## SUPERCONDUCTING THERMAL SWITCHES

V. P. PESHKOV and A. Ya. PARSHIN

Institute of Physics Problems, Academy of Sciences, U.S.S.R.

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The thermal conductivity of a number of thermal switches is measured at temperatures from 0.3 to  $2^{\circ}$  K in various magnetic fields. A theoretical estimate is made of the operating efficiency of superconducting thermal switches and of the influence of the sample purity and dimensions on the thermal conductivity.

A proposal that superconductors be used as thermal switches was first expressed by Heer and Daunt.<sup>[1]</sup> According to their measurements, at  $T = 0.55^{\circ}K$  the ratio of the thermal conductivity Kn of Ta in the normal state to its thermal conductivity K<sub>S</sub> in the superconducting state is equal to 60, and for tin at  $T = 0.65^{\circ}K$  this ratio is equal to 40. One would expect even larger values of  $K_n/K_s$  for metals with higher superconducting transition temperatures. However, the measurements of Montgomery<sup>[2]</sup> on Pb and Connoly and Mendelssohn<sup>[3]</sup> on Nb showed that this is not so. In addition, for operation of superconducting thermal switches with a He<sup>3</sup> bath, it is necessary to use them at temperatures from 0.3 to 0.5°K in different magnetic fields. However, although many measurements of the thermal conductivity of superconductors have been performed, the only data published for  $K_n/K_s$  at  $T \sim 0.5^{\circ}K$  is that of Zavaritskii<sup>[4]</sup> for Al and Zn and that of Satterthwaite <sup>[5]</sup> for Al, Zn, and Cd, and the magnetic field dependence of K in this temperature range has only been determined for Pb by Reese and Stevert;<sup>[6]</sup> moreover, the results of the latter authors are extremely strange. The results of a number of authors [2-4,7,8] with regard to the measurement of the thermal conductivity of various superconductors are presented in Fig. 1.

Since it is impossible to reach a definite conclusion about the advantages of one or the other material on the basis of the existing data, we measured the thermal conductivity of superconductors under conditions similar to the operating conductions for thermal switches. A metal cryostat containing He<sup>3</sup>, previously described, <sup>[9]</sup> was used to obtain and maintain low temperatures. The bath containing He<sup>3</sup> and the alternate versions for the arrangement of samples, thermometers, and heaters are shown in Fig. 2. In the first version



FIG. 1. Thermal conductivity of various superconductors: the dashed curves refer to the normal state, the solid curves refer to the superconducting state. Curves 1a are for a single crystal of Sn, d = 2.53 mm,  $\rho_0 = 4.1 \times 10^{-9}$  ohm-cm,  $H_{\parallel} \rightarrow 0$ ;[<sup>8</sup>] curves 1b are for a single crystal of Sn + 3.1% impurities, d = 2.7 mm,  $H_{\parallel} \rightarrow 0$ ;[<sup>8</sup>] curves 2 are for a single crystal of Sn + 2 ×  $10^{-3}\%$  impurities,[<sup>7</sup>] d = 1.4 mm; curves 3 are for a single crystal of Nb, d = 2.2 mm,  $\rho(273^{\circ}K)/\rho_0 = 120$ ;[<sup>3</sup>] curves 4 are for a single crystal of Pb, d = 2.5 mm,  $\rho_0 = 8 \times 10^{-9}$  ohm-cm, H = 1000 Oe;[<sup>2</sup>] curves 5 are for Pb foil, d = 0.07 mm,  $\rho_0 = 1 \times 10^{-8}$ ohm-cm, H = 1000 Oe; [<sup>2</sup>] curves 6 are for large crystals of Al + 0.01% impurities, d = 1.5 mm, H = 115 Oe;[<sup>4</sup>] curves 7 are for a single crystal of Zn, d = 1.5 mm, H = 60 Oe[<sup>4</sup>]

(a), two samples were soldered to the copper cold finger M, which was tightly screwed into the copper bottom of the He<sup>3</sup> bath. During the course of the experiments, however, it became clear that such a connection does not guarantee good thermal contact with the bath. The sample was already appreciably overheated even at a power release of



FIG. 2. Alternate versions of the apparatus for measurement of thermal conductivity:  $R_1$  and  $R_2$  are resistance thermometers,  $H_1$  and  $H_2$  are heaters, M is a copper cold finger.

