SOVIET PHYSICS JETP VOLUME 21, NUMBER 1 JULY, 1965

SOME FEATURES OF THE ENERGY SPECTRUM WITHIN THE MAIN BAND OF DISORDERED SOLID SOLUTIONS

G. G. SERGEEVA

Khar'kov State University

Submitted to JETP editor June 10, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 158-166 (January, 1965)

The effect of impurities on the structure of the energy spectrum and on the spectral density of the unperturbed spectrum band of a disordered solid solution is considered. It is shown that a singularity corresponding to states of an almost localized type may appear within the main band. This may result in some peculiarities in the behavior of the heat capacity, thermal conductivity, and absorption of ultrasound by crystals with impurities.

I. M. LIFSHITZ[1] investigated the structure of the energy spectrum of disordered solid solutions. He considered those regions of the spectrum of disordered solid solutions, in which the structure of the spectrum differs noticeably from the energy spectrum of ideal crystals or of fully-ordered solid solutions. Thus, he investigated the structure of the spectrum and the behavior of the spectral density near the unperturbed boundary of the spectrum; a method was proposed for investigating the structure of the spectrum inside the band corresponding to the unperturbed operator. We shall show in the present paper with the aid of the method developed in[1] that inside the spectrum band corresponding to the unperturbed operator there exist singular points of the energy spectrum of a disordered solid solution with low impurity concentration c, and that this leads to certain anomalies in the behavior of the first-approximation increment to the spectral density of the disordered solid solution. We shall indicate the singularities in the behavior of the specific heat, thermal conductivity, and ultrasound absorption resulting from the appearance of this singular point of the spectrum.

The simplest model describing the spectrum of the elementary excitations in a disordered solid solution corresponds to the equation

$$(H_0 - E)\psi + \sum_i U(\mathbf{r} - \mathbf{r}_i)\psi = 0, \qquad (1)$$

where H_0—Hamiltonian of the quasi-particles in an ideal crystal and $U(\mathbf{r} - \mathbf{r}_i)$—potential of local perturbation due to the impurity atom located at the point $\mathbf{r}_i$. The action of the perturbation $U(\mathbf{r})$ extends over distances on the order of atomic; the points $\mathbf{r}_i$ are random, having an average density determined by the concentration c of the impurity atoms.

As in[1], we confine ourselves here to the case when the unperturbed operator for the ideal crystal has a single spectral branch. The change in the spectral density within the region of the unperturbed spectrum is determined by the averaged influence of all the impurity centers, and in the first approximation in the concentration it is described by the equation[1]:

$$[1 - U_0F_0(E)]\tau(\mathbf{r}) = cU_0\int F(E, \mathbf{r} - \mathbf{r}')W(\mathbf{r} - \mathbf{r}')\tau(\mathbf{r}')d\mathbf{r}',$$
$$\tau(\mathbf{r}) = \psi(\mathbf{r}). \qquad (2)$$

This equation is obtained from (1) by replacing the summation over the random points $\mathbf{r}_i$ by integration over $\mathbf{r}'$, with average distribution density $cW(\mathbf{r} - \mathbf{r}')$ for the impurity atoms at the point $\mathbf{r}'$, if one of the atoms is located at the point $\mathbf{r}$. The function $F(E, \mathbf{r})$ takes the form

$$F(E, \mathbf{r}) = \int \frac{e^{i\mathbf{k}\mathbf{r}}\,d\mathbf{k}}{E - E(\mathbf{k})}, \qquad F_0(E) = \int \frac{\nu_0(E')\,dE'}{E - E'};$$

where $\nu_0(E)$—spectral density of ideal crystal. The validity of the assumptions made is confirmed by the fact that the usual method of expanding the spectral density in powers of the concentration leads to the same results (see below).

