EXCITATION OF MODES AND OSCILLATION KINETICS IN A RUBY LASER WITH A CONCENTRIC RESONATOR

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The excitation of modes and the oscillation kinetics in a ruby laser with concentric external reflectors were studied using an SFR-2M high-speed camera; the radiation spectrum was investigated with a Fabry-Perot interferometer. By studying the distribution of the radiation over the reflectors and in the center of the resonator it was found that in the case of oscillation conditions involving regular damping down to a certain level a large number of high-order transverse modes are excited, whereas only a small number of modes are excited under irregular conditions (depending on the positions of the reflectors, the crystal, and pump energy). The generated frequency bandwidth is 0.5—0.1 cm^{-1} for a regular regime, and 0.1 cm^{-1} for an irregular regime. The mean frequency usually decreases during oscillation. Besides a smooth frequency variation, 0.1—0.4 cm^{-1} jumps are observed. These results and those in the literature for regular and irregular oscillation kinetics in solids can be explained on the basis of energy balance considerations when many modes with different quality factors are generated.

INTRODUCTION

SOLID state lasers usually emit radiation in the form of spikes succeeding each other irregularly with almost no damping. In some instances, however, the oscillation kinetics are regular and the spikes are damped down to a certain level. Such regular or almost regular oscillation kinetics were observed in a laser with confocal reflectors at the ends of the ruby,[1] in a ruby ring laser,[2,3] in optically inhomogeneous neodymium glass,[4] in a traveling wave laser,[5] and in a ruby laser with flat reflectors at low temperatures.[6]

Similar oscillation has been produced in a ruby laser with a concentric resonator[7] consisting of spherical reflectors with coincident centers of curvature. Similar results have also been obtained in[8], where the resonator was almost equivalent to a concentric cavity although the reflector spacing was large (from 5 to 15 m). In both[7] and[8] the experimental results were compared with the theory given in[9]; it was shown that the oscillation kinetics satisfy the simplest equations of field energy balance in an active medium.

The theory expounded in[9] does not consider the spatial inhomogeneity of the field caused by the simultaneous excitation of several modes, nor the intrinsic optical frequency range of resonators. It was shown in[10] that these conditions can lead to unstable excitation of a single mode. The excitation of several modes having different axial orders was studied theoretically in[11], where it was concluded that if the quality factors of the modes are close the oscillation regime remains regular although a different set of modes will be excited in each spike. Thus modes with close quality factors become mutually adjusted (locked) in the different spikes. This conclusion was confirmed experimentally in[12], although the possibility of exciting several modes of different transverse orders was not considered.

The present work was undertaken for the purpose of learning which modes can be excited in a laser with a concentric resonator under different conditions of oscillation. It follows from the theory of resonators[13,14] that for a spherical resonator the diameter D of a mode excitation volume in the center of the resonator and the angle of spread φ of the generated beam are related by

$$\varphi D = (2\lambda / \pi)\varkappa_m^2, \quad (1)$$

where κ_m^2 is the a coefficient which increases with the transverse mode order m. Values of κ_m were calculated in[15] for m = 0—14. The dependence of κ_m^2 on m can be derived approximately from the following considerations. The field pattern in a spherical resonator is described by Chebyshev-Hermite polynomials, which are known to be the wave functions of a harmonic oscillator. Utilizing the properties of these functions for large values of m, when the transition from a quantum-mechanical to a classical oscillator occurs, we obtain an approximate expression for the diameter D of the mode volume:

$$D^2 = (2m+1)L_0\lambda/\pi,$$

where, according to[13], $L_0 = 2D/\varphi$, and therefore

$$\varphi D = (2\lambda/\pi)(2m+1). \quad (1a)$$

Comparing this expression with (1), we obtain

$$\varkappa_m^2 = 2m+1.$$

The extrapolation of the values of κ_m^2 calculated in[15] at 0.03 maximum intensity gives for large m the similar formula

$$\varkappa_m^2 = 2.09m + 5.9. \quad (2)$$

It can be shown that (1a) applies to any spherical resonator, and particularly to the concentric type.

It thus follows from (1a) that the transverse properties of an excited mode can be determined by observing the field pattern at the center of the resonator and the directional distribution of the radiation. The directional distribution can be replaced by the distribution over the resonator reflectors, because for a concentric resonator we have $\varphi = 2D_1/L$, where D_1 is the diameter of the mode region on the reflector and L is the reflector spacing. The mode frequency is determined with a Fabry-Perot interferometer.

