INELASTIC PROCESSES AT HIGH ENERGIES

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Various inelastic processes at high energies are considered in that region of the other kinematic variables where the asymptotic amplitudes are determined by fermion Regge poles. The amplitudes for the corresponding processes are obtained under the assumption that the elastic and inelastic amplitudes have the same type of structure and that they can be described by specific pole graphs corresponding to reggeon exchange. In the single-pole approximation, relations between the asymptotic differential cross sections and a number of isotopic relations are obtained.

1. In the investigation of the spin structure of the amplitudes for various processes at high energies and at angles corresponding to forward and backward scattering on the basis of the Regge pole hypothesis, it was found that the asymptotic amplitude can be described by a specific second order pole graph, if only the contribution of a single pole is taken into account.[1] The internal line of this graph corresponds to the reggeon exchanged between the particles participating in the reaction. The vertices of the graph correspond to certain quantities whose spin structure describes the transition of the pair of particles into the reggeon. The mass of the reggeon is given by $\sqrt{u}$ or $\sqrt{t}$, depending on the scattering angle (u and t are the usual Mandelstam variables). The form factors in the vertices are determined by the residues of the partial amplitudes.

One can say that the scattering at high energies is described by a pole graph corresponding to the exchange of a particle with variable mass $\sqrt{u}\,(\sqrt{t})$, variable spin j (j is the function describing the trajectory of the leading pole) and variable interaction constant, both depending on $\sqrt{u}\,(\sqrt{t})$.

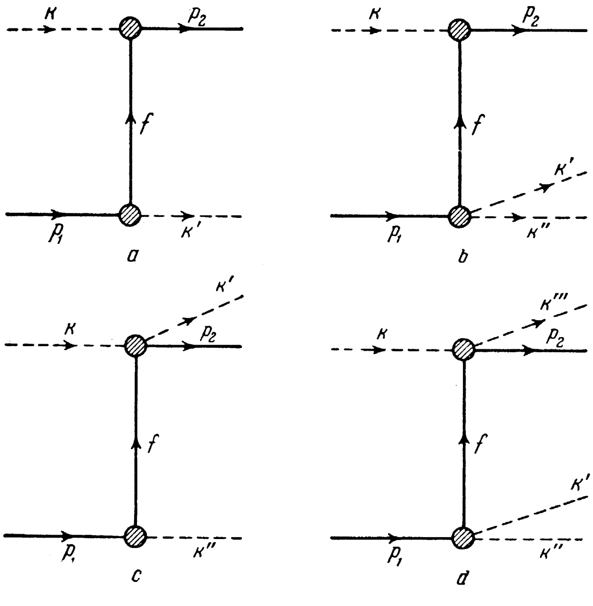
The asymptotic amplitude for processes of the type $\pi + N \to \pi + N$, $\gamma + N \to N + \pi$, $\gamma + N \to \gamma + N$, $\gamma + N \to Y + K$, etc., at high energies and angles $\sim 180°$ has some specific characteristics which have to do with the fact that the partial amplitudes with opposite parity have complex conjugate poles.[2] Owing to this circumstance, the amplitudes of the above-mentioned processes can be written in the form of a sum of contributions from two pole graphs: one graph corresponds to the exchange of a reggeon with mass $\sqrt{u}$ and spin $j(\sqrt{u})$, the other, to the exchange of a reggeon with mass $-\sqrt{u}$ and spin j^*.[1, 3,4]

2. Let us assume that the amplitudes for inelastic processes (i.e., processes which do not conserve the number of particles) in the corresponding asymptotic regions are determined, in the same way as the elastic amplitudes, by pole graphs with the characteristics described above. This can be justified as follows. Let us consider, for example, the process $\pi + N \to N + \pi + \pi$ and let us assume that it goes through two stages: first $\pi + N \to N^* + \pi$, and then the isobar decays, $N^* \to N + \pi$. Thus we have reduced the inelastic process (five-line graph) to a four-particle process. The asymptotic amplitude for this "reduced" process can now be determined in the standard manner by expanding in partial amplitudes, transforming the sum into an integral and separation of the contribution from the leading pole.