Solution of (2) in the first approximation c leads to a shift of all the points of the continuous spectrum of the unperturbed operator:

$$E = E(\mathbf{k}) + c\beta(E), \qquad \beta(E) = U_0 / [1 - U_0F_0(E)]. \quad (3)$$

For the values of the energy inside the band of the unperturbed spectrum $E_0 \leq E \leq E_m$, where E_0—left end point of the spectrum and E_m—right end point, we can represent the function $F_0(E)$ in

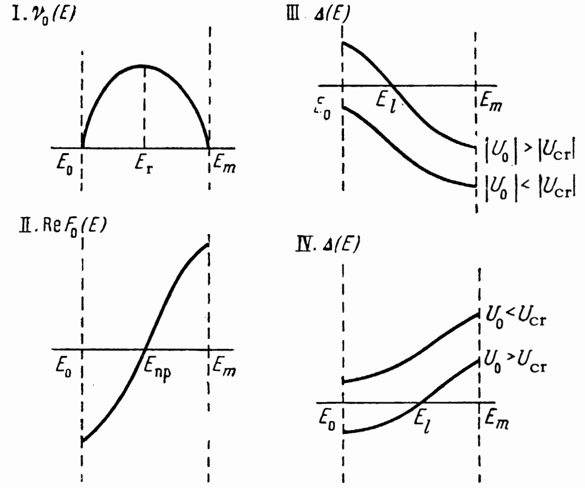

FIG. 1. Energy dependence of the following quantities: I – spectral density of ideal lattice, II – real part of the function $F_0(E)$, III – shifts of the values of the energy for $U_0 < 0$, IV – shifts of the values of the energy for $U_0 > 0$.

the form

$$F_0(E) = \mathrm{P}\int \frac{\nu_0(E')\,dE'}{E-E'} + i\pi\nu_0(E) = K_0(E) + i\pi\nu_0(E).$$

The energy dependence of the real part is determined by the character of the spectral density of the unperturbed crystal. Let us assume that $\nu_0(E)$ is a sufficiently smooth function (Fig. 1, I); then $K_0(E)$ changes with changing energy in the manner shown in Fig. 1, II. At the point $E = E_r$, where $d\nu_0(E)/dE$, we have $K_0(E) = 0$. The function $\beta(E)$ has for these values of the energy likewise an imaginary and a real part:

$$\beta(E) = \operatorname{Re}\beta(E) + i\operatorname{Im}\beta(E) = \Delta(E) + i\Gamma(E);$$

$$\Delta(E) = \operatorname{Re}\beta(E) = U_0[1 - U_0K_0(E)] \,/\, \{[1 - U_0K_0(E)]^2 + [U_0\pi\nu_0(E)]^2\},$$

$$\Gamma(E) = \operatorname{Im}\beta(E) = U_0^2\pi\nu_0(E) \,/\, \{[1 - U_0K_0(E)]^2 + [U_0\pi\nu_0(E)]^2\}. \quad (4)$$

An investigation of the dependence of $\Delta(E)$ on the energy leads to the following results (Fig. 1, III, IV): when the perturbations are smaller than critical

$$|U_0| < |U_{\mathrm{cr}}| = |1/K_0(E_g)|, \qquad E_g = E_0, E_m,$$

$\Delta(E)$ is a sufficiently smooth function, monotonic, and does not reverse sign inside the energy interval $\{E_0, E_m\}$, i.e., the shift of all the values of the energy inside the unperturbed-spectrum band, occurring under the influence of all the impurity centers, has a constant sign: $\Delta(E) > 0$ when $U_0 > 0$ and $\Delta(E) < 0$ when $U_0 < 0$. For perturbations larger than critical, $|U_0| > |U_{\mathrm{cr}}|$, the function $\Delta(E)$ is also smooth and monotonic, but reverses sign inside the energy interval $\{E_0, E_m\}$ at a value $E = E_l$, which satisfies the following equation:

$$1 - U_0K_0(E) = 0. \quad (5)$$

When $E = E_l$, the shift of this value of the energy $\Delta(E)$ vanishes. At this point of the spectrum the energy shift reverses sign (Fig. 2). To the left of this point $\Delta(E) > 0$ and to the right $\Delta(E) < 0$.

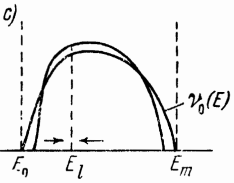

FIG. 2. Shift of the values of the energy when $|U_0| > |U_{\mathrm{cr}}|$.

