{\partial r_\beta}\frac{\partial M_\alpha}{\partial r_\gamma} + \frac{1}{2}B_{\beta\gamma}\frac{\partial L_\alpha}{\partial r_\beta}\frac{\partial L_\alpha}{\partial r_\gamma} + p_{\alpha\beta\gamma}M_\alpha\frac{\partial L_\beta}{\partial r_\gamma} + q_{\alpha\beta\gamma}L_\alpha\frac{\partial M_\beta}{\partial r_\gamma}, \tag{3.2}$$

where A and b are the parameters of exchange interaction and of magnetic anisotropy. The last two inhomogeneous terms can occur only in the case of substances in which effects of type B are possible.

The electric energy is

$$\mathcal{H}_{\mathrm{e}} = \frac{1}{2}\varkappa_1(P_x^2 + P_y^2) + \frac{1}{2}\varkappa_2P_z^2 - \mathbf{PE}, \tag{3.3}$$

where κ_1 and κ_2 are the components of the inverse dielectric susceptibility tensor.

The magnetoelectric energy will be defined here by five constants γ_i:

$$\mathcal{H}_{\mathrm{me}} = -\{\gamma_1[(M_xL_y + M_yL_x)P_x + (M_xL_x - M_yL_y)P_y] + \gamma_2M_z(L_xP_x + L_yP_y) + \gamma_3(M_xP_x + M_yP_y)L_z + \gamma_4(M_xL_x + M_yL_y)P_z + \gamma_5M_zL_zP_z\}. \tag{3.4}$$

We investigate two cases, when $\mathbf{L} \parallel \mathbf{z}$ and when $\mathbf{L} \perp \mathbf{z}$.

1. For the state with $\mathbf{L} \parallel \mathbf{z}$, which is realized in Cr_2O_3, we get the following equilibrium values for the magnetization and the electric polarization:

$$M_x^{(0)} = \frac{\varkappa_1H_x - \gamma_3L_0E_x}{A\varkappa_1 - \gamma_3^2L_0^2}, \qquad M_y^{(0)} = \frac{\varkappa_1H_y - \gamma_3L_0E_y}{A\varkappa_1 - \gamma_3^2L_0^2},$$
$$M_z^{(0)} = 0, \quad P_x^{(0)} = \frac{AE_x - \gamma_3L_0H_x}{A\varkappa_1 - \gamma_3^2L_0^2},$$
$$P_y^{(0)} = \frac{AE_y - \gamma_3L_0H_y}{A\varkappa_1 - \gamma_3^2L_0^2}, \qquad P_z^{(0)} = \frac{E_z}{\varkappa_2}. \tag{3.5}$$

When $\mathbf{E} \parallel \mathbf{z}$ and $\mathbf{H} \parallel \mathbf{z}$, the spin-wave spectrum will have the form

$$\varepsilon_{\mathbf{k}}^{(1,2)} = \mu\{[H_E(H_A + H_{\mathbf{k}}')]^{1/2} \pm (H + H_{\mathrm{me}})\}. \tag{3.6}$$

Here $H_E = AL_0$, $H_A = bL_0$;

$$H_{\mathrm{me}} = aE, \qquad a = (\gamma_5 - \gamma_4)L_0/\varkappa_2, \tag{3.7}$$
$$H_{\mathbf{k}}' = L_0[(B_\perp + f_1^2/A)(k_x^2 + k_y^2) + (B_z - f_2^2/A)k_z^2], \tag{3.8}$$

where $B_\perp = B_{xx} = B_{yy}$, $B_z = B_{zz}$, $f_1 = f_{xxy} = f_{xyx} = f_{yxx} = -f_{yyy}$, $f_2 = f_{xxz} = f_{yyz}$; in the present case, $f_{\alpha\beta\gamma} = p_{\alpha\beta\gamma} - q_{\alpha\beta\gamma}$.

