an exact expression for ρ_M in the region of the maximum; we can only give the order-of-magnitude estimate:

$$\rho_{M\,max} \sim \rho_{M0}(\varepsilon_F / J_1)^2. \qquad (4)$$

This means that if magnetic impurities are important, the magnetic part of the resistivity ρ_M is comparable in order of magnitude to the non-magnetic part at the maximum. With further decrease in temperature ρ_M tends to zero.

4. The hypothesis that owing to collective effects a resonance might occur in the scattering of electrons in the neighborhood of the Fermi surface has been previously advanced in the literature [3]. This hypothesis, then, is to some extent confirmed; however, it has turned out, in contradiction to the ideas of Korringa and Gerritsen, that the resonance occurs only as a result of the exchange interaction of the electrons with impurity atoms and only when this interaction is of antiferromagnetic sign. The resonance energy and the temperature T_r (which

are of the same order of magnitude) do not depend on the impurity concentration for small concentrations.

Details of this calculation will be published subsequently.

I should like to take this opportunity to express my gratitude to I. E. Dzyaloshinskiĭ for numerous discussions.

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RADIATIVE CORRECTIONS TO PAIR PHOTOPRODUCTION[1)]

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IN a paper by E. G. Vekshteĭn a statement was made concerning the anomalously strong energy dependence of relativistic radiative corrections to the cross section for pair photoproduction in the limiting case

$$\varepsilon_+ = \varepsilon_- = \omega / 2 \gg m, \quad \theta_+ = \theta_- = m / \omega \ll 1$$

$$(\theta_\pm = \widehat{\mathbf{k}\mathbf{p}_\pm}) \qquad (1)$$

where the momenta $\mathbf{k}$, $\mathbf{p}_+$ and $\mathbf{p}_-$ lie in a plane. In particular it is asserted that

$$\delta \equiv d\sigma^{(4)} / d\sigma^{(2)} = A a \omega^2 / \pi m^2, \quad A \sim 1. \qquad (2)$$

However, as will be shown below, this conclusion is in error and in fact

$$\delta = B a / \pi, \quad B \sim 1. \qquad (3)$$

The source of this error is the following. Of the ten diagrams describing the radiative correc-

tions to photoproduction (diagrams a–k in Fig. 3 of [1]), only the first eight are taken into account in [1]. Thus it was found that the contributions of diagrams c–d gave

$$\delta^{a}{}_{+}{}^{z} = 0.54\, a\omega^2 / \pi m^2. \qquad (4)$$

while diagrams a–b and e–h did not make a contribution of order ω^2/m^2. The contributions from diagrams j–k were not obtained because of the difficulty of the calculation. It was assumed instead that they could not compensate for the result in (4) because of the "independence" of diagrams j–k and c–d.

However, this assumption is incorrect. First, the indicated diagrams are mutually dependent and, second, the contributions of diagrams j–k cancel the result in (4). Both these assertions are proven below.

The existence of a relationship between diagrams j–k and c–d follows immediately from the fact that under the gradient transformation of the Coulomb potential

$$a_\mu(q) \to a_\mu(q) + iq_\mu f(q) \qquad (5)$$

the contribution from each of them changes, but the sum of all four remains invariant. Indeed, putting $S(p) = (i\hat{p} + m)^{-1}$ and using the obvious relations

$$\bar{u}_2 i\hat{q} S(p_2 + q) = \bar{u}_2, \quad S(p_1 - q) i\hat{q} u_1 = -u_1,$$
$$S(p)i\hat{q}S(p + q) = S(p) - S(p + q),$$

we find that under the transformation (5) the contributions of diagrams c, d, j, and k to the matrix element are proportional to the quantities

$$\Delta_c = Cf(q) \int dt \cdot t^{-2} \bar{u}_2 \gamma_\mu S (p_2 - t + q) \hat{e} S (p_1 - t) \gamma_\mu u_1,$$

$$\Delta_d = - Cf(q) \int dt \cdot t^{-2} \bar{u}_2 \gamma_\mu S (p_2 - t) \hat{e} S (p_1 - t - q) \gamma_\mu u_1,$$