The number of the states in the vicinity of the energy E_l is considerably larger than the number of the states of the unperturbed spectrum, since states with mixed values of the energy $E_l = E(\mathbf{k}) \pm c\Delta(E)$ are added to the states of the unperturbed spectrum. Thus, if the perturbation is sufficiently large, $|U_0| > |U_{\mathrm{cr}}|$, then the influence of all the impurity centers leads not only to a shift in all values of the energy, but also to the appearance inside the unperturbed-spectrum band of a point which in a certain sense is "singular" near $E = E_l$.

The damping $\Gamma(E)$ of the plane waves inside the band of the continuous spectrum of the unperturbed operator changes with the energy in the manner shown in Fig. 3. On the ends of the band, the damping vanishes, since the spectral density is equal to zero at these points. It is easy to show that when $|U_0| > |U_{\mathrm{cr}}|$ we have $d\Gamma(E)/dE|_{E=E_l} = 0$, i.e., the damping has a maximum when $E = E_l$ and is equal to $\Gamma(E_l) = 1/\pi\nu_0(E_l)$.

When $|U_0| < |U_{\mathrm{cr}}|$, the function $\Gamma(E)$ is smooth, and the maximum value is reached somewhere in the vicinity of E_r, to the right or to the left, depending on the character of the perturbation.

The nature of the states with energies in the vicinity of E_l, in spite of the fact that this energy lies inside the unperturbed spectrum band, differs greatly from plane waves. These states correspond to modulated plane waves, which attenuate

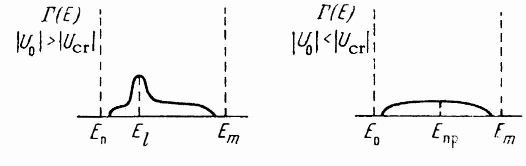

FIG. 3. Dependence of the damping on the energy.

over a certain distance λ. From the relaxation time $\tau = 1/c\Gamma(E_l) = c^{-1}\pi\nu_0(E_l)$ we can estimate the distance over which they attenuate:

$$\lambda = \left(\frac{2E_l}{m}\right)^{1/2}\tau = \left(\frac{2E_l}{m}\right)^{1/2} c^{-1}\pi\nu_0(E_l).$$

Thus, states of this type can be regarded in some sense as "localized"—they attenuate within a finite distance λ from the impurity atom. The value of the energy of the "localized" state E_l is a characteristic of the perturbation potential; the attenuation of modulated plane waves of such states is a characteristic of the unperturbed spectrum of an ideal lattice and does not depend explicitly on the perturbation.

The presence of such a point inside the unperturbed spectrum band of the vibrations of a lattice with heavy impurity atoms was already noted in some papers [2,3]. The nature and the cause of the appearance of such "localized" states were not indicated in these papers. As can be seen from the formulas presented above, the "localized" states always occur when $|U_0| > |U_{cr}|$, both when $U_0 > 0$ and when $U_0 < 0$. The sign and the value of the potential of the perturbation consist of the value of the energy of such a "localized" state. This state appears under the same conditions as the impurity level: to the left of the maximum of the spectral density, if $U_0 < 0$, and to the right if $U_0 > 0$, i.e., in that end of the spectrum from which the impurity band corresponding to these perturbation potentials is split off, and the larger U_0 the closer the state to the center of the band.

Let us now find the spectral density for $E = E_l$. The spectral density inside the unperturbed spectrum band is given by the formula [1]

$$\nu(E, c) = \nu_0(E) + \sum_{n=1}^{\infty} c^n\nu_n(E), \qquad \nu_1(E) = \frac{d}{dE}\mu_1(E), \quad (6)$$

where

$$\mu_1(E) = -\frac{1}{\pi}\tan^{-1}\frac{\pi U_0\nu_0(E)}{1 - U_0K_0(E)}.$$

Let us find the first-approximation increment to the spectral density

$$\nu_1(E) = \frac{1}{\pi}\,\frac{\pi U_0[1 - U_0K_0(E)]\,d\nu_0(E)\,/\,dE + U_0^2\pi\nu_0(E)\,dK_0(E)/\,dE}{[1 - U_0K_0(E)]^2 + [\pi U_0\nu_0(E)]^2}. \quad (7)$$