However, since some particles of the "reduced" process are unstable, anomalous singularities occur. One might think that these singularities do not alter the character of the asymptotic scattering into large angles, since the fact that the Regge trajectories with opposite parity coincide for $u = 0$ and become complex conjugates for $u < 0$ follows from the absence of a singularity at $u = 0$. In the presence of anomalous thresholds one cannot guarantee the absence of a singularity at $u = 0$ for arbitrary mass ratios of the unstable particles, but one can always move the singularity out from the point $u = 0$ by changing the mass of the unstable particle.

3. Let us consider the processes

$$\pi + N \to \pi + N, \quad \pi + N \to N + (\pi + \pi),$$
$$\pi + N \to (N + \pi) + \pi, \quad \pi + N \to (N + \pi) + (\pi + \pi).$$



The particles enclosed in the parentheses have a finite total energy in their center of mass system (c.m.s.), i.e., these groups of particles are formed as a whole at high incident energies. At high energies and for angles of the final particles $\sim 180°$ in the c.m.s. of the entire reaction, these processes are described by the graphs shown in the figure; the internal line of the graphs corresponds to a Fermi reggeon with the finite "mass" equal to $\sqrt{u}$. At high incident energies this corresponds to the formation of an N or an ($N\pi$) system at angles $\sim 180°$.

The amplitude for the elastic scattering of π mesons on nucleons at high energies and scattering angles $\sim 180°$ in the c.m.s. has the form

$$A_{\pi N} = A^{(+)}_{\pi N} + A^{(-)}_{\pi N};$$
$$A^{(+)}_{\pi N} = \bar{u}(p_2)(i\hat{f} + \sqrt{u})u(p_1)\alpha^2(\sqrt{u})\Delta_j,$$
$$A^{(-)}_{\pi N} = \bar{u}(p_2)(i\hat{f} - \sqrt{u})u(p_1)\alpha^{*2}(\sqrt{u})\Delta_{j^*},$$
$$\Delta_j = [s^{j-1/2} \pm (-s)^{j-1/2}] / \cos \pi j, \qquad (1)$$

where f is the momentum of the reggeon, $(i\hat{f} \pm \sqrt{u})\Delta_j$ corresponds to the propagator of the reggeon, $\alpha(\sqrt{u})$ is a quantity corresponding to the vertex $\pi + N \rightarrow$ "reggeon", $j(\sqrt{u})$ is the trajectory, and the $\pm$ signs correspond to different signatures; $u = f^2$, $s = -(k + p_1)^2 \rightarrow \infty$.

The amplitude corresponding to the graph b in the figure can be written in the form

$$A_{\pi^* N} = A^{(+)}_{\pi^* N} + A^{(-)}_{\pi^* N},$$
$$A^{(+)}_{\pi^* N} = \bar{u}(p_2)\gamma_5(i\hat{f} - \sqrt{u})[a(s', u', \sqrt{u})$$
$$+ ib(s', u' \sqrt{u})\hat{q}]u(p_1)\alpha(\sqrt{u})\Delta_j,$$
$$A^{(-)}_{\pi^* N} = \bar{u}(p_2)\gamma_5(i\hat{f} + \sqrt{u})[a^*(s', u' \sqrt{u})$$
$$+ ib^*(s', u', \sqrt{u})\hat{q}]u(p_1)\alpha^*(\sqrt{u})\Delta_{j^*}, \qquad (2)$$

where $q = k' - k''$, k', k'' are the momenta of the produced π mesons; the quantity $a + ib\hat{q}$ corresponds to the lower vertex of graph b and describes the process of πN scattering, with one of the nucleons replaced by the reggeon; $s' = -(k' + k'')^2$, $u' = -(k' - p_1)^2$. We consider the case where s' has a finite value, i.e., the mesons are emitted under a small relative angle; then u' is also finite.

