PION AND BETA-RAY EMISSION BY PROTONS MOVING IN A MAGNETIC FIELD

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The problem considered is that of cyclotron β-ray and pion emission from protons, that is, we deal with the conversion of a proton into a neutron in a magnetic field with the emission of a π meson or a β-ray positron. It is shown that under actual conditions the energy losses of protons owing to these processes are small in comparison with the loss to synchrotron radiation.

THE electromagnetic radiation which arises from the motion of charged particles in a magnetic field, which has been given the name of magnetic-acceleration radiation or synchrotron radiation, is sometimes very important and determines the energy losses both under laboratory conditions and under cosmic conditions. The loss of this type per unit time is given by the well known expression

$$I_e = \frac{2}{3}\frac{e^4H^2}{M^2c^3}\left(\frac{E}{Mc^2}\right)^2. \tag{1}$$

Here e, M, and E are the charge, mass, and total energy of the particle, and H is the component of the magnetic field perpendicular to its velocity. In the derivation of (1) it is assumed that $E/Mc^2 \gg 1$, and also, which is very important, the classical approximation is used; this means that

$$\chi = \frac{\hbar\omega_m}{E} = \frac{\hbar eHE}{M^3c^5} \ll 1; \qquad \omega_m = \frac{eH}{Mc}\left(\frac{E}{Mc^2}\right)^2. \tag{2}$$

The meaning of this inequality is clear if we recall that the frequency ω_m is close to that corresponding to the maximum in the spectrum of the synchrotron radiation (cf. e.g.,[1,2]).

In its motion in a magnetic field a particle with sufficiently high energy can emit not only photons, but also quanta of any field with which it interacts. Moreover, as is already clear from general considerations, the energy dependence shown in (1) is not a universal one, and consequently at large energies nonelectromagnetic magnetic-acceleration losses might in principle exceed the electromagnetic losses. It was this that led us to examine the question of the energy losses associated with the following processes which can occur in the motion of relativistic protons in a magnetic field:

$$p \to p + \pi^0, \tag{3}$$

$$p \to n + \pi^+, \tag{4}$$

$$p \to n + e^+ + \nu. \tag{5}$$

There are no grounds a priori for believing that such processes are not important, for example, in cosmic rays, where the energy of the particles, probably including protons, reaches values $E \sim 10^{20}$ eV. It will be clear from what follows, however, that even at such energies the processes (3)–(5) in cosmic fields $H \sim 10^{-5}$ are still very improbable.[1)] Nevertheless it seemed to us appropriate to give a brief discussion of the relevant calculations, since they are rather interesting from a physical point of view.

1. Let us consider a particle which moves along a prescribed trajectory $\mathbf{r}_e(t)$ and is the source of a neutral scalar field φ. The Lagrangian density is then

$$L = -\frac{1}{8\pi}\left\{\varkappa^2\varphi^2 + (\nabla\varphi)^2 - \frac{1}{c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2\right\}$$

$$- g\delta(\mathbf{r} - \mathbf{r}_e(t))\sqrt{1-\beta^2}\,\varphi. \tag{6}$$

The resulting equation of motion is

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right)\varphi = 4\pi g\sqrt{1-\beta^2}\,\delta(\mathbf{r} - \mathbf{r}_e(t)). \tag{7}$$

Here $\kappa = mc/\hbar$, m is the mass of the neutral scalar meson which is the quantum of the field in question, $\beta = v/c$, v is the speed of the particle, and g is its mesonic charge.

We shall be interested in the case in which the particle (of mass M) also has the electric charge e and is moving in an external magnetic field along a helical path with the frequency $\omega_H = eHc/E$. The

[1)] These results had mainly been obtained as early as 1960, and were mentioned in a book by Syrovat-skiĭ and one of the writers[2] (see Sec. 7), and also earlier in [3]. A detailed quantum-mechanical calculation of the effects (3)–(5), including the influence of spins, is presented in[4].

meson field can then be represented in the form

$$\varphi(\mathbf{r}, t) = \sum \varphi_n \exp(-i\omega_H n t),$$

and here, by (7), we have

$$\left(\Delta - \frac{n^2\omega_H{}^2}{c_n{}^2}\right)\varphi_n = 4\pi g \frac{Mc^2}{E}\frac{\omega_H}{2\pi}\int_0^{2\pi/\omega_H} \delta(\mathbf{r} - \mathbf{r}_e(t))\, e^{in\omega_H t}\, dt, \quad (8)$$

$$c_n{}^2 = \frac{c^2}{1 - n_0{}^2/n^2}, \qquad n_0 = \frac{\varkappa c}{\omega_H} = \frac{mc^2}{\hbar\omega_H}. \quad (9)$$

If we were dealing not with a scalar, but with a vector meson field, the right member of Eq. (8) would be changed in an appropriate way, but the operator $\Delta - n^2\omega^2/c_n^2$ would appear on the left as before. For m = 0 this equation for the vector meson goes over into the equation of electrodynamics.