When $E = E_l$ we have $1 - U_0K_0(E) = 0$, so that the first term in the expansion is equal to

$$\nu_1(E_l) = \left.\frac{dK_0(E)}{dE}\right|_{E=E_l} \Big/ \pi^2\nu_0(E_l).$$

Taking into account only the first approximation in c, we get for the spectral density $\nu(E, c)$

$$\nu(E_l, c) = \nu_0(E_l) + \frac{cf(E_l)}{\pi^2\nu_0(E_l)},$$

$$f(E_l) = \left.\frac{dK_0(E)}{dE}\right|_{E=E_l}. \quad (8)$$

Thus, the spectral density of the "localized" state, like the attenuation of the modulated plane waves of this state, does not depend explicitly on the perturbation; the entire dependence on the perturbation lies in the dependence of the energy of such a "localized" state on the perturbation potential U_0:

$$1 - U_0K_0(E_l) = 0, \qquad E_l = \varphi(U_0).$$

In the general case we can find only the derivative

$$dE_l\,/\,dU_0 = K_0(E_l)\,/\,U_0f(E_l).$$

A simple estimate of such a dependence was obtained by Kagan and Iosilevskiĭ [2] for the resonant frequency of the vibrational spectrum of a crystal with heavy impurity atoms:

$$\omega_l^2 = \omega_m^2\,/\,|\varepsilon|\,\langle\omega^{-2}\rangle, \qquad \varepsilon - 1 - m'/m,$$

where m′—mass of the impurity atom, m—mass of the atoms of the ideal lattice, ω_m—maximum frequency of the spectrum of the ideal lattice, and the averaging of ω^{-2} over the phonon spectrum of the ideal lattice is given in the angle brackets.

Let us investigate now the behavior of the spectral density in the vicinity of E_l. The expression $1 - U_0K_0(E)$ in the vicinity of E_l can be expanded in powers of the deviation from E_l:

$$1 - U_0K_0(E) = -U_0(E - E_l)\left.\frac{dK_0(E)}{dE}\right|_{E=E_l} = -U_0(E - E_l)f(E_l).$$

The first-approximation addition to the spectral density $\nu_1(E)$ has in this energy region the form

$$\nu_1(E) = \frac{-(E - E_l)d\nu_0(E)/dE + \nu_0(E)}{(E - E_l)^2f^2(E_l) + \pi^2\nu_0^2(E)}f(E_l). \quad (9)$$

For energy values satisfying the condition

$$-(E - E_l)d\nu_0(E)\,/\,dE + \nu_0(E) = 0, \quad (10)$$

the addition to the spectral density vanishes, and reverses sign for large values of the energy. A simple estimate of the first-approximation addition to the spectral density $\nu_1(E)$ can be obtained for such perturbations, at which the value of the energy of only the "localized" state lies near one of the ends of the band. Since such "localized" states arise only at sufficiently large perturbations, for almost all sensible perturbations $|U_0| \gtrsim |U_{cr}|$ we always have E_l close to the edge of the band—right or left, depending on the sign of the perturbation potential. In this region of the spectrum we can assume that the following character of the spectral density is valid:

$$\nu_0(E) = \begin{cases} \alpha(E-E_0)^{1/2}; & E \gtrsim E_0 = 0 \\ \alpha(E_m - E)^{1/2}, & E \lesssim E_m \end{cases}.$$

The value of the energy E at which Eq. (10) vanishes, is equal to in the zeroth approximation in E

$$E = E_l + \frac{\nu_0(E)}{d\nu_0(E)/dE} \approx \begin{cases} 3E_l \quad , & U_0 < 0 \\ 3E_l - 2E_m, & U_0 > 0 \end{cases}.$$