 $5 \times 10^{-6}$  W. Therefore, in the final version of the apparatus (b) the tin sample was directly soldered to the He<sup>3</sup> bath with tin of the same purity. This allowed a power input of  $10^{-4}$  W to the sample without significantly increasing the temperature.

Pb, Sn, and Al of 99.99% purity were initially chosen as the objects of investigation. The samples were of 0.05 mm foil obtained by rolling of single crystals, rolled into rods of length 100 mm and diameter close to 1.5 mm. In subsequent measurements three more samples, prepared from tin with  $\rho(4.2^{\circ} \text{ K}) = 1.7 \times 10^{-10}$  ohm-cm, were investigated. One made out of 0.05 mm foil as in the preceding investigations, another made of the same foil, but cleansed beforehand in acid, and the third sample was a single crystal of diameter 2.2 mm, length 100 mm, with tetragonal axis along the sample, which was grown <sup>1)</sup> by Kapitza's method <sup>[10]</sup> in a quartz capillary with the quartz subsequently etched off.

During the work with the single crystal, special attention was taken so that the sample was not subjected to mechanical stresses and overheatings capable of damaging the crystal lattice. The transverse cross section area S of the samples was determined from their weight, length, and the density of the material. In the case of the foil samples,  $S = 0.02 \text{ cm}^2$ .

The temperature was measured with carbon resistance thermometers. In order to decrease

the heat capacity and improve thermal contact. the thermometers were prepared in the following manner. The points at which the thermometers were attached were covered with a thin layer (0.1 mm) of BF-2 glue, and polymerization was carried out at 160°C. Then these points on the sample were once more covered with BF-2 glue. and on them, close to each other, four  $50\,\mu$  bare copper wires were wound in a helix. The overall length of the spiral was about 5 mm, and the distance between the centers of the thermometers was about 60 mm. After heating to 120°C and cooling, two non-adjacent wires were removed, and the other two wires served as the contacts of the thermometer. A thin layer of carbon suspended in glycerin was deposited between them. Then drying in vacuum at 100°C was carried out for about two hours. For a measuring current of  $20\,\mu\text{A}$ , thermometers prepared in this fashion had a resistance of order 3 ohms at room temperature, 70 ohms at 1.2°K, and 340 ohms at 0.4°K.

The thermometers were calibrated against the vapor pressure of He<sup>3</sup> while the samples were located in a magnetic field somewhat exceeding the critical field. In a magnetic field of 10 kOe the resistance of the thermometers decreased by 2% at 0.5°K and by 1% at 1.2°K. The accuracy of the absolute temperature determination by resistance thermometer was  $5 \times 10^{-3}$  °K over the entire temperature interval investigated. Upon warming the thermometers up to room temperature and repeated cooling, their resistance increased somewhat (about 5%), but the overall temperature response remained unchanged. Therefore, in order to determine the absolute temperature, it was enough to fix the foregoing calibration at one point. The derivative  $\partial R/\partial T$  was determined by differentiating the fourth-order polynomial R(T) (fifth-order for large intervals) whose value differed from the calibration curve over the entire interval of measurements by less than the temperature-measurement error  $(0.005^{\circ})$ . Inside the apparatus, the electric wires ahead of the  $He^{\delta}$ bath were constant n of  $50\,\mu$  diameter, thus reducing parasitic heat supply to negligibly small amounts.

Heaters made of 50  $\mu$  diameter constantan wire, of resistance 290 ohms in the case of the single crystal sample and 400 ohms in the case of the foil samples, were wound on the samples and attached to them with BF-2 glue. The foil-sample ends on which the heaters were wound were fused. The resistance of the heaters decreased by 3% upon cooling from room temperature to liquid helium temperature and by 0.04% upon application

<sup>&</sup>lt;sup>1)</sup>The single crystals from which the samples were prepared were grown in the Technological Section of the Institute of Physics Problems under the direction of N. N. Mikhaĭlov.

of a 10 kOe field. The leads from the heater were constantan wire of 0.3 mm diameter in the case of the single crystal sample, and 0.1 mm diameter in the case of the foil samples.

