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ELECTRIC CONDUCTIVITY OF SEMICONDUCTORS WITH A NARROW ENERGY BAND IN A STRONG ELECTRIC FIELD

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The dependence of the conductivity of a semiconductor on the electric field was investigated. Semiconductors were considered with a sufficiently narrow energy band. In this case the heating of the electrons in the strong electric field (the effective temperature of the electrons exceeds the width of the allowed band) leads to inverse proportionality of the current and the electric field intensity.

THE behavior of semiconductors in strong electric fields has been the subject of many theoretical and experimental papers, a sufficiently detailed review of which can be found in the paper by Gunn.[1] However, the theoretical analysis was limited to the case of a quadratic carrier dispersion, and no account was taken of the finite width of the energy band. As will be shown below, allowance for the latter circumstance leads to an entirely different dependence of the current j on the field intensity E. Namely, in sufficiently strong fields, when the effective temperature T_{eff} of the electrons becomes larger than the width of the allowed energy band, the current decreases with increasing electric field, and when $T_{eff} \gg \Delta$ we have $j \sim 1/E$.

Let us trace qualitatively the transition from the relation $j \sim E$ to $j \sim 1/E$. As is well known, a noticeable deviation from Ohm's law occurs when the energy acquired by the electron per unit time in the electric field, $j \cdot E$, becomes comparable with the energy given up in inelastic collisions. For concreteness we shall consider collisions between electrons and phonons. Scattering by impurities is taken into account analogously and leads to similar results. We shall assume that the initial temperature T_0 of the electrons and the phonons (the latter is assumed to be invariant) is small compared with the Debye temperature Θ. Then for $T_{eff} \ll \Theta$ we can disregard the scattering by the optical phonons. In scattering by acoustical phonons, the electrons lose on the average per unit time an energy equal to $ms^2\epsilon_T/\tau kT_0$ (ϵ_T is the thermal energy of the electrons, the factor $ms^2/kT_0 \ll 1$ determines the degree of inelasticity of the collision, s is the speed of sound, and m is the electron mass).

For simplicity we consider first a model in which the electrons have a quadratic dispersion law, but the assumed values of the energy are bounded from above by a certain value Δ (width of the band). Assume that the field is already strong enough to satisfy the equality

$$\mathbf{Ej} \approx \frac{n}{\tau}\frac{ms^2}{kT_0}\varepsilon_T, \tag{1}$$

where n is the number of carriers. We set

$$\tau = l / v, \tag{2}$$

and assume the mean free path l to be independent of the energy. Using (2) and the expression for the current

$$j = e^2 n\tau E / m, \tag{3}$$

we obtain from (1)

$$\varepsilon_T \approx eEl\left(\frac{kT_0}{ms^2}\right)^{1/2}. \tag{4}$$

We see therefore that $v_T \sim \sqrt{E}$, and consequently $j \sim \sqrt{E}$. In the intermediate region, a transition takes place from $j \sim E$ to $j \sim \sqrt{E}$.

With further increase in the field, T_{eff} becomes $\sim \Theta$. This leads to a turning on of optical phonon emission processes. The electron can then lose per unit time an energy $\hbar\omega_0/\tau_{op}$ (τ_{op} is the radiation time of the optical phonon and $\hbar\omega_0$ is its characteristic energy). As seen from (1), the following inequality is satisfied

$$\mathbf{Ej} \ll n\hbar\omega_0/\tau_{op} \tag{5}$$

and the dependence of the current on the field again becomes linear.

With further increase in the current, T_{eff} remains $\sim \Theta$ until the inequality (5) turns into an equality. Then we see from (3) and (5) that the current in this region does not depend on the field (saturates). The electron energy E_T continues to increase in this region, and this in usual semiconductors leads to one of the forms of breakdown.

We wish to call attention to an effect which can occur in semiconductors with a narrow allowed band, separated from the next allowed band by a wide gap. In this case, the electron energy can become comparable with the width of the allowed band even in the pre-breakdown region. The energy which the electron can transfer to the lattice per unit time (C) will then be determined by characteristics that are averaged over the band, and ceases to depend on the field. From the energy balance we then obtain

$$j \approx C / E, \tag{6}$$

i.e., the current in this region of fields should decrease in inverse proportion to the electric field.