The first-approximation addition to the spectral density $\nu_1(E)$ in the vicinity of E_l has in this approximation the form

$$\nu_1(E) = \begin{cases} \dfrac{\alpha}{2E^{1/2}f^2(E_l)} \dfrac{3E_l - E}{\pi^2\alpha^2 f^{-2}(E_l)E_l + (E-E_l)^2}, & U_0 < 0 \\ \dfrac{\alpha}{2E^{1/2}f^2(E_l)} \dfrac{E + 2E_m - 3E_l}{\pi^2\alpha^2 f^{-2}(E_l)E_l + (E-E_l)^2}, & U_0 > 0 \end{cases}.$$

Thus, the addition to the spectral density has in the first approximation in E, in the vicinity of E_l, a δ-function character, reaching a maximum value for energies close to E_l, and when $E = 3E_l$ ($U_0 < 0$) or when $E = 3E_l - 2E_m$ ($U_0 > 0$) it vanishes and then reverses sign. Regardless of the sign of the perturbation potential, the values $\nu_1(E) > 0$ lie in a narrow frequency interval: from 0 to $3E_l$ when $U_0 < 0$ and from $3E_l - 2E_m$ to E_m when $U_0 > 0$. It must be remembered here that the estimate given above for the addition to the spectral density is valid if E_l lies near the start or the end of the band. The closer to the middle of the band is this quasi-local level, the larger the smoothing of the δ-function character of the addition to the spectral density.

The δ-function character of the addition to the spectral density of the vibrational spectrum of the disordered lattice can lead to certain singularities in the behavior of the thermal conductivity, specific heat, and absorption of ultrasound of the disordered lattice. To clarify the character of these singularities, it is necessary to investigate the vibrational spectrum of the frequencies of the disordered lattice. Equation (1) has for elastic displacements u of atoms in a single-atom lattice the following form [1]:

$$[\omega^2(\mathbf{k}) - \omega^2]\mathbf{u} + \sum_i U(\mathbf{r} - \mathbf{r}_i)\mathbf{u} = 0.$$

All the formulas given above are valid also for the vibrational frequency spectrum of a disordered lattice. For the phonon spectrum, the role of the energy E is played by the frequencies ω^2, and the role of the perturbation potential is taken by $U_0 = \omega^2(m' - m)/m$, if the impurity atoms differ only in mass.

Behavior of the specific heat of a crystal with impurities. The influence of heavy impurity atoms on the specific heat of a crystal was investigated by Kagan and Iosilevskiĭ [2]. The presence of heavy impurity atoms leads to a considerable relative change in the specific heat of the crystal.

Since the phonon spectrum of a crystal with impurity atoms has a resonant character near the energies of the "localized" states, both for heavy impurity atoms and for light ones, we should expect a crystal with light impurity atoms to have likewise an anomalous specific heat behavior. The foregoing analysis shows that the relative change in the specific heat of a crystal with light impurity atoms is of the form

$$\frac{\Delta C(T)}{C_0(T)} = c\omega_m^2 \frac{\omega_m - \omega_l}{\omega_l^3}, \qquad \omega_m^2 \frac{\omega_m - \omega_l}{\omega_l^3} \lesssim 1,$$

i.e., in this case the relative change of the specific heat is of the same order of magnitude or smaller than the concentration c of the light impurity atoms.

Effect of impurities on the thermal conductivity. Many experiments [4,5] on the measurement of the thermal conductivity of crystals with impurities show that their thermal conductivity at low temperatures differs appreciably from the thermal conductivity of ideal crystals. The usual Rayleigh mechanism of scattering by impurity atoms is not sufficient to explain the experimental data. The results of the experiment can be understood if in addition to the usual Rayleigh scattering mechanism we assume that there exists scattering (similar to resonant scattering) of phonons by changes in the lattice vibrations, due to the introduction of the impurities. Walker and Pohl [5] made an extensive theoretical investigation of the coefficient of thermal conductivity of crystals with impurities. They reached the conclusion that to explain the experimental data it is necessary to introduce the relaxation time for the process of scattering of the phonons by the changes in the lattice vibrations, which should have the following frequency dependence:

$$\tau^{-1}(\omega) = A\omega^2 / [(\omega_0^2 - \omega^2)^2 + B\omega^2\omega_0^2],$$

where ω_0^2 is some characteristic frequency.