Thus the existence of a rest mass of the field quanta is taken into account by the factor $(1 - n_0^2/n^2)^{-1}$ in (9). From the physical point of view the effect of this factor on the radiation becomes clear at once if we recall that a precisely similar factor appears in electrodynamics when the charge moves not in vacuum but in an isotropic plasma. In fact, the dielectric constant of plasma is $\epsilon = 1 - \omega_0^2/\omega^2$, and in the wave equation c^2 must be replaced by c^2/ϵ. Consequently, the plasma frequency $\omega_0 = (4\pi e^2 N/m)^{1/2}$ plays the role of the frequency $mc^2/\hbar$ [cf. (9)], where $\omega = n\omega_H$. The main difference between the electromagnetic radiation in the medium and that in vacuum is due to the difference between the factors $(1 - \beta\epsilon^{1/2}\cos\theta)^{-1}$ and $(1 - \beta\cos\theta)^{-1}$ (cf. e.g., [5]). Indeed, for $\epsilon < 1$ this factor $(1 - \beta\epsilon^{1/2}\cos\theta)^{-1}$ does not go to infinity even for $\cos\theta = 1$ and $\beta \to 1$. Therefore as long as $\epsilon < 1$ the ultra-relativistic case (in the sense of the nature of the emission of radiation) is not realized even for $\beta \to 1$. In order not to encumber the exposition we shall confine ourselves to these remarks, and shall give below the results of the calculation without going into the analogy with the case of synchrotron radiation in a plasma.

The field φ in the wave zone is found in just the same way as in electrodynamics.[1] There is no reason to repeat these calculations here. We only use the fact that the amount of energy radiated in unit time through the element of the surface of a sphere of radius r that corresponds to the solid angle $d\Omega$ is given by

$$dI = \frac{1}{4\pi}\frac{\partial\varphi}{\partial t}\frac{\partial\varphi}{\partial r} r^2 d\Omega \quad (10)$$

[this follows from the expression for the energy flux for the field described by the Lagrangian (6)].

We have for the total emitted intensity

$$I = \sum_{n=n_0}^{\infty} g^2(1 - \beta^2)\frac{n^2\omega_H{}^2}{c_n}\int_0^{\pi} J_n{}^2\left(n\frac{v}{c_n}\sin\theta\right)\sin\theta\, d\theta, \quad (11)$$

where J_n is the Bessel function and the restriction to harmonics with $n \geq n_0$ is due to the fact that for $n < n_0$ the components φ_n fall off exponentially with increasing distance, and therefore do not contribute to the radiation in the wave zone (for $n < n_0$ the energy $n\hbar\omega_H < mc^2$, and it is clear that a meson cannot be emitted).

Using the formula

$$\int_0^{\pi} J_n{}^2(a\sin\theta)\sin\theta\, d\theta = \frac{1}{a}\int_0^{2a} J_{2n}(x)\, dx,$$

expressing J_{2n} for large n approximately in terms of the function $K_{1/3}$, and going over from summation over n to an integration (we set $n = n_0\epsilon x$, $\epsilon = E/Mc^2$), we find

$$I = \frac{g^2}{\sqrt{3}\,\pi}\frac{m^2c^3}{\hbar^2}\int_0^{\infty} x\, dx \int_{y(x)}^{\infty} K_{1/3}(\xi)\, d\xi,$$

$$y(x) = \frac{2}{3}\frac{m}{M}\frac{H_0}{H\varepsilon}x\left(1 + \frac{1}{x^2}\right)^{3/2}, \quad H_0 = \frac{M^2c^3}{e\hbar}, \quad \varepsilon = \frac{E}{Mc^2}. \quad (12)$$

The function y(x) is at its minimum value, $y_{min} = 3^{1/2} m H_0/MH\epsilon$, for $x = 2^{1/2}$. If the energy of the proton is such that $y_{min} \gg 1$, then we can set

$$K_{1/3}(\xi) = (\pi/2\xi)^{1/2}e^{-\xi} \quad (\xi \gg 1),$$

and then the integral over ξ in (12) can be obtained at once. The integral over x can then be calculated by expanding the function y(x) which appears in the exponent in a series around its minimum value. The result is

$$I = \frac{g^2}{\sqrt{3}}\frac{mMc^3}{\hbar^2}\frac{H}{H_0}\varepsilon \exp\left\{-\sqrt{3}\frac{m}{M}\frac{H_0}{H\varepsilon}\right\}, \quad (13)$$

where it is assumed that

$$\frac{M}{m}\frac{H\varepsilon}{H_0} = \frac{M}{m}\frac{\hbar\omega_m}{E} \equiv \frac{M}{m}\chi \ll 1. \quad (14)$$

This condition differs from (2) only by the factor M/m, and when it holds the classical theory can surely be used. In agreement with this, the constant $\hbar$ appears in (13) only in the combination $\kappa = mc/\hbar$, which in the classical region is to be regarded as a parameter which characterizes the field in question. We note that Eq. (13) and our other formulas actually involve not e, but |e|, and consequently the result does not depend on the sign of the charge.