EXPERIMENT

The laser used in the present work consisted of a resonator, and a ruby crystal with a pumping arrangement. The resonator consisted of external spherical reflectors with 30-cm and 50-cm radii of curvature and dielectric coatings; the respective spacings were 60 cm and 100 cm. The ruby crystal (with 0.015% Cr concentration) was a polished cylinder 75 mm long and 10 mm in diameter with plane parallel ends. For pumping we used an IFK-1500 helical xenon flash lamp energized by a 900-μF condenser bank; the pump energies were 1.3 kJ (threshold) to 4 kJ.

In order to determine the field patterns on the resonator reflectors at different times and to study the time dependence of the generated frequency we used an SFR-2M high-speed sequence camera. The optical arrangement was similar to that used in[6]. The streak photographs recorded a vertical strip defined by the SFR slit from the image of the reflector or of the resonator center. The time resolution in the photographs was 0.5 μsec. The radiation spectrum was determined with an IT-51-30 Fabry-Perot interferometer having dielectric reflectors and 5-, 10-, and 20-mm plate spacings;

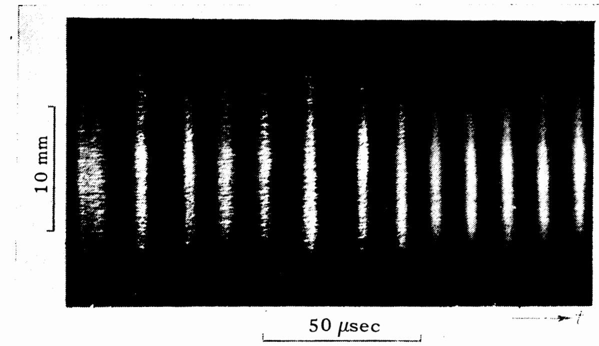

FIG. 1. Distribution of generated radiation on reflector of a concentric resonator, photographed with SFR high-speed camera 600 μsec after onset of oscillation. Pump energy 4 kJ.

SFR streak photographs were also obtained. In this case a matte scattering plate was placed at some distance behind the reflector so that field nonuniformity on the reflectors would not be taken as the structure of the radiation spectrum.

In some experiments we also used the SFR as a single-frame camera; the arrangement has been described in[15].

RESULTS

A. Coherence. In studying the excitation of laser modes it is important to know what modes are excited immediately within the entire volume of the resonator or only in some part of the latter, i.e., to determine whether the radiation is coherent over the entire cross section of the generated beam. It is known that in a laser with flat reflectors the radiation is coherent over the entire cross section of the beam.[16,6]

In the present work we investigated coherence by studying Fraunhofer diffraction with a two-hole diaphragm located directly behind a spherical reflector. The holes were 0.3 mm in diameter and were separated by a distance of 11.5 mm. Very clear interference fringes are observed in the SFR frame photographs of the directional distribution of radiation. This indicates that the radiation was coherent over the entire volume of the concentric resonator, i.e., that in a concentric resonator modes are excited within the entire volume of the resonator simultaneously. Interference fringes are not obtained if one of the holes is covered.

B. Radiation pattern on the reflectors. A streak camera photograph of the radiation pattern on a reflector is shown in Fig. 1. Regions of both irregular and regular oscillation kinetics are seen against a continuous background. In both cases the diameter of the mode region on the reflector is 15—20 mm, corresponding to the divergence angle $\varphi = 1°$—$1.5°$.

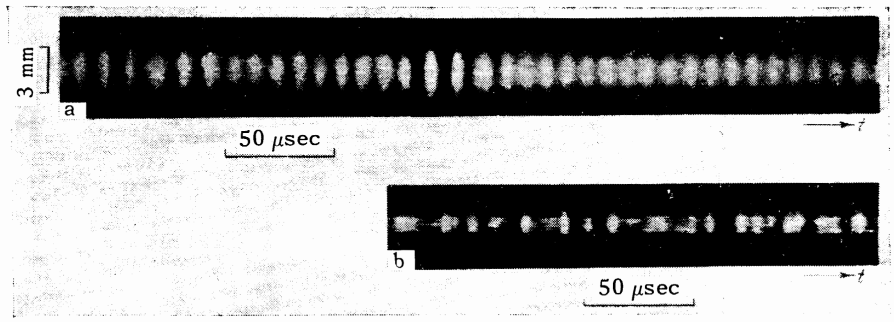

FIG. 2. Distribution of generated radiation at the center of a concentric resonator, photographed with SFR high-speed camera; pump energy 4kJ. a – Crystal at center of resonator, 600 μsec after onset of oscillation; b – crystal shifted 20 cm from center of resonator, 750 μsec after onset of radiation.