The process pictured in graph c is described by the amplitude

$$A_{\pi N^*} = A^{(+)}_{\pi N^*} + A^{(-)}_{\pi N^*},$$
$$A^{(+)}_{\pi N^*} = \bar{u}(p_2)[a'(s'', u'', \sqrt{u})$$
$$+ ib'(s'', u'', \sqrt{u})\hat{q}'](i\hat{f} - \sqrt{u})\gamma_5 u(p_1)\alpha(\sqrt{u})\Delta_j,$$
$$A^{(-)}_{\pi N^*} = \bar{u}(p_2)[a'^*(s'', u'', \sqrt{u})$$
$$+ ib'^*(s'', u'', \sqrt{u})\hat{q}'](i\hat{f} + \sqrt{u})\gamma_5 u(p_1)\alpha^*(\sqrt{u})\Delta_{j^*}, \qquad (3)$$

where $q = k + k'$; $a' + ib'\hat{q}$ corresponds to the upper vertex of graph c which describes πN scattering, again with one nucleon replaced by the reggeon; the quantity $s'' = -(k' + p_2)^2$ is assumed to have a finite value, i.e., the angle of emission of the final nucleon and the meson with momentum k' is very small; $u'' = -(k - p_2)^2$.

Finally, the amplitude corresponding to graph d is written in the form

$$A_{\pi^* N^*} = A^{(+)}_{\pi^* N^*} + A^{(-)}_{\pi^* N^*};$$
$$A^{(+)}_{\pi^* N^*} = \bar{u}(p_2)[a'(s'', u'', \sqrt{u}) + ib'(s'', u'', \sqrt{u})\hat{q}'](i\hat{f} - \sqrt{u})$$
$$\times [a(s', u', \sqrt{u}) + ib(s', u', \sqrt{u})\hat{q}]u(p_1)\Delta_j,$$
$$A^{(-)}_{\pi^* N^*} = \bar{u}(p_2)[a'^*(s'', u'', \sqrt{u}) + ib'^*(s'', u'', \sqrt{u})\hat{q}'](i\hat{f} + \sqrt{u})$$
$$\times [a^*(s', u', \sqrt{u}) + ib^*(s', u', \sqrt{u})\hat{q}]u(p_1)\Delta_{j^*}, \qquad (4)$$

where s' and u' take the same values as in (2), while s'' and u'' agree with the corresponding values in (3).

Using (1) to (4), we now compute the differential cross sections for these processes, summed and averaged over the polarizations of the nucleons. Retaining only the terms which are important asymptotically, we obtain a relation between the differential cross sections:

$$(d\sigma/dP)_{\pi N}(d\sigma/dP)_{\pi^* N^*} = (d\sigma/dP)_{\pi^* N}(d\sigma/dP)_{\pi N^*}, \qquad (5)$$

where dP is the density of final states in the corresponding reactions.

In an analogous manner we can obtain a relation between the asymptotic differential cross sections for the processes

$$\gamma + N \to N + \pi, \quad \gamma + N \to N + (\pi + \pi),$$
$$\pi + N \to \pi + N, \quad \pi + N \to N + (\pi + \pi). \qquad (6)$$

Denoting the amplitudes for the two first processes by $(d\sigma/dP)_{\gamma\pi}$ and $(d\sigma/dP)_{\gamma\pi^*}$, respectively, we have

$$(d\sigma/dP)_{\gamma\pi}(d\sigma/dP)_{\pi^*N} = (d\sigma/dP)_{\pi N}(d\sigma/dP)_{\gamma\pi^*}.$$

If we also consider the scattering of photons on nucleons, we have still another relation:

$$d\sigma_{\gamma\gamma}/d\sigma_{\pi N} = 2(d\sigma_{\gamma\pi^*}/d\sigma_{\pi^*N})^2. \qquad (7)$$