To calculate the intensity of the radiation at large energies it is convenient to do an integration by parts in (12); this leads to the expression

$$I=\frac{g^2}{\sqrt{3}\pi}\frac{m^2c^3}{3\hbar^2}\frac{m}{M}\frac{H_0}{\varepsilon H}\int_0^\infty (x^2-2)\left(1+\frac{1}{x^2}\right)^{1/2} K_{1/3}(y(x))\,dx. \quad (15)$$

For large energies the important part of the integral is that from large values of x. Therefore, using the formula

$$\int_0^\infty x^2 K_{1/3}(ax)\,dx=\frac{8\pi}{9\sqrt{3}\,a^3},$$

we get from (15)

$$I=\frac{g^2e^2H^2}{3M^2c^3}\left(\frac{E}{Mc^2}\right)^2, \quad (16)$$

where it has been assumed that

$$\frac{M}{m}\frac{\hbar\omega_m}{E}=\frac{M}{m}\chi\gg 1. \quad (17)$$

The loss (16) differs from (1) by the replacement of the factor $2e^2$ by g^2, and thus for sufficiently large g it will be larger than the electromagnetic loss. Moreover, if the meson field is not a scalar but a vector field, one gets a formula of the type of (16) but containing the factor $(E/Mc^2)^4$. In the electromagnetic case, owing to the fact that the wave field is transverse, there is an additional factor $(mc^2/E)^2$, and it is only for this reason that the energy dependence is given by the factor $(E/Mc^2)^2$ [cf. (1)]. An analogous situation in regard to the effect of transversality of the field is characteristic of magnetic-acceleration gravitational radiation[6]; for this case the intensity (the loss) is given by (1) with e^2 replaced by $13\kappa M^2/4$, where κ is the gravitational constant. For our scalar field the additional factor $(Mc^2/E)^2 = 1-\beta^2$ in the intensity appears from the beginning [see Eqs. (6) and (7)]; it is due to the fact that the quantity that is a scalar is $(1-\beta^2)^{1/2}dt$, not dt.

The meaning of the condition (17) is that the speed of a meson emitted with the energy $\hbar\omega_n$ is larger than the speed of the emitting particle. In fact, according to (17),

$$(1-\beta^2)^{1/2}=Mc^2/E\gg mc^2/\hbar\omega_m=(1-\beta^2_{\text{mes}})^{1/2}.$$

Under such conditions the fact that the rest mass of the meson is $m \neq 0$ is unimportant, and in agreement with this the mass m does not appear in (16). This statement about the high-energy region is valid, however, only under conditions in which the classical calculation can be used. This means that we can neglect recoil and make no use of the inequality $n\hbar\omega_H < E$. In other words, Eq. (16) is valid not simply under the condition (17), but at the same time subject to the condition (2).[2)] For $M/m \gg 1$ the conditions (2) and (17) can be compatible.

2. The formula (16) gives the correct result in order of magnitude right up to values $\chi = H\xi/H_0 \sim 1$, for which

$$I_{max}\sim g^2M^2c^3/\hbar^2. \quad (18)$$

With further increase of the energy (for $\chi > 1$) the energy loss of a proton through emission of scalar mesons, like that in other cases (electromagnetic interaction, pseudoscalar interaction, and so on) begins to increase owing to spin effects (it is assumed that the emitting particle has a spin). Another important point is that the process we are considering is partly electromagnetic. Therefore the fact that the proton has a spin is important even in the emission of scalar mesons.[4]