$$\Delta_j = Cf(q) \int dt \cdot t^{-2} u_2 \gamma_\mu [S (p_2 - t) - S (p_2 - t + q)]$$

$$\times \hat{e} S (p_1 - t) \gamma_\mu u_1,$$

$$\Delta_k = Cf(q) \int dt \cdot t^{-2} \bar{u}_2 \gamma_\mu S (p_2 - t)$$

$$\times \hat{e} [S (p_1 - t - q) - S (p_1 - t)] \gamma_\mu u_1,$$

where C is a common factor for all the diagrams. It follows from this that

$$\Delta_c + \Delta_d + \Delta_j + \Delta_k = 0, \tag{6}$$

q. e. d.

The result in (4), which involves only diagrams c-d, in view of the above, is not gauge-invariant and, consequently, taken by itself has no physical meaning. It is necessary to take j-k into account along with c-d.

We now prove that when all the diagrams are considered the contributions of order ω^2/m^2 appearing in the individual terms completely cancel each other and are absent from the final expression. In other words we shall show that (3) is the correct result. This is done easily for the limiting case (1) of the exact expression for the radiative corrections to pair photoproduction obtained earlier in [2] using the mass operator method [cf. (9)—(12) of reference [2]]. In the following we make use of the notation and units introduced in [2,3], in particular m = 1. We also recall that the expression for pair production is obtained from that for bremsstrahlung by the change of momenta:

$$k \to -k, \quad p_1 \to -p_+, \quad p_2 \to p_-. \tag{7}$$

and, in addition, $\omega \to -\omega$.

In the limit (1) the vector $p_+ + p_-$ is collinear with the vector k, $p_+ - p_-$ is orthogonal to k and the fundamental parameters have the approximate values

$$\varkappa = \tau = {}^5/_2, \quad \rho = 25/4\omega^2 \ll 1. \tag{8}$$

In addition

$$R_1 = 2[kp_+], \quad R_2 = -2[kp_-], \quad R_1 = R_2, \quad R = T = 0,$$

$$S = [k, p_- - p_+] / \omega\varkappa, \quad U_0 = -\rho S^2 = -\omega^{-2} \ll 1. \tag{9}*$$

All the integrals J (. . .) which enter into (12)

of [2] and their derivatives with respect to the parameters have finite values of order unity for $\rho \ll 1$, $\kappa = \tau = {}^5/_2$. This is easily seen both from the definition (35) of [3] (see also the appendix to [3]) and from the resulting formulas (37)—(40) of [3] and (13) of [2].

The factors S_i for photoproduction are obtained from (41) of [3] by the momentum substitution (7)[2]. It is easily seen that in the limiting case (1) all

$$S_i = O(\omega^{-2}), \quad i = 1, 2, \ldots, 15. \tag{10}$$

As an example we show this for S_1. Taking into account (7)—(9) we have

$$S_1 = \varkappa\tau(\varkappa + \tau)S^2 + 2\rho\omega SR_2 + O(\omega^{-2}).$$

From (8)-(9) we find $S \cdot R_2 = -\omega\kappa S^2$ and $\tau(\kappa + \tau) - 2\rho\omega^2 = 0$. Therefore the terms of order unity cancel and we obtain $S_1 = 0 (\omega^{-2})$. It is just as easy to prove (10) for the remaining S_i.

It may therefore be asserted that $S_i/U_0 \sim 1$ and as a consequence relation (3) is correct.

The fact, that in the limit of (1) U_0, together with all S_i is small is not accidental. It is connected, in particular, with the circumstance that the considered radiative corrections ($\sim \alpha$) to the cross section are a result of the interference of the radiative corrections to the matrix element with the first approximation to the matrix element.

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[1] From the editors. The editors regret that E. G. Vekshtein's erroneous article was published. This occurred as a result of incorrect information on the part of the author about the character of his discussion of the given problem with P. I. Fomin.

[2] In (41) of [3] all products of the type RT, SR_2, etc. are three-dimensional scalar products of the vectors R, T, S, etc.

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*$[kp] = k \times p$.