4. Let us further consider the isotopic relations for these inelastic reaction mechanisms in terms of the isotopic spin of the leading pole and the final particles. These relations are more general than the Regge pole theory, as they are a consequence of the exchange of a state with definite isospin.[1]

First we find relations between the cross sections for the following processes:

$$\pi^- + p \to (p + \pi^0) + \pi^- \qquad \sigma(p^{-0}),$$
$$\pi^- + p \to (n + \pi^0) + \pi^0 \qquad \sigma(p^{0-}),$$
$$\pi^- + p \to (p + \pi^-) + \pi^0 \qquad \sigma(n^{00}),$$
$$\pi^- + p \to (n + \pi^-) + \pi^+ \qquad \sigma(n^{-+}),$$
$$\pi^- + p \to (n + \pi^+) + \pi^- \qquad \sigma(n^{+-}).$$

The group of particles enclosed in parentheses is emitted in the direction of the momentum of the incident π meson in the c.m.s. of the reaction.

With positive mesons, the following processes are possible:

$$\pi^+ + p \to (p + \pi^0) + \pi^+ \qquad \sigma(p^{0+}),$$
$$\pi^+ + p \to (p + \pi^+) + \pi^0 \qquad \sigma(p^{+0}),$$
$$\pi^+ + p \to (n + \pi^+) + \pi^+ \qquad \sigma(n^{++}).$$

The allowed values of the isotopic spin of the reggeon are $^1/_2$ and $^3/_2$. Let us first consider the case of isospin $^1/_2$. If the pair of particles in parentheses has isospin $^1/_2$, we have the following relations

$$\sigma(p^{-0}) : \sigma(p^{0-}) : \sigma(n^{00}) : \sigma(n^{-+}) : \sigma(n^{+-}) = 2:0:1:0:0,$$
$$\sigma(p^{0+}) : \sigma(p^{+0}) : \sigma(n^{++}) = 1:0:2, \quad \sigma(p^{0+}) = 2\sigma(n^{00}), \qquad (8a)$$

if the pair has spin $^3/_2$, we have

$$\sigma(p^{-0}) : \sigma(p^{0-}) : \sigma(n^{00}) : \sigma(n^{-+}) : \sigma(n^{+-}) = 1:0:2:9:0,$$
$$\sigma(p^{0+}) : \sigma(p^{+0}) : \sigma(n^{++}) = 8:9:4, \quad \sigma(n^{-+}) = 9\sigma(n^{++}). \qquad (8b)$$

If a reggeon with isospin $^3/_2$ is exchanged, the cross sections for the formation of a πN system with isospin $^1/_2$ satisfy the relations

$$\sigma(p^{-0}) : \sigma(p^{0-}) : \sigma(n^{00}) : \sigma(n^{-+}) : \sigma(n^{+-}) = 4:9:2:0:16,$$
$$\sigma(p^{0+}) : \sigma(p^{+0}) : \sigma(n^{++}) = 2:0:1, \qquad (8c)$$

and the cross sections for the formation of a πN system with isosopin $^3/_2$ satisfy

$$\sigma(p^{-0}) : \sigma(p^{0-}) : \sigma(n^{00}) : \sigma(n^{-+}) : \sigma(n^{+-}) = 18:9:8:16:9,$$
$$\sigma(p^{0+}) : \sigma(p^{+0}) : \sigma(n^{++}) = 4:9:2. \qquad (8d)$$

It should be noted that a πN system with isospin $^1/_2$ can not be formed in the reactions $\pi^+ + p \to (p + \pi^+) + \pi^0$, $\pi^- + p \to (n + \pi^-) + \pi^+$, since $(n\pi^-)$ and $(p\pi^+)$ have isospin $^3/_2$; the fact that the reactions $\pi^+ + p \to (p + \pi^0) + \pi^-$, $\pi^- + p \to (n + \pi^+) + \pi^-$ are forbidden independent of the final isospin is a consequence of the exchange of a particle with isospin $^1/_2$.