As we see, calculation of the last two terms in (3.2) leads to certain additions to the coefficients $B_{\alpha\beta}$ in (3.8). Here the conditions $B_\perp + f_1^2/A > 0$ and $B_z - f_2^2/A > 0$ must be satisfied, since otherwise the state with a uniform distribution of **M** and **L** proves to be unstable.

From the expression for the thermodynamic potential,

$$\Omega = kT\sum_{\mathbf{k}j}\ln\left[1 - \exp\left(-\frac{\varepsilon_{\mathbf{k}j}}{kT}\right)\right]$$

it is easy to find the temperature dependence of the m. e. modulus α, defined by the relation $M_z = \alpha E_z$:

$$M_z = -\frac{\partial\Omega}{\partial H} = a\chi_{\|}E_z, \tag{3.9}$$

that is, α depends on T in the same way as does the susceptibility $\chi_{\|}$. A similar result in the molecular-field approximation was obtained by Rado[9,10]. Such a dependence agrees well with experimental data.

2. We consider the state with $\mathbf{L} \perp \mathbf{z}$, which probably occurs in Ti_2O_3 (with the reservations stated above). If in such a substance the anisot-

ropy in the basal plane is small, then, as is known, one of the branches of the spin-wave spectrum lacks in this case the usual gap connected with magnetic anisotropy energy[13], and there occurs only a gap of magnetostrictive origin[15,16]. Therefore in the complete Hamiltonian (3.1) it is necessary to include the energy of magnetostriction,

$$\mathscr{H}_{\text{ms}} = d_{\alpha\beta\gamma\delta} L_\alpha L_\beta u_{\gamma\delta} \tag{3.10}$$

and the energy of the elastic subsystem,

$$\mathscr{H}_{\text{el}} = c_{\alpha\beta\gamma\delta} u_{\alpha\beta} u_{\gamma\delta} - u_{\alpha\beta}\sigma_{\alpha\beta}, \tag{3.11}$$

where $u_{\alpha\beta}$ is the elastic deformation tensor (explicit expressions for these energies are given in the Appendix).

When **H** lies in the basal plane at angle φ to the x axis, minimization of the total energy—which consists of the expressions (3.1), (3.10), and (3.11)—with respect to the parameters M_α, L_α, P_α, and $u_{\alpha\beta}$ leads to the following equilibrium values of these parameters: $\mathbf{M} \parallel \mathbf{H}$, $\mathbf{L} \approx \mathbf{L}_0$, $\mathbf{L} \perp \mathbf{H}$,

$$M = \frac{\varkappa_1 H + \gamma_1 L_0 (E_X \cos 3\varphi - E_Y \sin 3\varphi)}{A\varkappa_1 - \gamma_1^2 L_0^2},$$

$$P_X^{(0)} = \frac{(A\varkappa_1 - \gamma_1^2 L_0^2 \sin^2 3\varphi) E_X - \gamma_1^2 L_0^2 E_Y \sin 3\varphi \cos 3\varphi + \varkappa_1 \gamma_1 L_0 H \cos 3\varphi}{\varkappa_1 (A\varkappa_1 - \gamma_1^2 L_0^2)},$$

$$P_Y^{(0)} = \frac{(A\varkappa_1 - \gamma_1^2 L_0^2 \cos^2 3\varphi) E_Y - \gamma_1^2 L_0^2 E_X \sin 3\varphi \cos 3\varphi - \varkappa_1 \gamma_1 L_0 H \sin 3\varphi}{\varkappa_1 (A\varkappa_1 - \gamma_1^2 L_0^2)}$$

$$P_z^0 = \frac{E_z}{\varkappa_2}. \tag{3.12}$$

Here we have gone over to a new system of coordinates XYz, rotated with respect to the old through an angle φ about the z axis, so that now $\mathbf{H} \parallel \mathbf{X}$.