It is easy to show that such a formula for the relaxation time can be derived relatively simply, if account is taken of the influence of the impurity atoms on the vibrational spectrum of the crystal. The averaged influence of the impurities on the vibrational spectrum of the crystal leads to a frequency shift, to attenuation of the modulated plane waves corresponding to these frequencies, and to the appearance of states of an almost "localized" type:

$$\omega^2(\mathbf{k}) = \omega_0^2(\mathbf{k}) + c[\operatorname{Re}\beta(\omega^2) + i\operatorname{Im}\beta(\omega^2)],$$

$$\operatorname{Im}\beta(\omega^2) = c^{-1}\Gamma(\omega^2).$$

The functions $\beta(\omega^2)$ and $\Gamma(\omega^2)$ are determined from the general formulas (4). For the lattice vibrations the attenuation of the modulated plane waves is of the form

$$\Gamma(\omega^2) = c\varepsilon^2\pi\nu_0(\omega^2)\omega^2\left\{\left[1 - \varepsilon\omega^2 \mathrm{P}\int_0^{\omega_m^2}\frac{\nu_0(\omega'^2)\,d\omega'^2}{\omega'^2-\omega^2}\right]^2 + \varepsilon^2\pi^2\nu_0^2(\omega^2)\omega^4\right\}^{-1}.$$

At a frequency $\omega^2 = \omega_l^2$, satisfying the condition

$$1 - \varepsilon\omega^2\mathrm{P}\int_0^{\omega_m^2}\frac{\nu_0(\omega'^2)\,d\omega'^2}{\omega'^2-\omega^2} = 0,$$

a state of almost "localized" type is produced. The attenuation of the modulated plane waves for states of this type is equal to

$$\Gamma(\omega_l^2) = c/\pi\nu_0(\omega_l^2).$$

For oscillations with frequencies lying in the vicinity of ω_l^2, the attenuation of the modulated plane waves can be expanded in powers of the deviation from ω_l^2:

$$\Gamma(\omega^2) = \frac{c\varepsilon^2\pi\nu_0(\omega_l^2)\omega^2}{\varepsilon^2(\omega^2-\omega_l^2)^2 f^2(\omega_l^2) + \varepsilon^2\pi^2\omega^4\nu_0^2(\omega_l^2)}$$

$$= \frac{A\omega^2}{(\omega^2-\omega_l^2)^2 + B\omega^2\omega_l^2}.$$

If we recognize that $\Gamma(\omega^2) = \tau^{-1}(\omega^2)$, this result leads to just the same formula for the relaxation time of the scattering of phonons by changes in the crystal lattice vibrations of a crystal with impurities, as was obtained phenomenologically by Pohl and Walker [4,5].

Scattering of ultrasound by the vibrations of a disordered lattice. The coefficient of absorption of ultrasound is determined by the general formula

$$\gamma = \omega^2\tau/(1+\omega^2\tau^2),$$

where $\tau(\omega)$—relaxation time of the dissipative processes which cause ultrasound absorption. We are interested in the process of scattering of ultrasound by the vibrations of the disordered lattice, namely: scattering of ultrasound by changes in the lattice vibrations, due to introduction of the impurity atoms. In the case of an ideal lattice, the dimensionless ultrasound absorption coefficient is determined by a certain mean free path and reaches a maximum when $\omega\tau = 1$:

$$\gamma/\omega = \omega\tau/(1+\omega^2\tau^2), \quad (\gamma/\omega)_{max} = {}^1/_2, \quad \omega_{max} = 1/\tau.$$

This is the so-called kinematic resonance.

By way of a model of a disordered lattice, we take a single-atom lattice with low concentration c of impurity atoms. The averaged influence of all the impurity atoms on the vibrational spectrum of the crystal leads to a shift of the frequencies, to the appearance of states of almost "localized" type, and to attenuation of the oscillations corresponding to these frequencies. For oscillations with frequencies lying in the vicinity of ω_l^2, the oscillation attenuation coefficient $\Gamma(\omega^2)$ is of the form

$$\Gamma(\omega^2) = \frac{A\omega^2}{(\omega^2-\omega_l^2)^2 + B\omega^4}; \qquad A = \frac{c\pi\nu_0(\omega_l^2)}{f^2(\omega_l^2)},$$

$$B = \frac{\pi^2\nu_0^2(\omega_l^2)}{f^2(\omega_l^2)}, \qquad f(\omega_l^2) = \frac{d}{d\omega^2}\int_0^{\omega_m^2}\frac{\nu_0(\omega'^2)\,d\omega'^2}{\omega'^2-\omega^2}.$$