5. Formation of strange particles in the interaction of π mesons with nucleons occurs in the following reactions:

$$\pi^+ + p \to (\Lambda + K^+) + \pi^+ \qquad \sigma_\Lambda(++),$$
$$\pi^- + p \to (\Lambda + K^0) + \pi^0 \qquad \sigma_\Lambda(00),$$
$$\pi^- + p \to (\Lambda + K^+) + \pi^- \qquad \sigma_\Lambda(+-),$$
$$\pi^+ + p \to (\Sigma^0 + K^+) + \pi^+ \quad \sigma_1,$$
$$\pi^+ + p \to (\Sigma^+ + K^0) + \pi^+ \quad \sigma_2,$$
$$\pi^+ + p \to (\Sigma^+ + K^+) + \pi^0 \quad \sigma_3,$$
$$\pi^- + p \to (\Sigma^0 + K^0) + \pi^0, \quad \sigma_4,$$
$$\pi^- + p \to (\Sigma^0 + K^+) + \pi^- \quad \sigma_5,$$
$$\pi^- + p \to (\Sigma^- + K^+) + \pi^0 \quad \sigma_6,$$
$$\pi^- + p \to (\Sigma^- + K^0) + \pi^+ \quad \sigma_7,$$
$$\pi^- + p \to (\Sigma^+ + K^0) + \pi^- \quad \sigma_8.$$

The particles enclosed in parentheses are emitted in the direction of the incident π mesons; this group of particles may have isospin $^1/_2$ and $^3/_2$, the reggeon has isospin $^1/_2$ or $^3/_2$ and zero strangeness.

In the reactions with formation of a Λ hyperon the group of strange particles is formed with isospin $^1/_2$, and depending on the isospin of the reggeon we have

$$\sigma_\Lambda(++) : \sigma_\Lambda(00) : \sigma_\Lambda(+-) = 2:1:0, \text{ if } T_R = {}^1/_2,$$
$$\sigma_\Lambda(++) : \sigma_\Lambda(00) : \sigma_\Lambda(+-) = 1:2:9, \text{ if } T_R = {}^3/_2. \qquad (9a)$$

Table I

T_R	$T_{K\Sigma}$	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
$1/2$	$1/2$	2	4	0	1	0	2	0	0
	$3/2$	4	2	9	2	0	1	18	0
$3/2$	$1/2$	1	2	0	4	9	4	0	18
	$3/2$	8	4	18	16	18	8	9	9

Table II

T_R	$T_{K\pi}$	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
0	$1/2$	0	2	1	1	2	1	2	0
1	$1/2$	0	2	1	2	4	1	2	0
	$3/2$	9	2	4	2	1	4	2	0

The relations between the cross sections for the formation of Σ hyperons are given in Table I for different isospins of the system of strange particles $T_{\Sigma K}$ and of the reggeon.

If only the hyperon is emitted in the direction of the momentum of the incident π meson, the asymptotic amplitude for these reactions is determined by the contribution from the poles with negative strangeness −1 and isospin 0 and 1. The formation of the Λ hyperons goes through a reggeon with $T_R = 1$, and depending on the isospin of the mesons, we have the relations

$$\sigma_\Lambda(++) : \sigma_\Lambda(00) : \sigma_\Lambda(+-) = 9:2:1, \quad \text{if } T_{K\pi} = 3/2,$$

$$\sigma_\Lambda(++) : \sigma_\Lambda(00) : \sigma_\Lambda(+-) = 0:1:2, \quad \text{if } T_{K\pi} = 1/2. \tag{9b}$$

The relations between the cross sections for the formation of Σ hyperons are given in Table II. It is interesting to note that the reaction $\pi^- + p \to \Sigma^+ + (K^0 + \pi^-)$ takes place only via the exchange of a reggeon with isospin 2.