The values of the equilibrium deformations $u_{\alpha\beta}^{(0)}$ are given in the Appendix. We consider only the diagonal components of the stress tensor with respect to the new system of coordinates. Thus we get the following expressions for the energy of the spin waves:

$$\varepsilon_{\mathrm{k}}^{(1)2} = \mu^2 \left\{ H_E (H_1 + H_{\mathrm{k}}) - \frac{H_E H_2^2}{4H_A} \sin^2 3\varphi + (H + H'_{\text{me}})(H + 4H'_{\text{me}}) \right\}, \tag{3.13}$$

$$\varepsilon_{\mathrm{k}}^{(2)2} = \mu^2 \left\{ H_E (H_A + H_{\mathrm{k}}) \left(1 - \frac{H^2}{H_E^2} \right) + H_E H_3 - H_A H_4 + \frac{H_E H_2^2}{4H_A} \sin^2 3\varphi + H'_{\text{me}} (H'_{\text{me}} + H) \right\}, \tag{3.14}$$

where the effective magnetoelectric field is

$$H'_{\text{me}} = a' (E_X \cos 3\varphi - E_Y \sin 3\varphi), \qquad a' = \frac{\gamma_1 L_0}{\varkappa_1}; \tag{3.15}$$

H_{k} coincides with H'_{k} of (3.8) when $f_i = 0$ (here we have disregarded in (3.2) the terms linear in the gradients). For the values of the magnetostriction fields H_i ($i = 1, 2, 3, 4$), see the Appendix.

Now we can at once write the formulas for the antiferromagnetic resonance frequencies (at $\mathbf{H} = 0$):

$$\omega = \gamma (H_{EA} \pm H_{\text{me}}), \qquad H_{EA} = (H_E H_A)^{1/2} \tag{3.16}$$

in the case $\mathbf{L} \parallel \mathbf{z}$, and

$$\omega_1^2 = \gamma^2 (H_E H_1 + 4H'^2_{\text{me}}) \tag{3.17}$$

in the case $\mathbf{L} \perp \mathbf{z}$ (the low-frequency branch). In (3.17) we have omitted the anisotropic magnetostriction term (determined by the field H_2), since it is small in comparison with the first (isotropic) term[16,17].

We introduce a certain limiting electric field E_l, at which the magnetoelectric terms in (3.16) and (3.17) become comparable with the size of the gap determined by the first term in these expressions. At values close to E_l, the electric field will have an appreciable influence on the antiferromagnetic resonance frequency 5). In the first case, $\mathbf{L} \parallel \mathbf{z}$, this relates to the region of unattainably large fields $E_l^{(\mathrm{I})} = H_{EA}/a \sim 10^8$ v/cm ($H_{EA} \approx 60$ kOe[13], $a \sim 10^{-1}$[7]). In the second case, $\mathbf{L} \perp \mathbf{z}$, $E_l^{(\mathrm{II})} \approx (H_E H_1)^{1/2}/2a'$; that is, $E_l^{(\mathrm{II})}$ may be appreciably smaller than $E_l^{(\mathrm{I})}$, since usually $H_1 \ll H_A$[16,17].

The magnetoelectric part of the gap in (3.17) depends both on the magnitude of the field E and on the angle φ. The specific anisotropy of this

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5) Date, Kanamori, and Tachiki[18], to elucidate the mechanism of the m. e., investigated experimentally the influence of an electric field on the paramagnetic resonance frequency in Cr_2O_3 (see also[19]).

gap, determined by formula (3.15), is also favorable to a separation of it against the background of the magnetoelastic gap. Furthermore, in the experimental observation of the influence of an electric field on the resonance frequency, the magnetostriction gap can first be diminished either by application of external stresses [in definite directions, see (A.3) and (A.5)], or by introduction of definite impurities, which would show an effect similar to that of stresses. Thus in uniaxial antiferromagnets in which the m. e. exists and the axis of antiferromagnetism lies in the basal plane, there is a substantial possibility of observing experimentally the dependence of the antiferromagnetic resonance frequency on the external electric field.