Let us now proceed to a quantitative analysis of the problem. We assume that the drift velocity v_D is much smaller than the thermal velocity v_T. This enables us to represent the electron distribution function in the form

$$f = f_0(\varepsilon) - e\mathbf{E}\mathbf{v}\tau\frac{df_0(\varepsilon)}{d\varepsilon} \equiv f_0 + f_1, \quad f_1 \ll f_0, \tag{7}$$

where $f_0(\epsilon)$ is the sought function, which depends only on the electron energy. In the case of weak fields $f_0 = \exp(-\epsilon/kT_0)$. The problem consists of finding $f_0(\epsilon)$ and the coefficient $\tau(\epsilon)$ connected with the drift velocity of an electron possessing an energy ϵ.

To this end we write down the kinetic equation[1)]

$$e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} + \left(\frac{\partial f}{\partial t}\right)_{\text{ac}} + \left(\frac{\partial f}{\partial t}\right)_{\text{op}} = 0, \tag{8}$$

where $(\partial f/\partial t)_{\text{ac}}$ is the integral of collisions with the acoustical phonons, which has the following form $(\mathbf{q} = \mathbf{p} - \mathbf{p}')$

$$\begin{aligned}\left(\frac{\partial f}{\partial t}\right)_{\text{ac}} = \int |c_{\mathbf{q}}{}^{\text{ac}}|^2 \{&f(\mathbf{p})(N_{\mathbf{q}}+1)\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}}' - \hbar\omega_{\mathbf{q}}) \\ &+ f(\mathbf{p}) N_{\mathbf{q}}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}}' + \hbar\omega_{\mathbf{q}}) \\ &- f(\mathbf{p}')(N_{\mathbf{q}}+1)\,\delta(\varepsilon_{\mathbf{p}}' - \varepsilon_{\mathbf{p}} - \hbar\omega_{\mathbf{q}}) \\ &- f(\mathbf{p}') N_{\mathbf{q}}\delta(\varepsilon_{\mathbf{p}}' - \varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{q}})\}\, d\mathbf{p}'\end{aligned} \tag{9}$$

and $(\partial f/\partial t)_{\text{op}}$ is the integral of collisions with optical phonons, equal to

$$\begin{aligned}\int |c_{\mathbf{q}}{}^{\text{op}}|^2 \{&f(\mathbf{p})\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}}' - \hbar\omega_0) \\ &- f(\mathbf{p}')\,\delta(\varepsilon_{\mathbf{p}}' - \varepsilon_{\mathbf{p}} - \hbar\omega_0)\}\, d\mathbf{p}'\end{aligned} \tag{10}$$

Here $|c_{\mathbf{q}}{}^{\text{ac,op}}|^2$ is the suitably normalized square of the modulus of the transition matrix element, ω is the frequency of the acoustical phonon, ω_0—the frequency of the optical phonon which for simplicity is assumed to be independent of $\mathbf{q}$. In (10) we have taken into account only effects connected with the spontaneous emission of the optical phonon, which is valid when $T_0 \ll \Theta$.

Substituting (7) in (8) and separating the symmetrical and asymmetrical parts, we obtain two equations for the two corresponding functions of the energy $f_0(\epsilon)$ and $\tau(\epsilon)$. In the derivation of the asymmetrical part of the equation we disregard in $(\partial f/\partial t)_{\text{ac}}$ the small inelasticity, that is, we put $\omega = 0$. As to (10), we shall assume that $|c^{\text{op}}|^2 = A$ does not depend on $\mathbf{q}$, and that the equal-energy surface of the electron has a center of inversion.