The measuring system consisted of two potentiometers and two galvanometers. Measurements were made in the following way. The resistance  $R_2$  was measured while the heater  $H_1$  (see Fig. 2) was turned on and for a fixed value of the resistance  $R_1$ . Then the heater  $H_1$  was turned off and the heater  $H_2$  was turned on with the current chosen so that  $R_1$  remained unchanged, and the value of  $R_2$  was measured again. Such an operation was repeated many times. The unknown temperature difference  $\Delta T$  was determined from the difference in the values of  $R_2 (\Delta R_2)$  and from the value of the derivative  $\partial R/\partial T$  for the average temperature in this interval; viz.,  $\Delta T$ =  $\Delta R_2 (\partial R_2/\partial T)^{-1}$ .

The total error in the determination of the coefficient of thermal conductivity did not exceed 10% over the entire range of temperatures and fields. In the superconducting state it was primarily determined by the accuracy of the absolute temperature measurement, and in the normal state by the accuracy of the determination of a temperature difference, which sometimes amounted to only 0.001°. In such cases  $\Delta R_2$  was determined from the difference between the curves of  $R_2$  as a function of time,  $R_2$  (t), for  $H_1$  turned on and turned off, and the same  $R_1$ . Usually from 10 to 20 measurements were made.

Preliminary measurements on Pb, Sn, and Al samples made out of 0.05 mm foil of 99.99% purity showed that, at T = 0.5°K, the ratio K<sub>n</sub>/K<sub>s</sub> is equal to 300 for Sn, 40 for Pb, and 10 for Al. It was therefore decided to investigate Sn in more detail, but to use Sn containing a smaller concentration of impurities. The following measurements pertain to samples of Sn with  $\rho$  (4.2° K) = 1.7 × 10<sup>-10</sup> ohm-cm which, on the basis of estimates according to the data collected by Zernov and Sharvin <sup>[11]</sup> [c = 0.2  $\rho$  (0°K)/ $\rho$  (300°K),  $\rho$  (300°K) = 1.21 × 10<sup>-5</sup> ohm-cm], corresponds to 99.9998% purity.

## SUPERCONDUCTING STATE

One can describe the temperature dependence of  $K_{\rm S}$  for  $T>0.6^{\circ}{\rm K}$  for all three samples, within the limits of accuracy of the measurements, by the formula  $K_{\rm S}\sim \exp{(-\beta T_{\rm C}/T)}$ , where  $\beta$  = 1.53  $\pm$  0.05, in agreement with the measurements of other authors.<sup>[7,8]</sup> The large difference between the absolute values of  $K_{\rm S}$  for single



FIG. 3. Thermal conductivity of tin:  $\Delta$  - single crystal, superconducting state; o - single crystal, H<sub>II</sub> = 500 Oe; • single crystal, H<sub>I</sub> = 500 Oe; •,  $\Box$  - foil, superconducting state; x, + - foil, H<sub>II</sub> = 330 Oe.

crystal and foil samples permits us to assume that in the second case the value of  $K_S$  is limited by the scattering of electrons from the grain boundaries. It is necessary to attribute the change in the character of the temperature dependence of  $K_s$  for  $T \stackrel{<}{\sim} 0.5^{\circ}K$  to a manifestation of the lattice thermal conductivity, which plays a decisive role in superconductors at very low temperatures, as already noted by Zavaritskii. [7] The temperature dependence of the thermal conductivity in the region of the "phonon tails" is close to cubic, as it must be (in Fig. 3 the straight lines 4 and 5 correspond to  $K \sim T^3$ ). Upon comparison of the two foil samples, it is evident that whereas in the "electron" region the differences between the samples are insignificant, the phonon thermal conductivity of the etched foil is half that of the unetched foil. From this we can conclude that an appreciable fraction of the phonons are specularly reflected from the surface of the foil obtained by rolling between well polished rollers. The mean free paths of the phonons turn out to be 1.5 mm for the single crystal and 0.11 mm for the etched foil, which is in quite good agreement with the dimensions of the samples.