Finally, we consider the relations for the case when the K meson is emitted in the direction of the momentum of the initial nucleon and the hyperon and the π meson in the direction of the initial π meson. The asymptotic amplitude for these processes is determined by the poles with negative strangeness −1 and isospin 0 or 1. If $T_R = 0$, only charged K mesons are formed, and the relations between the cross sections have in this case the form

$$\sigma_1 = \sigma_3 = \sigma_5 = \sigma_6, \qquad \sigma_\Lambda(++) = \sigma_\Lambda(+-). \tag{9c}$$

If $T_R = 1$, the $(\Sigma\pi)$ system can be formed with isospin $T_{\Sigma\pi} = 0$, 1, and 2, and the following relations hold:

$$\sigma_1 : \sigma_2 : \sigma_3 : \sigma_4 : \sigma_5 : \sigma_6 : \sigma_7 : \sigma_8 = 0:0:0:1:0:0:1:1, \quad \text{if } T_{\Sigma\pi} = 0,$$

$$\sigma_1 : \sigma_2 : \sigma_3 : \sigma_4 : \sigma_5 : \sigma_6 : \sigma_7 : \sigma_8 = 1:0:1:0:1:1:2:2, \quad \text{if } T_{\Sigma\pi} = 1,$$

$$\sigma_1 : \sigma_2 : \sigma_3 : \sigma_4 : \sigma_5 : \sigma_6 : \sigma_7 : \sigma_8 = 9:72:9:4:9:9:2:2, \quad \text{if } T_{\Sigma\pi} = 2. \tag{9d}$$

In conclusion I thank A. I. Akhiezer and D. V. Volkov for a discussion of the results of this work.

Addendum (August 20, 1964). At present it is impossible to verify relations (6) and (7) owing to the absence of the necessary experimental data on the differential cross sections for inelastic processes. As for the isotopic relations a test is possible in some cases. Thus the investigation of the reactions [5]

$$\pi^- + p \to \pi^- + \pi^0 + p, \tag{I}$$

$$\pi^- + p \to \pi^- + \pi^+ + n \tag{II}$$

at incident pion momenta of 2.16 BeV/c revealed an appreciable number of events of nucleon isobar formation in the forward hemisphere with respect to the momentum of the incident pion in the c.m.s. of the reaction. This leads one to conclude that the reactions go via the exchange of a baryon state. The fact that in the above-mentioned processes only negative isobars and no positively charged ones are formed implies, according to (8a) to (8d), that a baryon with isospin ½ is exchanged.

If we take into account that the formation of isobars in the reaction $\pi^- + p \to \pi^+ + \pi^0 + p$ is more intensive than in the reaction $\pi^- + p \to \pi^- + \pi^+ + n$, we can conclude from (8a) and (8b) that the isobar is formed with isospin 3/2. The investigation of the mass spectrum of the (πN) system confirms this result; [6] in these reactions the isobar Δ (1920) is formed.

An analogous situation was also observed by the group in Berkeley, [7] who investigated the reaction (11) with an initial pion energy between 360 and 780 MeV. At these energies, the isobar Δ (1238) should be produced copiously through the exchange of a nucleon with $T = 1/2$ (this follows from the requirement that the pole be as close as possible to the physical region [6]). Therefore one should observe the negative isobar but no positive isobar. This was the case in this experiment.

Note added in proof. (November 30, 1964). After this paper had been submitted to press, the author became acquainted with the work of Ter-Martirosyan and co-workers, [8] in which the asymptotic amplitude of inelastic processes for scalar particles is investigated. It is shown in these papers that the functions a and b in (2) to (4) of the present paper must also depend on the angle between the momenta of the particles entering in the various groups. The account of this dependence does not affect the results obtained in this paper. The author thanks K. A. Ter-Martirosyan for directing his attention to this question.

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