The radiation pattern on the reflector (vertically in Fig. 1) depends essentially on whether the oscillation regime is regular or irregular.

In the case of an irregular regime the strip corresponding to each spike is broken up into a large number of separate spots, indicating that one or more modes of high transverse order m are excited. The number of spots on the reflector is about 100 and corresponds to the transverse order m of the mode. In the regular region the distribution is almost uniform within each spike and does not break up into separate spots, although the mode occupies a region of the same size. This indicates either the excitation of purely axial modes without nodes at the reflector, or that so many modes of high transverse orders are excited that the average field distribution is uniform.

Having determined the diameter of the mode region in the center of the resonator, we easily find the transverse order of the mode from (1a). Figure 2 shows the radiation distribution at the center of the resonator in a streak camera photograph when the image of the region was focused on the film in the SFR. Figure 2a shows identical approximately 3-mm diameters for the mode region at the center of the resonator in the cases of both regular and irregular regimes. We thus obtain $m \approx 100$ from (1a), in agreement with the number of spots on the reflector for the irregular regime. Therefore a large number of modes of high transverse order are excited in the regular regime.

In the irregular regime beats result when the excited modes have close frequencies and close transverse characteristics. This effect appears to account for the photographs in which the spots change even within a single spike. In the regular regime many high transverse order modes are excited, the beat frequency becomes large, and the time-averaged distribution is uniform.

All the foregoing data pertain to the case in which the ruby is located at the center of the resonator and the latter is well adjusted. When the crystal is moved from the resonator center toward one of the reflectors the oscillation kinetics become irregular. The diameter of the excited region on the reflector changes very little, but the diameter of the excited region at the center decreases (Fig. 2b); this corresponds to a reduction of the mode order to $m = 20-30$. The regularity of the regime is also impaired when the reflector spacing becomes smaller than twice the reflector radius of curvature.

The pump energy was 4 kJ during the foregoing observations. The regularity is also impaired by reduced pump energy; large irregular regions appear and the regime is irregular near the oscillation threshold.

C. Spectra. The emission spectra were recorded on both frame and smear photographs with the aid of the Fabry-Perot interferometer. The reflector surface was focused on the plane of the film. Distinct Fabry-Perot rings are visible in the frame photographs, indicating that the same frequencies are excited at different places on the reflector.

In the smear photographs the slit defined a vertical strip cut out of the Fabry-Perot rings and several orders of interference were registered (Fig. 3). These photographs indicate that the width of the emission spectrum is less than or equal to $0.1\ cm^{-1}$ in the irregular regions at each instant (Fig. 3a). In the regular regions (Fig. 3b) the spectra are usually wider (up to $0.5\ cm^{-1}$). This indicates that in a regular regime modes are excited over a greater frequency range than in an irregular regime, thus confirming to some extent the conclusion stated in the preceding section, that a larger number of modes are excited in a regular regime.

The width of the spectrum and the mean emitted frequency vary with time. At the onset of oscillation an increase of the frequency is ordinarily observed extending over four to six successive spikes (Figs. 3 and 4), after which the mean frequency de-