For $\chi \gg 1$ the magnetic acceleration spin radiation of the proton increases in proportion to $\chi^{2/3}$, i.e., in proportion to $E^{2/3}$ (cf.[4,7,8]). With a view to application to actual conditions, we can confine our discussion to a range of energies which satisfies the condition (2). In fact, the condition $\chi \sim 1$ means that

$$\frac{E_0}{Mc^2}\sim\frac{H_0}{H}\sim\frac{10^{20}\,\text{Oe}}{H}, \quad (19)$$

where the last expression is for the case of protons. In interstellar space $H \lesssim 10^{-5}$ Oe and $E_0 \gtrsim 10^{34}$ eV. The place where the strongest fields can arise is near collapsing stars, where values $H \sim 10^{10}$ Oe are possible (cf.[9]). Even in this case $E_0 \sim 10^{19}$ eV. The energy E_0 is so large that in what follows we shall assume that the more severe condition (14) is also satisfied, and also for charged mesons and for β decay the condition

$$(m/M)^2\chi\ll 1. \quad (20)$$

In the region of (14) a quantum calculation[4] for a Dirac particle interacting with a neutral scalar field leads exactly to (13), as was indeed to be expected. For the π^0 meson [process (3)], which is pseudoscalar, a factor $5m^2/4M^2$ appears in the formula (13), and g must be the constant for which the value $g^2/\hbar c \approx 14$ is accepted (since in[4] the calculation was made in first-order perturbation theory, this value $g^2/\hbar c \approx 14$ can be used only for an order-of-magnitude estimate).

2)We are assuming that the emitting particle is not receiving energy from outside. If, on the other hand, we assume, as can be done under certain conditions in classical physics, that the velocity of the emitting particle is prescribed, the restriction given by (2) does not apply.

For the emission of pseudoscalar π^+ mesons [process (4)] a formula was obtained in[4] which under the condition (20) and with neglect of quantities of the order m/M differs from (13) by a factor $2^{-5/2}$ and replacement of the factor $3^{1/2}(m/M)(H_0/H\epsilon)$ in the exponent[3)] by the factor $3^{1/2}(m/M)^2(H_0/H\epsilon)$. Finally, for β decay [process (5)], under the condition (20) and with the mass $m = m_e = 9 \times 10^{-28}$ g, one gets the expression

$$I_\beta = \frac{G^2M^6c^6}{3\sqrt{3}(2\pi)^3\hbar^7}\nu^2\left(\frac{\chi}{\nu}\right)^{7/2}\exp\left\{-\sqrt{3}\frac{\nu}{\chi}\right\},$$
$$\nu = m_e^2/M^2, \quad \chi \ll \nu, \tag{21}$$

where G is the weak-interaction constant, $GM^2c/\hbar^3 = 1 \times 10^{-5}$.

In the energy ranges (14) or (20) the most important factor is of course the exponential [cf. (13) and (21)]. We can compare the energy loss of a proton by emission of π^0 mesons with the loss by electromagnetic synchrotron radiation. The ratio of (13) and (1) is

$$\frac{I_{\pi^0}}{I_e} = \frac{g^2}{2e^2}xe^{-x}, \quad x = \sqrt{3}\frac{m}{M}\frac{H_0}{H\varepsilon}, \quad \varepsilon = \frac{E}{Mc^2}. \tag{22}$$

Taking

$$\frac{g^2}{\hbar c} \sim 14, \quad \frac{e^2}{\hbar c} \sim \frac{1}{137}, \quad H_0 = \frac{M^2c^3}{e\hbar} \sim 10^{20}\,\text{Oe},$$
$$H \sim 10^{-5}\,\text{Oe}, \quad \frac{m}{M} \sim \frac{1}{6},$$

we find that $I_{\pi^0}/I_e \sim 1$ for $E \sim 10^{33}$ eV. Meanwhile, no particles with energy larger than 10^{20} eV have been observed in cosmic rays, and particles with much larger energies are undoubtedly extremely rare. On the other hand, for energy $E \sim 10^{20}$ eV and the indicated values of the parameters the ratio (22) gives the negligibly small value $I_{\pi^0}/I_e \sim \exp(-10^{14})$. For π^+ mesons the additional factor m/M in the exponent makes no qualitative difference in the general situation, changing the energy scale by about a factor six.

Finally, with the weak-interaction constant $GM^2c/\hbar^3 \sim 10^{-5}$ and with $m_e^2/M^2 \sim 3 \times 10^{-7}$ and $H \sim 10^{-5}$ Oe we get from (1) and (21) as the ratio I_β/I_e at energy $E \sim 10^{28}$ eV (which corresponds to $\chi m_e^2/M^2 \sim 1$) the value $I_\beta/I_e \sim 10^{-10}$. For smaller energies the ratio is very much smaller.

Accordingly, under actual conditions the energy losses of a proton owing to the reactions (3)—(5) are negligibly small.

3)The emission of charged particles differs from that of neutral particles owing to the fact that in the former case the charged particle in the final state has mass m, and in the latter case this mass is M. Since the magnetic field has more effect on the lighter particle, it is quite natural that there is an additional small factor m/M in the exponent, which facilitates the emission of the charged meson.

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