## NORMAL STATE

The dependence of  $K_n$  on H and T was investigated for two directions of the magnetic

FIG. 4. Dependence of the thermal conductivity of tin on the transverse field;  $\times -$  single crystal, T =  $0.46^{\circ}$ K;  $\bullet -$  single crystal, T =  $0.60^{\circ}$ K;  $\Delta -$  foil, T =  $0.345^{\circ}$ K;  $\Box -$  foil, T =  $0.83^{\circ}$ K; curve 1 - increasing field, curve 2 - decreasing field.



field: along the axis of the sample and perpendicular to it. The thermal conductivity of the foil turned out to be independent of the direction of the field; conversely, the single crystal is very sensitive to it.

In a constant field, the thermal conductivity of all three samples was roughly proportional to the temperature. In Fig. 3 the straight line 1 corresponds to a thermal conductivity  $K_n = 75 T$ [W/cm-deg] for the single crystal in a longitudinal field  $H_{\parallel} = 500$  Oe; straight line 2 corresponds to a thermal conductivity  $K_n = 12 T^{1.1}$ [W/cm-deg] for the single crystal in a transverse field  $H_{\perp} = 500$  Oe; straight line 3 corresponds to a thermal conductivity  $K_n = 6.8 T$ [W/cm-deg] for etched foil in a longitudinal field of 330 Oe. The deviation from a linear law, which is noticeable for the single crystal in a longitudinal field for  $T > 1^{\circ}K$ , should be ascribed to a decrease of the electron mean free paths because of scattering by lattice vibrations.

In the temperature region below  $0.4^{\circ}$ K and in a transverse magnetic field, systematic deviations from the linear law, amounting to 30%, towards larger values of the thermal conductivity (Fig. 3), were observed. However, at very low values of the temperatures investigated, the possible appearance of more significant measurement errors than those indicated above cannot be excluded.

Therefore, our measurements do not allow us to assert that deviations from the linear law actually do occur in this temperature range.

The results of measurements of  $K_n$  in a transverse field are presented in Fig. 4. It is evident that the similarity law established by Kohler <sup>[12]</sup> is satisfied (for a constant field, the ratio  $K_n/T$  does not vary with temperature). For large magnetic fields,  $K_n \sim H_{\perp}^{-2}$  in the case of a single crystal, and  $K_n \sim H_{\perp}^{-1}$  in the case of a foil. The dependence of the thermal conductivity of a single crystal on the magnitude of the longitudinal field is negligible: for an increase of  $H_{\parallel}$  from 300 to 750 Oe, it is decreased by roughly 20%.

The dependence of the thermal conductivity of a single crystal on the previous history of the sample is shown in Fig. 4 for fields smaller than the critical field. With increasing field (curve 1), the thermal conductivity of a sample in the intermediate state turns out to be several times smaller than in the superconducting state, in agreement with the measurements of Zavaritskiĭ.<sup>[13]</sup> Hysteresis is observed upon lowering the field out of the trans-critical region (curve 2) the field is "frozen" in the sample, as is evident from the value of K when  $H_{\perp} = 0$ , which turns out to be less than it was in the initial state before application of the field. Such hysteresis was not observed in the foil. Analyzing the results obtained, and also the results of other authors working with sufficiently pure metals, it is possible to reach the conclusion that, at temperatures of the order of tenths of a degree and below and for metal purities attainable under contemporary conditions (99.9999%), the thermal conductivity in the superconducting state is determined by the phonons and is limited by the scattering of phonons at the boundaries of the sample (on Figs. 1 and 3—the points on the curves indicated by the arrows A). In the normal state, the thermal conductivity is determined by the electrons, and for sufficiently thin samples it is also limited by boundary scattering.

However, if the samples are thick or the metals are not sufficiently pure, then the electrons are scattered by impurities, and the thermal conductivity in the normal state falls abruptly. At the same time, at low temperatures the long wavelength phonons are almost not scattered at all by impurities, but are scattered by "normal" electrons. Therefore, for a tin sample containing 3.1% in impurities, at certain temperatures the thermal conductivity in the normal state turns out to be even lower than in the superconducting state (Fig. 1, curve 1b). With increasing temperature, the fraction of "normal" electrons losing momentum to the indium atoms increases, and the scattering of phonons by "normal" electrons also increases. The latter leads to a decrease of the thermal conductivity in the superconducting state with increasing temperature, and explains the presence of the peaks in curves 1b and 3 of Fig. 1.