Under these assumptions, the asymmetrical part of (8) reduces to an equation for

$$1/\tau = 1/\tau_{\text{ac}} + 1/\tau_{\text{op}}; \tag{11}$$

$$\frac{1}{\tau_{\text{op}}} = A\frac{dN(\varepsilon_{\mathbf{p}} - \hbar\omega_0)}{d\varepsilon_{\mathbf{p}}}\,\theta(\varepsilon_{\mathbf{p}} - \hbar\omega_0), \tag{12}$$

$$\frac{1}{\tau_{\text{ac}}} = \int |c_{\mathbf{q}}{}^{\text{ac}}|^2\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}}')(2N_{\mathbf{q}}+1)\,d\mathbf{p}', \tag{13}$$

where $dN/d\epsilon$—density of the number of states, and

$$\theta(x) = \begin{cases}1, & x > 0\\ 0, & x < 0\end{cases}$$

If we put

$$|c_{\mathbf{q}}{}^{\text{ac}}|^2 = B\omega_{\mathbf{q}}\hbar \qquad (\omega_{\mathbf{q}} = sq)$$

and recognize that

$$\frac{\hbar\omega_{\mathbf{q}}}{kT_0} \sim \frac{s\sqrt{m\varepsilon}}{kT_0} \ll 1,$$

then we obtain from (13)

$$\frac{1}{\tau_{\text{ac}}} = 2BkT_0\frac{dN(\varepsilon)}{d\varepsilon}. \tag{14}$$

To derive the symmetrical part of equation (8), it is simplest to average it over the constant-energy surface. We have

$$e\mathbf{E}\int \frac{\partial f}{\partial \mathbf{p}}\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon)\,d\mathbf{p} = \mathbf{E}\frac{d}{d\varepsilon}\mathbf{j}(\varepsilon), \tag{15}$$

where

$$\mathbf{j}(\varepsilon) = -\frac{1}{3}\tau e^2\mathbf{E}\overline{v^2}\frac{df_0(\varepsilon)}{d\varepsilon}\frac{dN(\varepsilon)}{d\varepsilon} \tag{16}$$

is the current density in energy space. In the derivation of (16) we have put

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = {}^1/_3\,\overline{v^2}.$$

Averaging the integral of collisions with the acoustic phonons, we obtain

$$-\frac{d}{d\varepsilon}\left[\frac{1}{2}f_0(\varepsilon)\frac{s^2\overline{(\mathbf{p}-\mathbf{p}')^2}}{kT_0}\frac{1}{\tau_{\text{ac}}}\right]. \tag{17}$$

[1)]The scattering of the electrons by the impurities is taken into account in analogy with scattering by acoustical phonons.

In the derivation of (17) we have assumed for simplicity that $\omega_q = sq$, and, since we shall be henceforth interested only in the case $T_{eff} \gg T_0$, we have discarded the term $\sim kT_0 df_0/d\epsilon$ compared with f_0. The averaging of the integral of collisions with the optical phonons yields

$$A\left[f_0(\varepsilon)\frac{dN(\varepsilon)}{d\varepsilon}\frac{dN(\varepsilon-\hbar\omega_0)}{d\varepsilon}\theta(\varepsilon-\hbar\omega_0) - f_0(\varepsilon+\hbar\omega_0)\frac{dN(\varepsilon)}{d\varepsilon}\frac{dN(\varepsilon+\hbar\omega_0)}{d\varepsilon}\right]. \tag{18}$$

Ultimately we obtain the following equation for $f_0(\epsilon)$

$$\frac{d}{d\varepsilon}\left\{\frac{dN}{d\varepsilon}\left[\frac{1}{2}f_0(\varepsilon)\frac{s^2\overline{(\mathbf{p}-\mathbf{p}')^2}}{kT_0}\frac{1}{\tau_{ac}}+\frac{\tau e^2E^2}{3}\overline{v^2}\frac{df_0(\varepsilon)}{d\varepsilon}\right]\right\} - A\left[f_0(\varepsilon)\frac{dN(\varepsilon-\hbar\omega_0)}{d\varepsilon}\theta(\varepsilon-\hbar\omega_0)-f(\varepsilon+\hbar\omega_0) \times \frac{dN(\varepsilon+\hbar\omega_0)}{d\varepsilon}\right]\frac{dN(\varepsilon)}{d\varepsilon}=0. \tag{19}$$