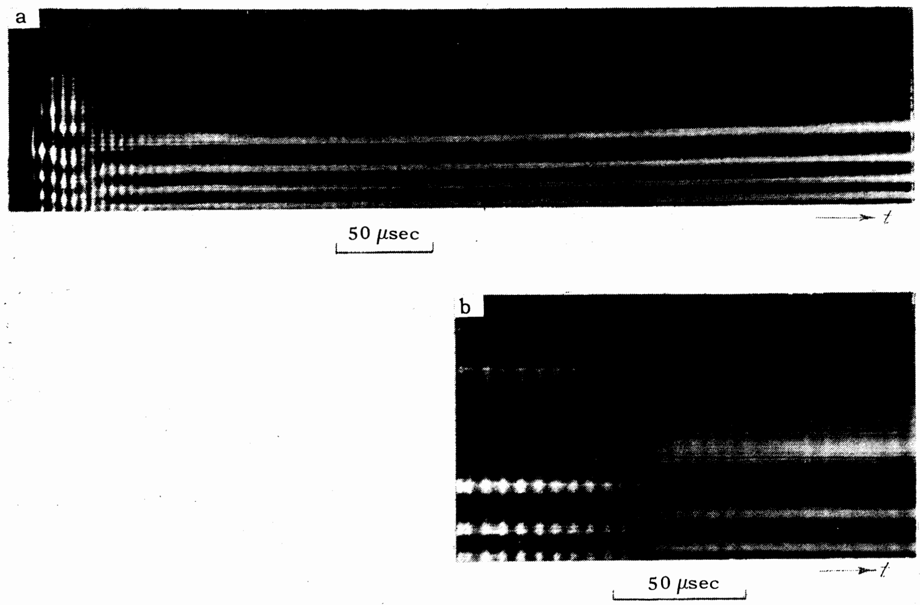

FIG. 3. Emission spectrum in a concentric resonator, photographed with SFR camera. a – example of sharp frequency change 200 μsec after onset of oscillation; b – example of smooth frequency change at onset of oscillation. Plate spacing in Fabry-Perot interferometer – 10 mm; pump energy 4 kJ.

creases. In addition to the smooth frequency variation we observe abrupt changes (as in Fig. 3a), [1)] which are usually decreases; sudden increases occur less frequently. Both regular and irregular excited regions are often observed on a single photograph. The mean frequency does not vary in the irregular regions, and the frequency band is not narrowed. The transition to a regular region usually involves an abrupt frequency change of 0.1–0.4 cm^{-1} (Fig. 3a). Such jumps sometimes occur at transitions from one regular regime to another, but with a different degree of intensity modulation; in this case the period of the spikes remains practically unchanged. At times a smooth, rather than an abrupt, frequency variation is observed at the transition from a less to a more regular regime.

All the foregoing data pertain to spectra obtained using reflectors with a 50-cm radius of curvature. The excited frequency band is about 1.5 times wider in the case of reflectors with a 30-cm radius of curvature.

DISCUSSION

It follows from the foregoing results that the kinetics of oscillation depend strongly on the type and number of modes that are excited. When many modes of high transverse order are excited the oscillation regime is regular and damped; when few modes are excited the regime is less regular. We note that in a laser with a plane parallel resonator only a few modes of low transverse order are excited,[15] and the spikes are irregular.

It follows from energy balance[9] that regular and damped spikes can be accounted for as follows. The degree of population inversion brought about by pumping is reduced through induced emission; during this time the number of photons in the resonator increases. When as the result of pumping the inversion again reaches the threshold, the cycle starts again. It is here assumed that all photons in the resonant cavity have the same lifetime. This is a realistic analysis only if the excited modes have the same Q, in which case the regime is regular. If the quality factors of modes having different spatial field patterns are not identical but very similar, a mode of large Q is excited in the first spike; either no other mode or only a weak one is excited in addition. The degree of

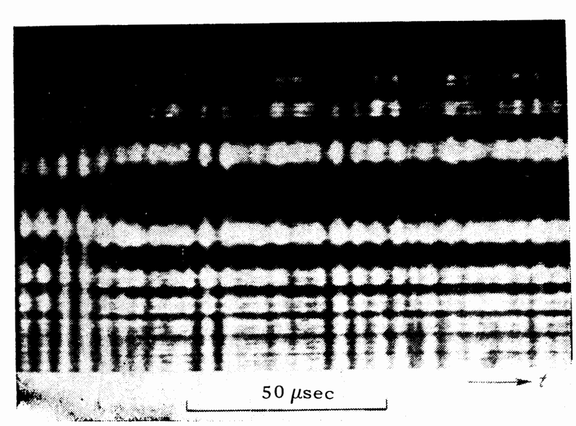

FIG. 4. Emission spectrum at onset of oscillation for an irregular regime. 20 mm spacing of Fabry-Perot interferometer plates; 2 kJ pump energy.

1)This effect was also observed in [17], but the interpretation proposed there does not seem applicable to all cases.

population inversion diminishes wherever the given mode is excited. The field of the mode excited in the next spike most probably does not coincide with that of the mode in the first spike even if its Q is smaller, because the population inversion will be of greater degree. The period between the spikes will be almost the same as in the case of modes having an identical Q. We thus have an overall regular regime, the different modes being adjusted to each other. Obviously, the closer the quality factors of the modes, the better the "substitution" of one mode for another and the better the mutual adjustment of the modes. This qualitative picture has been confirmed by calculations,[11,12] which showed that a regular regime exists when the excited modes have the same transverse orders but different axial orders. It is here essential that the different excited modes occupy, on the average, almost the same volume in the resonator (or more accurately, in the active medium of the resonator). If they occupy different volumes they will not be related by a common population inversion and will be excited almost independently of each other, especially when they do not have the same Q.