For thermal switches it is important that the ratio  $K_n/K_s$  be a maximum; therefore it is desirable to achieve those conditions under which the phonon mean free path will be the same as that of the "normal" electrons. It is possible to achieve the latter if very pure superconductors of very small thickness are obtained, that is, if we use a thin foil or wire. For working with fields up to a few kilo-oersteds, it is particularly convenient to employ polycrystals with grain dimensions such that they contain only a few impurity atoms each. In this case the thermal conductivity of the polycrystal in the normal state, for significant magnetic fields, turns out to be even larger than that of a single crystal along certain directions, but in the superconducting state it is an order of magnitude lower (Figs. 3 and 4-foil and single crystal). To be sure, if we use very pure single crystals in a longitudinal magnetic field, then their thermal conductivity will exceed that obtained by extrapolating  $K_n$  to the point  $H_{\parallel} = 0$ . Here the electrons will move in helical paths without being scattered by the boundaries. A similar phenomenon was observed in tin and zinc by Aleksandrov.<sup>[14]</sup> In this case the ratio  $K_n/K_s$  for a single crystal will be larger than for a polycrystal; however, measures must be taken to prevent a frozen magnetic field from remaining in the superconducting state.

At the present time there are no complete data about the properties of superconductors, but even from the existing characteristics we can estimate the operating efficiency of superconductors as thermal switches.

We shall assume that: 1) the temperature is so low that in the normal state the thermal conductivity is entirely due to electrons and the scattering by phonons is small, but in the superconducting state the thermal conductivity is due to phonons, and scattering by electrons is small; 2) the dimensions of the crystal are so small (or, correspondingly, the purity is so great) that the thermal conductivity is limited by boundary scattering; 3) the magnetic field is not too large in comparison with the critical field H<sub>c</sub> and has a weak influence on the thermal conductivity of the normal state. We shall neglect the anisotropy of the thermal conductivity and, in accordance with our assumptions, we shall regard the mean free path l of electrons and phonons as equal to the smallest dimension d of the crystal. Since almost no reliable data exists on the thermal conductivity of pure superconductors in the normal state at low temperatures, in order to estimate the value of Kn we shall use the model of almost free electrons, according to which (see, for example, the book by Kittel<sup>[15]</sup>)</sup>

$$K_n = \frac{1}{3} C u_{\rm F} l,$$

where C is the heat capacity of the electrons,  $u_F$  is the Fermi velocity of the electrons, and l is their mean free path. Since

$$C = \frac{\pi^2}{2} \frac{nk^2T}{E_{\rm F}}, \quad \frac{m^* u_{\rm F}^2}{2} = E_{\rm F} = \frac{h^2}{2m^*} (3\pi^2 n)^{2/3},$$

then

$$K_n = lT \frac{k^2}{h} \left(\frac{\pi}{3}\right)^{4/3} n^{2/3},$$

where n is the number of free electrons per unit volume, m\* is the effective mass of an electron,  $E_F$  is the Fermi energy of the electrons, T is the absolute temperature, and k and h are the constants of Boltzmann and Planck. The number of free electrons per unit volume is not precisely known; however, from measurements of the ano-malous skin effect according to the estimates of Chambers <sup>[16]</sup> it is necessary to have 1.24 free

electrons per atom for Pb, 1.12 for Sn, 2.18 for Al, and 0.42 for Cd, and according to estimates from optical data 1.2 for Pb,  $^{\left[17\right]}$  1.35 for Al,  $^{\left[18\right]}$  and 2 for In.  $^{\left[19\right]}$  We see that the results differ appreciably, but with regard to order of magnitude one can assume n = N/V<sub>mol</sub>, where N is Avogadro's number and V<sub>mol</sub> is the molar volume; therefore

$$\frac{K_n}{lT} = \frac{k^2}{h} \left(\frac{\pi}{3}\right)^{4/3} \left(\frac{N}{V_{mol}}\right)^{2/3} = 1.38 \cdot 10^4 V_{mol}^{-2/3} \text{ W/cm}^2 \cdot \deg^2.$$
(1)