We now proceed to an investigation of (19). If $kT_{eff} \ll \hbar\omega_0$, then we can disregard optical phonons. Then for a quadratic dispersion law $\overline{(\mathbf{p}-\mathbf{p}')^2} = 4m\epsilon$ we get

$$f_0(\varepsilon) = c\exp\left\{-\frac{3ms^2\varepsilon^2}{kT_0(eEl_{ac})^2}\right\}, \tag{20}$$

where l_{ac} is the mean free path on acoustical phonons. This expression for f_0 leads to $j \sim \sqrt{E}$. Formula (20) is valid so long as

$$T_{eff} \equiv eEl_{ac}\left(\frac{kT_0}{ms^2}\right)^{1/2} \ll \hbar\omega_0. \tag{21}$$

The case $T_{eff} \gtrsim \hbar\omega_0$ can be considered only qualitatively. We note that we have here essentially different energy regions. When $\epsilon < \hbar\omega_0$ the principal role is played by collisions with acoustical phonons, and the solution of equation (19) in this region is of the form (20). This solution for $\epsilon = \hbar\omega_0$ must be joined with the solution for $\epsilon > \hbar\omega_0$. In the latter region, the principal role is played by processes of emission of optical phonons, owing to the appreciable energy lost by the electron when it radiates optical phonons. Equation (19) reduces in this region to the following:

$$\frac{d}{d\varepsilon}\left\{\frac{dN(\varepsilon)}{d\varepsilon}\left[\frac{1}{2}f_0(\varepsilon)\frac{s^2\overline{(\mathbf{p}-\mathbf{p}')^2}}{kT_0}\frac{1}{\tau_{ac}}+\frac{\tau e^2E^2}{3}\overline{v^2}\frac{df_0(\varepsilon)}{d\varepsilon}\right]\right\} - \frac{1}{\tau_{op}}\frac{dN(\varepsilon)}{d\varepsilon}f_0(\varepsilon)=0. \tag{22}$$

This equation can be approximately solved in the region

$$(ms^2/kT_0)^2\hbar\omega_0 \ll \varepsilon-\hbar\omega_0 \ll \hbar\omega_0.$$

In this region all the quantities except τ_{op} vary little, and they can therefore be regarded as constant and the acoustical phonons can be neglected. Taking this into account, we write (22) in the form

$$\frac{1}{3}\tau^2(Ee)^2\overline{v^2}f_0''(\varepsilon)-\frac{\tau}{\tau_{op}}f_0(\varepsilon)=0. \tag{23}$$

The coefficient of $f_0''(\epsilon)$ in (23) has an order of magnitude $(eEl)^2$, which for $T_{eff} \sim \hbar\omega_0$ is equal to $\sim ms^2(\hbar\omega_0)^2/kT_0$. The quantity τ/τ_{op} is small when $\epsilon - \hbar\omega_0 \ll \hbar\omega_0$ and becomes of the order of unity when $\epsilon - \hbar\omega_0 \sim \hbar\omega_0$.

Thus, for not too small $\epsilon - \hbar\omega_0$, equation (23) contains a small parameter preceding the higher-order derivative. Its "quasiclassical" solution is of the form

$$f_0(\varepsilon) = f_0(\hbar\omega_0)\exp\left\{\frac{1}{eE\tau}\left(\frac{\overline{v^2}}{3}\right)^{-1/2}\int_{\hbar\omega_0}^{\varepsilon}\left(\frac{\tau}{\tau_{op}}\right)^{1/2}d\varepsilon'\right\}. \tag{24}$$

We assume for simplicity that $1/\tau_{op}$ is proportional to $(\epsilon - \hbar\omega_0)^{1/2}$, and then obtain

$$f_0(\varepsilon) = f_0(\hbar\omega_0)\exp\left\{-a\frac{\hbar\omega_0}{eEl}\left(\frac{\varepsilon-\hbar\omega_0}{\hbar\omega_0}\right)^{5/4}\right\}. \tag{25}$$

Here a is a number of the order of unity, which depends on the concrete structure of the energy band.