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APPENDIX

The expressions $\mathcal{H}_{\text{ms}}$ and $\mathcal{H}_{\text{el}}$ for the case of a rhombohedral antiferromagnet have the following form:

$$\begin{aligned}\mathcal{H}_{\text{ms}} = {} & (d_1L_x^2 + d_2L_y^2 + d_3L_z^2 + d_5L_yL_z)u_{xx} \\ & + (d_2L_x^2 + d_1L_y^2 + d_3L_z^2 - d_5L_yL_z)u_{yy} \\ & + [d_4(L_x^2 + L_y^2) + d_7L_z^2]u_{zz} + [d_6(L_x^2 - L_y^2) \\ & + d_8L_yL_z]u_{yz} + (2d_6L_xL_y + d_8L_zL_x)u_{zx} \\ & + [2(d_1 - d_2)L_xL_y + 2d_5L_zL_x]u_{xy}, \end{aligned} \tag{A.1}$$

$$\begin{aligned}\mathcal{H}_{\text{el}} = {} & \frac{c_1}{2}(u_{xx}^2 + u_{yy}^2) + c_2u_{xx}u_{yy} + \frac{c_3}{2}u_{zz}^2 \\ & + c_4(u_{xx} + u_{yy})u_{zz} + (c_1 - c_2)u_{xy}^2 + \frac{c_5}{2}(u_{yz}^2 + u_{zx}^2) \\ & + c_6(u_{xx}u_{yz} - u_{yy}u_{yz} + 2u_{xy}u_{zx}) - u_{\alpha\beta}\sigma_{\alpha\beta}. \end{aligned} \tag{A.2}$$

The equilibrium values of $u_{\alpha\beta}^{(0)}$ are:

$$\begin{aligned} u_{\substack{XX\\YY}}^{(0)} = {} & -\frac{L_0^2}{2}\left(\frac{(cd)_+}{(cc)_+} \mp \frac{(cd)_-}{(cc)_-}\right) \\ & \pm \frac{c_5}{2(cc)_-}(\sigma_{XX} - \sigma_{YY}) + \frac{1}{2(cc)_+}[c_3(\sigma_{XX} + \sigma_{YY}) \\ & - 2c_4\sigma_{zz}], \\ u_{zz}^{(0)} = {} & -L_0^2\frac{[cd]_+}{(cc)_+} + \frac{(c_1 + c_2)\sigma_{zz} - c_4(\sigma_{XX} + \sigma_{YY})}{(cc)_+}, \\ u_{Yz}^{(0)} = {} & \delta\cos 3\varphi, \quad u_{zX}^{(0)} = \delta\sin 3\varphi, \quad u_{XY}^{(0)} = 0, \\ \delta = {} & L_0^2\frac{[cd]_-}{(cc)_-} - \frac{c_6}{(cc)_-}(\sigma_{XX} - \sigma_{YY}); \end{aligned} \tag{A.3}$$

here the following notation has been introduced:

$$\begin{aligned} & (cn)_+ = c_3(n_1 + n_2) - 2c_4n_4, \qquad (cn)_- = c_5(n_1 - n_2) - 2c_6n_6, \\ & [np]_+ = (n_1 + n_2)p_4 - (p_1 + p_2)n_4, \qquad [np]_- = (n_1 - n_2)p_6 \\ & \quad - (p_1 - p_2)n_6, \\ & n, p \equiv c, d. \end{aligned} \tag{A.4}$$

The values of the effective magnetostriction fields are the following:

$$\begin{aligned} H_1 & = 2L_0[(d_1 - d_2)(u_{XX}^{(0)} - u_{YY}^{(0)}) + 2d_6\delta], \\ H_2 & = 2L_0[d_5(u_{XX}^{(0)} - u_{YY}^{(0)}) + d_8\delta], \\ H_3 & = 2L_0[(d_3 - d_2)u_{XX}^{(0)} + (d_3 - d_1)u_{YY}^{(0)} \\ & \quad + (d_7 - d_4)u_{ZZ}^{(0)} + d_6\delta], \\ H_4 & = 2L_0[d_2u_{XX}^{(0)} + d_1u_{YY}^{(0)} + d_4u_{ZZ}^{(0)} - d_6\delta]. \end{aligned} \tag{A.5}$$

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