In the superconducting state, at temperatures much below the critical temperature, one can estimate the thermal conductivity according to the formula

$$K_s = \frac{1}{3}l(C_l u_l + 2C_l u_l) = \frac{1}{3}lC\widetilde{u}.$$

where *l* is the mean free path of the phonons, Cland 2  $C_t$  are, respectively, the heat capacities of longitudinal and transverse phonons; ul and  $u_t$ are the velocities of propagation of longitudinal and transverse sound vibrations; C is the total heat capacity per unit volume, determined by the well known expression  $C = 1944 V_{mol}^{-1} (T/\Theta)^3$ Joule/cm<sup>3</sup>-deg. The quantity  $\tilde{u}$ , as is easily shown,<sup>2)</sup> coincides to within a few percent with the velocity  $\bar{u}$  which appears in the definition of the Debye temperature:  $\Theta = 2.51 \times 10^{-3} V_{mol}^{-1/3} \bar{u}$ , and is determined from the expression

$$3 / \bar{u}^3 = 1 / u_l^3 + 2 / u_t^3$$
.

Finally, assuming  $\tilde{u} = \bar{u}$ , we obtain

$$K_s/lT^3 = 2.58 \cdot 10^5 V_{mol}^{-2/3} \Theta^{-2} \text{ W/ cm} \cdot \deg^4.$$

Since, under the assumed conditions, the mean free paths of the phonons and electrons are equal and are limited by the dimensions of the sample, the ratio  $K_n/K_s$  of interest to us is determined in the following way:

$$K_n T^2 / K_s = 0.053 \ \Theta^2. \tag{3}$$

The temperature  $\rm T_0$ , below which it is possible to neglect in the superconducting state the contribution of the electrons to the quantity  $\rm K_S$  (corresponding in pure samples to the points analogous

<sup>2)</sup>Since 
$$C_l:C_t:C = u^{-3}:u^{-3}:\overline{u}^{-3}$$
, then  

$$\frac{\tilde{u}}{\overline{u}} = \frac{C_l u_l + 2C_t u_t}{C\overline{u}} = \frac{(u_l^{-2} + 2u_l^{-2}) \cdot 3^{-1/3}}{(u_l^{-3} + 2u_l^{-3})(u_l^{-3} + 2u_l^{-3})^{-1/3}}$$

$$= \left(\frac{2}{3}\right)^{1/3} \frac{1 + 1/2(u_l/u_l)^2}{[1 + 1/2(u_l/u_l)^3]^{1/3}}.$$

Usually  $u_l/u_t = 1.5$  to 2; for  $u_l/u_t = 1.5$  and 2 we have, respectively,  $\widetilde{u}/\overline{u} = 0.97$  and 0.94.

to A on Figs. 1 and 3), is determined from the condition

$$K_{es}(T_0) = 0.1 \ K_{gs}(T_0).$$
 (4)

Here we calculate the electron thermal conductivity in the superconducting state according to the formula

$$K_{es} = K(T_{c}) \exp\left[-\frac{\Delta}{T}\left(1-\frac{T}{T_{c}}\right)\right]$$

which, according to Zavaritskiĭ, <sup>[20]</sup> is justified with sufficient accuracy. For  $\Delta$  we assume the theoretical value  $\Delta = 1.76 \text{ T}_{\text{C}}$ , and the value of K (T<sub>C</sub>) we shall take from formula (1), substituting T = T<sub>C</sub> into it. We determine the lattice thermal conductivity K<sub>gs</sub> from formula (2). The temperature T<sub>0</sub> is estimated from here only approximately, since we neglect the mutual scattering of electrons and phonons; however, the error in the determination of T<sub>0</sub> will be small, since the number of "normal" electrons decreases very rapidly.

Next we estimate how pure the material must be in order to neglect scattering by impurities. It is possible to determine the order of magnitude of the mean free path  $l_{is}$  of the electrons for impurity scattering from the formula  $l_{is} = 1/n\sigma$ , where  $n = cn_0$  is the number of impurities per unit volume,  $\sigma$  is the cross section for the scattering of an electron by an impurity. According to the work of Kuz'menko, <sup>[21]</sup>  $\sigma \sim 3 \times 10^{-16}$  cm<sup>2</sup>, and  $n_0 = (3 \text{ to } 8) \times 10^{22}$  cm<sup>-3</sup>. The permissible amount of impurities is determined from the condition