Solution (25) is valid in the region where the exponent is large. We see from it that for $kT_{eff} \sim \hbar\omega_0$ the distribution function decreases rapidly when $\epsilon > \hbar\omega_0$. As can be seen from formula (7), this rapid decrease leads to an increase in the current, which, as can be seen from (3), is again linearly dependent on E. For further heating of the electrons it is necessary to have $eEl \gg \hbar\omega_0$. In such fields, the distribution function will no longer decrease in the energy region in question. Then $f_0(\epsilon)$ will differ from zero even for $\epsilon \gg \hbar\omega_0$. To find the distribution function in this region, it is necessary to take into account the absorption of optical phonons. Collisions with acoustical phonons can be disregarded as before, owing to the small energy exchange.

Expanding equation (10), we obtain for $\epsilon \gg \hbar\omega_0$

$$\tau e^2E^2\overline{v^2}\frac{df_0(\varepsilon)}{d\varepsilon}+\frac{\hbar\tilde{\omega}_0}{\tau}f_0(\varepsilon)=0, \quad \tilde{\omega}_0 \approx \frac{3A\omega_0}{A+2BkT_0}. \tag{26}$$

Here $\tilde{\omega}_0$ is the effective frequency, which is $\sim \omega_0$. The difference between $\tilde{\omega}_0$ and ω_0 is due to allowance for the acoustical phonons. For a quadratic dispersion law we obtain

$$\hbar\tilde{\omega}_0 f_0(\varepsilon)+(eEl)^2\frac{df_0}{d\varepsilon}=0.$$

Assuming the mean free path to be independent of the energy, we find that

$$f_0(\varepsilon)=\exp\left\{-\frac{\hbar\tilde{\omega}_0}{(eEl)^2}\varepsilon\right\}, \qquad \varepsilon \gg \hbar\omega_0. \tag{27}$$

Calculating the current with the aid of this distribution function, we find that the current is independent of the electric field (is saturated) if $eEl \gg \hbar\omega_0$.

A further change in the behavior of the field dependence of the current is obtained in fields for which

$$(eEl)^2/\hbar\omega \sim \Delta, \tag{28}$$

where Δ is the width of the energy band. The derivative $df_0/d\epsilon$ then decreases, which leads to a decrease in the current as a function of the electric field. The character of this decrease depends on the specific characteristics of the substance, and cannot be calculated in general form. We can, however, find the limiting form of the dependence of the current on the field in very strong fields.

When $(eEl)^2/\hbar\omega_0 \gg \Delta$, the distribution function tends to a constant value determined from the normalization condition. In this case the current tends to zero when $df_0/d\epsilon$ tends to zero. To calculate the current, it is necessary to take into account the difference between the distribution function and a constant. Expanding expression (27) for small values of the argument and substituting in (7), we obtain the following expression for the current:

$$j = \frac{\hbar\widetilde{\omega}_0}{3El}\int_0^\Delta v\frac{dN(\varepsilon)}{d\varepsilon}d\varepsilon. \tag{29}$$

This expression has been obtained under the assumption that the mean free path is independent of the energy. We can write (29) in the form

$$jE = \int_0^\Delta \frac{\hbar\widetilde{\omega}_0}{\tau}\frac{dN(\varepsilon)}{d\varepsilon}d\varepsilon. \tag{30}$$

The latter equation, which expresses the energy balance in an electron gas, is free of the assumption made above. Thus, in the (one-band) model considered, the decrease of the current in inverse proportion to the field is a universal law, independent of the characteristics of the substance.

The question of the choice of substance in which one can observe experimentally the described decrease of current with electric field calls for further study. The main requirements that govern the choice of substance are the following: a) the allowed energy band must be narrower than the forbidden band; 2) the breakdown field must be larger than the field determined from (28), at which the decrease in current begins.

[1] J. B. Gunn, Progress in Semiconductors **2**, 211 (1957).

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