A similar case occurs in a concentric resonator, at the center of which only high order modes occupy a large volume [see Eq. (1a)]; therefore the population inversion created in the active material must decay radiatively for these modes. There are many ($\sim m^2$) modes of high transverse order m occupying an identical excited region; therefore the generation regime is regular. The distribution on a reflector therefore becomes uniform, on the average, because a large number of modes are excited simultaneously. The modes of lower order occupy a smaller volume, and even when they are of higher Q their excitation is less probable because when higher order modes are excited the population inversion is also deactivated for lower order modes. When lower-order modes are excited the inversion for higher order modes is almost unaffected.

In our experiments we also sometimes observed oscillation regimes in which nearly axial modes were excited (Fig. 5a), although the excitation of these modes alternated with the excitation of higher order modes. In the later stages of oscillation only the high order modes were excited (Fig. 5b). In photographs of the emission distribution at the center of the resonator (Fig. 5a) the lower order modes occupy a smaller region.

The foregoing discussion shows that in order to produce a regular regime it must be possible to excite a large number of low-Q modes, with higher-Q modes occupying a much smaller volume. The principal condition for an irregular regime is the excitement of modes of different quality factors. The different modes have different excitation thresholds; for higher-Q modes the population inversion required for oscillation is smaller than for the lower-Q modes. Therefore the spike spacing will be governed, on the one hand, by the mode that is excited in the later spike, since its excitation threshold depends on its Q, and on the other hand, by the mode excited in the earlier spike and by the favorable or unfavorable population inversion created for the generation of a mode in the following spike. On the whole, it appears at first glance that the regime will be irregular and more weakly damped. It is obvious that in earlier stages of oscillation when "burning out" of the population inversion between spikes is more complete, this irregularity will be more pronounced, as has been observed experimentally.

However, in the case of high pumping power the difference between the quality factors will have less effect on the regularity of the regime, because the population inversion will be reestablished more rapidly between spikes,[9] the regime will be damped more rapidly and modes with different Q values will be excited simultaneously more easily. This is also observed experimentally; with enhanced pumping a regular regime is established more easily.

Other experiments can be understood in the same way. Thus, the irregular oscillation kinetics and the lower order of the modes occurring when the ruby crystal is shifted from the center of a concentric resonator are accounted for by the fact that the mode volumes in the active medium become of similar size for both high and low orders. A similar explanation can be given for the impaired regularity of the regime which we observed upon bring-

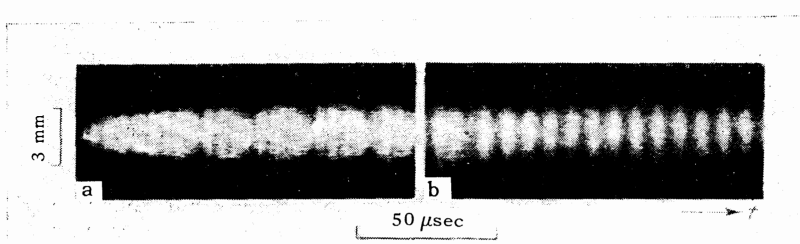

FIG. 5. Radiation distribution at the center of a concentric resonator with reflectors of 300-cm radius of curvature. a – at onset of oscillation; b – 650 μsec after onset. Pump energy 4 kJ.

ing the reflectors closer together, and which was observed in[7] when a diaphragm covered the crystal.

The regular regime in a small confocal ruby[1] is explained by the large excited volume compared with the volume of a pure axial mode ($D_{ax} \sim \sqrt{L_0 \lambda}$) and by the favorable conditions for the excitation of higher order modes. High order modes should also be excited in a ruby ring[2,3] because of the conditions for plane wave interference in a ring.

A similar explanation may possibly be found for the regular regimes in neodymium glass of poor optical quality. Here only modes of high transverse orders are excited, because the wave front is bent by optical inhomogeneities so that many nodes appear in the field. Modes of the lowest orders can also be excited in glass of good optical quality; therefore irregular regimes are more likely to occur in good glass.

The almost regular regime in a ruby laser at low temperatures[6] is accounted for by the higher probability of induced emission, which is equivalent to higher pumping power.

We can therefore conclude that in order to obtain a regular oscillation regime which is damped to a constant level conditions must exist for the excitation of many low-Q modes, while the higher-Q modes occupy a much smaller volume. An essential condition for an irregular regime is the excitation of modes with unequal Q.

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