$$l_{\rm is} \sim 5 \cdot 10^{-8} / c > d$$
 or  $c < 5 \cdot 10^{-6} / d$  %,

where d(cm) is, as before, the smallest dimension of the sample. It should be noted that this estimate is very rough, because the cross section  $\sigma$  for different impurities may vary up to a factor of ten; in addition, the measurement of such small impurity concentrations is very complicated. Therefore, an estimate of the permissible amount of impurities on the basis of residual resistance may serve as a more simple method for the determination of the suitability of a given material for a thermal switch. Since the Widemann-Franz law,  $K_n/T = L/\rho_0$ , where  $\rho_0$  is the residual resistivity and  $L = 2.45 \times 10^{-8}$  W-ohm-deg<sup>-2</sup>, must be satisfied under the conditions stated above, then using formula (1) we obtain  $\rho_0 l \approx 10^{-11}$ ohm-cm<sup>2</sup>. A criterion for the suitability of a given superconductor for manufacture of an effective switch of thickness d(cm) will be

$$\rho_0 < 10^{-11} / d \ \Omega \cdot \mathrm{cm} \ \mathrm{or} \ c < 5 \cdot 10^{-6} / d \ \%.$$
 (5)

| Metal   | T <sub>c</sub> , °K  | H <sub>o</sub> , Oe  | Θ, °K  | ${K_n}/{lT}, W/{cm^2-deg^2}$   | K <sub>s</sub> /lT <sup>3</sup><br>W/cm <sup>2-</sup> deg <sup>4</sup>  | $10^{-3}$ K <sub>n</sub> T <sup>2</sup> /K <sub>s</sub>  | T₀, °K  | ${ m H_0}^2/2\pi { m T_c}^2,$ erg/cm <sup>3</sup> -deg <sup>2</sup>  |
|---|--|--|--|--|---|--|---|--|
| Nb<br>Pb<br>V<br>Ta<br>Sn<br>In<br>Tl<br>Re<br>Al<br>Ga<br>Mo<br>Zn<br>Zr<br>Cd | $\begin{array}{c}9.09 \ [22]\\7.17 \ [23]\\5.03 \ [25]\\4.39 \ [25]\\3.72 \ [27]\\3.4 \ [23]\\2.4 \ [27]\\1.7 \ [40]\\1.163 \ [31]\\0.925 \ [32]\\0.852 \ [32]\\0.852 \ [32]\\0.55 \ [36]\\0.55 \ [36]\\0.557 \ [36]\\0.20 \ [31]\ [31]\\0.20 \ [31]\ [31$ | $\begin{array}{c} 2000 \begin{bmatrix} 22\\ 800 \begin{bmatrix} 23\\ 310 \end{bmatrix} \\ 800 \begin{bmatrix} 23\\ 28\\ 28\\ 28\\ 28\\ 304 \begin{bmatrix} 27\\ 28\\ 28\\ 30\\ 473 \end{bmatrix} \\ \begin{array}{c} 200 \begin{bmatrix} 40\\ 403 \end{bmatrix} \\ \begin{array}{c} 31\\ 59, 4 \end{bmatrix} \\ \begin{array}{c} 32\\ 98 \end{bmatrix} \\ \begin{array}{c} 33\\ 33\\ 53, 4 \end{bmatrix} \\ \begin{array}{c} 32\\ 47\\ 38\\ 28, 4 \end{bmatrix} \\ \begin{array}{c} 37\\ 20\\ 38\\ 38\\ 38\\ 38\\ 38\\ 38\\ 38\\ 38\\ 38\\ 38$ | $\begin{array}{c} 238 \\ 238 \\ 106 \\ 2^{11} \\ 338 \\ 2^{55} \\ 200 \\ 2^{86} \\ 109 \\ 2^{87} \\ 2^{99} \\ 417 \\ 309 \\ 428 \\ 425 \\ 309 \\ 425 \\ 309 \\ 425 \\ 292 \\ 339 \\ 425 \\ 292 \\ 309 \\ 425 \\ 309 \\ 32 \\ 292 \\ 309 \\ 32 \\ 292 \\ 30 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 32 \\ 309 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 3$ | 2.8<br>2.8<br>2.3<br>2.4<br>2.2<br>2.1<br>3.2<br>3.0<br>7<br>3.1<br>3.1<br>2.4<br>3.1<br>2.4<br>3.1<br>2.4<br>2.5<br>0 | $\begin{array}{c} 0.94\\ 3.3\\ 0.56\\ 0.8\\ 1.0\\ 3.5\\ 5.1\\ 0.35\\ 0.35\\ 0.30\\ 0.5\\ 0.62\\ 0.52\\ 1.32\\ 0.22\\ 1.32\end{array}$ | $\begin{array}{c} 3.0\\ 0.6\\ 6.1\\ 3.5\\ 2.4\\ 0.64\\ 9.3\\ 9.8\\ 5.4\\ 9.7\\ 5.1\\ 4.6\\ 1.9\\ 10\\ \end{array}$ | $\begin{array}{c} 1.2\\ 1.0\\ 0.55\\ 0.5\\ 0.4\\ 0.3\\ 0.16\\ 0.08\\ 0.075\\ 0.075\\ 0.045\\ 0.03\end{array}$ | $\begin{array}{c} 7700\\ 2000\\ 11000\\ 3200\\ 1100\\ 1100\\ 820\\ 2200\\ 1300\\ 480\\ 1800\\ 630\\ 1150\\ 430\\ 410\end{array}$ |