The oscillation attenuation coefficient $\Gamma(\omega^2)$ of a disordered lattice, and the relaxation time for the scattering of ultrasound by these oscillations, are connected by the usual relation

$$\Gamma(\omega^2) = \tau^{-1}(\omega^2).$$

The coefficient of ultrasound absorption by a lattice containing impurities is determined by the same formula as for the ideal lattice, but the dependence of the relaxation time on the frequency now becomes important. An investigation of the dependence of the absorption coefficient γ/ω on the frequency shows that the latter has a maximum if

$$\frac{d}{d\omega}\frac{\gamma}{\omega} = \frac{(\omega\tau)'[1-\omega^2\tau^2]}{[1+\omega^2\tau^2]^2} = 0,$$

i.e., the extremal points γ/ω are roots of the equations

$$(\omega\tau)' = 0, \qquad (\omega^2-\omega_l^2)C + B\omega^4 = 0,$$

$$1 - \omega^2\tau^2 = 0, \qquad \omega = \tau^{-1}(\omega). \tag{11}$$

Since $\tau^{-1}(\omega^2)$ has a cupola-shaped form for the vibrations of a lattice with impurities (Fig. 3), the first equation of (11) yields two roots, ω_1 and ω_2. The second equation yields for heavy impurity atoms a root ω_3 which is close to ωl. The coefficient of absorption of ultrasound at these frequencies is equal to

$$\frac{\gamma}{\omega}\bigg|_{\omega=\omega_1}=\frac{\gamma}{\omega}\bigg|_{\omega=\omega_2}=\frac{1}{2},$$

$$\frac{\gamma}{\omega}\bigg|_{\omega=\omega_3}=\frac{\omega_l\tau(\omega_l)}{1+\omega_l^2\tau^2(\omega_l)}=\frac{1}{c}\,\frac{\omega_l\pi\nu_0(\omega_l^2)}{1+\omega_l^2c^{-2}\pi^2\nu_0^2(\omega_l^2)}.$$

At low impurity atom concentrations $c \ll 1$ we have

$$\frac{\gamma}{\omega}\bigg|_{\omega=\omega_3}=\frac{c\pi\omega_l\nu_0(\omega_l^2)}{c^2+\omega_l^2\pi^2\nu_0^2(\omega_l^2)}\approx c\ll 1.$$

Thus, the coefficient of absorption of ultrasound by a lattice containing the impurity atoms has a split kinematic resonance, as shown in Fig. 4. At low temperatures, when the mechanism for the scattering of ultrasound by the changes in the lattice vibrations, due to the introduction of the heavy impurity atoms, is the only dissipative process, determining the absorption of ultrasound, this singularity in the behavior of the coefficient of absorption of ultrasound can be appreciable.

In conclusion, the author takes the occasion to thank I. M. Lifshitz for guidance and aid in the work.

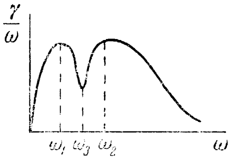

FIG. 4. Dependence of the coefficient of absorption of ultrasound on the frequency in scattering by vibrations of a disordered lattice.

[1] I. M. Lifshitz, UFN 83, 617 (1964), Soviet Phys. Uspekhi 7, 549 (1965).

[2] Yu. Kagan and Ya. Iosilevskiĭ, JETP 45, 819 (1963), Soviet Phys. JETP 18, 562 (1964).

[3] R. Brout and W. Vissner, Phys. Rev. Lett. 9, 54 (1962).

[4] R. O. Pohl, Phys. Rev. Lett. 8, 481 (1962).

[5] C. T. Walker and R. O. Pohl, Phys. Rev. 131, 1433 (1962).

Translated by J. G. Adashko
26