A few words should be said about the quantity of heat emitted during the transition of a superconductor from the normal state into the superconducting state. As is well known, the entropy difference between the normal and superconducting states is  $S_n - S_s = -(H_c/4\pi) dH_c/dT$ . The approximate relationship  $H_c = H_0 [1 - (T/T_c)^2]$ , where  $H_0$  is the value of  $H_c$  at T = 0, exists between the value of the critical field and the temperature. If this relation is used, and one takes the fact that  $(T/T_c) \ll 1$  into consideration, then  $S_n - S_s = (H_0^2/2\pi)T/T_c^2$ , and the minimum amount of heat emitted per unit volume of superconductor upon demagnetization is

$$\Delta Q = T(S_n - S_s) = \frac{H_0^2}{2\pi} \frac{T^2}{T_c^2}.$$
 (6)

Thus, the amount of heat emitted is smaller for those superconductors whose ratio of  $\rm H_0/T_C$  is smaller.

The characteristics of 15 superconductors are presented in the table. Values of  $K_n/lT$  are calculated according to formula (1), values of  $K_s/lT^3$  according to formula (2), the ratios  $K_nT^2/K_s$  according to formula (3), and the values of  $T_0$  are determined from condition (4).

Unfortunately, it is impossible to compare the tabular data with experiments, because there are no direct measurements of thermal conductivity at temperatures below  $T_0$  on samples of sufficient purity. Mendelssohn and Renton, <sup>[41]</sup> by extrapolating the value of  $K_n$  according to the linear law  $K \sim T$  from the temperature region above  $T_c$ , obtained for  $K_n T^2/K_s$  the value 100 for Pb, 350 for Sn, and 57 for In. As is evident, their data is still far from the corresponding values of the table; this means that their superconductors were not sufficiently pure and the phonon mean free paths in the samples were considerably larger than the electron mean free paths.

This also pertains to the measurements of Zavaritskii<sup>[4]</sup> on the thermal conductivity of Al and Zn. In our experiments (see Fig. 3), for a single crystal of tin  $K_n T^2/K_s = 510$  (the value of  $K_n$  is taken for a longitudinal field of 500 Oe), and  $K_n T^2/K_s = 620$  for the etched foil. If the value of  $K_n$  is extrapolated to zero field, then we obtain, respectively, 600 to 650 and 750 to 800. The remaining deviation by a factor of 2.5 to 3 can be attributed to both the crudeness of the theoretical model and to the fact that the mean free path of the electrons is still less than the mean free path of the phonons, even in etched foil.

Thus, there is reason to believe that the values presented in the table are of the correct order of magnitude, and it is possible to use them for selection of material for a thermal switch, and one can use formula (5) in order to estimate the necessary degree of purity of the material. Comparing the data presented in the table, Nb, Sn, Al, and Cd should be named as recommended materials, depending on the operating temperature. From the point of view of experimental convenience, one can recommend the use of foil or thin wires instead of single crystals. One should also keep in mind that when the thermal switch is open the magnetic field on the switch must not be too large in comparison with the critical field.

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